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Outage Probability for Multi-Cell Processing under Rayleigh Fading

Virgile Garcia¹, Nikolai Lebedev^{1,2}, Jean-Marie Gorce¹

¹University of Lyon, INRIA

INSA-Lyon, CITI, F-69621, France

²CPE Lyon, BP 2077, F-69616, France

Abstract—Multi-cell processing, also called Coordinated Multiple Point (CoMP), is a very promising distributed multi-antennas technique that uses neighbour cell’s antennas. This is expected to be part of next generation cellular networks standards such as LTE-A. Small cell networks in dense urban environment are mainly limited by interferences and CoMP can strongly take advantage of this fact to improve cell-edge users’ throughput. This paper provides an analytical derivation of the capacity outage probability for CoMP experiencing fast Rayleigh fading. Only the average received power (slow varying fading) has to be known, and perfect Channel State Information (CSI) is not required. An optimisation of the successfully received data-rate is then derived with respect to the number of cooperating stations and the outage probability, illustrated by numerical examples.

Keywords: Multicell processing, Capacity distribution, macro-diversity, Rayleigh fading.

I. INTRODUCTION

Today, it is commonly recognized that the solution comes with smaller cells and dense spatial frequency reuse, possibly with the whole spectrum resource made available within each cell (reuse 1)[REF]. However, this increasing number of base-station (BS), with shorter coverage range critically increases the inter-cell interference level, especially at cell boundaries.

The deployment of those small BS in urban area imposes some new difficulty for providers to plan their network, since they hardly control the location of the transmitters, their orientation or the propagation model. Self-Optimized and Self-Organised Networks (SON) [1] have been motivated by this context and by the necessity of an automated network planning and optimisation procedures with minimal human involvement.

CoMP was introduced in 4G systems, not in order to provide a huge increase of the total throughput [2], but rather to improve the SINR of cell edge users. A mobile located close to cell border will be able to decode a joint transmission from two or more BS in the same frequency band. That way, a better coverage is ensured with less users experiencing outage, and thus, a more intense reuse of the spectrum in dense areas is allowed. Several cooperation techniques (including CoMP) have been studied in [3], [4], [5], [6].

The main CoMP’s drawback is the need of coordination between cells: huge amount of data representing users channels and algorithms related parameters as well as synchronisation data, making it impractical due to backhaul limitations. For evolved MIMO-like coordination, a perfect time synchronisation is required to distributively generate beams or efficient

precoding [5]. Considering these problems, and looking for practical solutions, we focus on a simple Multi-Cell Processing that solely requires statistical users SINR, and can be used with a less precise synchronisation than any beamforming-based MIMO.

This paper proposes an analytic expression of the capacity outage probability in Rayleigh fading condition for CoMP. Average received powers are required as input of this theoretical expression. Such a derivation was made in [7] for Single-Input Single-Output(SISO) links. Here, we use the generalized chi-square random variable (see a derivation for communication in [8]) to derive this outage probability for distributed Multiple-Input Multiple-Output(MISO) links. We then use this expression to optimise the number of cooperating stations and the transmission rate to improve the “goodput” (i.e. the successfully received throughput). A numerical example in small cells conditions is then provided.

II. SIGNAL MODEL

At a given mobile, on a given channel, the received (complex) signal is:

$$y = \sum_{n=1}^M (h_n \sqrt{p_n} g_n x_n) + z, \quad (1)$$

where n refers to transmitting antennas, considering M total sites in the system; h_n are independent circular symmetric complex Gaussian Random Variables(RV), representing fast-fading; x_n are the transmitted symbols; p_n the transmitted power; g_n the path gain, including the path-loss from antenna n to user i and a log-normal shadowing (slowly varying with respect to fading); z is the thermal noise, normal variable of standard deviation σ_z . Users are by default attached to the BS with the strongest signal averaged over fading $\operatorname{argmax}_n(P_n)$, where $P_n = p_n g_n$.

We note \mathcal{N} the set of $N = \operatorname{card}(\mathcal{N})$ cells that serves this user in a coordinated manner, to create a distributed MISO link. \mathcal{N}_N refers to the set of N cooperating BSs. We can then write the received signal separating useful signal and interference as:

$$y = \sum_{n \in \mathcal{N}} (h_n \sqrt{P_n} x_n) + \sum_{k \notin \mathcal{N}} (h_k \sqrt{P_k} x_k) + z \quad (2)$$

We assume that BSs are aware of a partial CSI, consisting of the average received power fed back by the user, but not of

the instantaneous channel, since it would require a tremendous feedback and instant sharing of information between all cells. This implies that evolved CoMP techniques such as Zero-Forcing, Beam-Forming or Dirty Paper Coding [3], [5] can not be used. Perfect CSI knowledge and coherent detection is assumed at receiver side. The received SINR is then:

$$\gamma(\mathcal{N}) = \frac{\sum_{n \in \mathcal{N}} |h_n|^2 P_n}{\sum_{k \notin \mathcal{N}} |h_k|^2 P_k + \sigma_z^2} \quad (3)$$

One of the challenges in CoMP is to determine the best number of cooperating stations: the improvement of SINR is balanced by the increased cost of resource due to cooperation. This number depends, for each user, on the topology of the BSs surrounding it (values of $P_n, \forall n$).

III. OUTAGE IN RAYLEIGH FADING

A. Throughput outage probability

$\forall n, H_n = |h_n|^2 P_n$ is a chi-squared RV of order 2 with 0 mean and a variance P_n . Received SINR (3) is then a non-trivial random variable, composed by the fraction of sums of independent, non identically distributed chi-squared random variables.

In [7], authors derived the outage probability $\mathbb{P}(SIR \leq \gamma_{th})$ for a SISO link, also experiencing Rayleigh fading:

$$\mathbb{P}(SIR \leq \gamma_{th}) = \mathbb{P}\left(\frac{H_1}{\sum_{k=2}^N H_k} \leq \gamma_{th}\right) = 1 - \prod_{k=2}^N \frac{1}{1 + \gamma_{th} \frac{P_k}{P_1}} \quad (4)$$

Thermal noise has been neglected, looking at interference limited conditions. It can however be added without difficulty.

The pdf of the sum of generalized chi-square random variables [8] is:

$$\mathbb{P}\left(\sum_{n \in \mathcal{N}} H_n = x\right) = \sum_{n \in \mathcal{N}} \frac{e^{-x/P_n}}{P_n \prod_{j \in \mathcal{N}, j \neq n} (1 - \frac{P_j}{P_n})} \quad (5)$$

Note that mathematically, each received power must be different from another. We can consider without loss of generality that this condition is satisfied, since in our case, transmitting antennas are not co-located. The case where some received powers are equal has been derived in [9]: those RV have to be aggregated into higher order chi-square RV.

Let's derive the complementary cumulative density function:

$$\begin{aligned} \mathbb{P}\left(\sum_{n \in \mathcal{N}} H_n > x\right) &= \int_{u=x}^{\infty} \sum_{n \in \mathcal{N}} \frac{e^{-u/P_n}}{P_n \prod_{j \in \mathcal{N}, j \neq n} (1 - \frac{P_j}{P_n})} du \\ &= \sum_{n \in \mathcal{N}} \frac{e^{-x/P_n}}{\prod_{j \in \mathcal{N}, j \neq n} (1 - \frac{P_j}{P_n})} \end{aligned} \quad (6)$$

Using (3) as SINR and the same reasoning as [7]'s derivation for SISO, we derive the success probability to reach the threshold rate $R_{th} = \log_2(1 + \gamma_{th})$ for a distributed MISO

link, every H_n being independent:

$$\begin{aligned} \mathbb{P}(\log_2(1 + \gamma(\mathcal{N})) > R_{th}) &= \mathbb{P}(\gamma(\mathcal{N}) > \gamma_{th}) = \\ &= \mathbb{P}\left(\sum_{n \in \mathcal{N}} H_n > \gamma_{th} \left(\sum_{k \notin \mathcal{N}} H_k + \sigma_z^2\right)\right) = \\ &= \int_0^{\infty} \dots \int_0^{\infty} \mathbb{P}\left(\sum_{n \in \mathcal{N}} H_n > \gamma_{th} \left(\sum_{k \notin \mathcal{N}} t_k + \sigma_z^2\right)\right) \prod_{k \notin \mathcal{N}} \frac{e^{-t_k/P_k}}{P_k} dt_k \end{aligned} \quad (7)$$

Using (6),

$$\begin{aligned} \mathbb{P}(\gamma(\mathcal{N}) > \gamma_{th}) &= \\ &= \int_0^{\infty} \dots \int_0^{\infty} \sum_{n \in \mathcal{N}} \frac{e^{-\gamma_{th} (\sum_{k \notin \mathcal{N}} t_k + \sigma_z^2)/P_n}}{\prod_{j \in \mathcal{N}, j \neq n} (1 - \frac{P_j}{P_n})} \prod_{k \notin \mathcal{N}} \frac{e^{-t_k/P_k}}{P_k} dt_k = \\ &= \sum_{n \in \mathcal{N}} \frac{e^{-\gamma_{th} \sigma_z^2/P_n}}{\prod_{j \in \mathcal{N}, j \neq n} (1 - \frac{P_j}{P_n})} \prod_{k \notin \mathcal{N}} \int_{t_k=0}^{\infty} \frac{e^{-t_k(\gamma_{th}/P_n + 1/P_k)}}{P_k} dt_k = \\ &= \sum_{n \in \mathcal{N}} \frac{e^{-\gamma_{th} \sigma_z^2/P_n}}{\prod_{j \in \mathcal{N}, j \neq n} (1 - \frac{P_j}{P_n})} \prod_{k \notin \mathcal{N}} \frac{1}{(1 + \gamma_{th} \frac{P_k}{P_n})} \end{aligned} \quad (8)$$

The outage probability of a given user with a set \mathcal{N} of cooperating stations and requiring a SINR higher than γ_{th} can then be derived as:

$$\begin{aligned} P^{out}(\mathcal{N}, \gamma_{th}) &= \\ &= 1 - \sum_{n \in \mathcal{N}} \left(e^{-\frac{\gamma_{th} \sigma_z^2}{P_n}} \prod_{j \in \mathcal{N}, j \neq n} \frac{P_n}{P_n - P_j} \prod_{k \notin \mathcal{N}} \frac{P_n}{P_k \gamma_{th} + P_n} \right) \end{aligned} \quad (9)$$

The decomposition into rational fractions gives:

$$P^{out}(\mathcal{N}, \gamma_{th}) = 1 - \sum_{n \in \mathcal{N}} \left(e^{-\frac{\gamma_{th} \sigma_z^2}{P_n}} \alpha_n P_n \sum_{k \notin \mathcal{N}} \frac{\beta_k}{P_k \gamma_{th} + P_n} \right), \quad (10)$$

with $\alpha_n = \prod_{j \in \mathcal{N}, j \neq n} \frac{P_n}{P_n - P_j}$ and $\beta_k = \prod_{z \in \mathcal{N}, z \neq k} \frac{P_k}{P_k - P_z}$. The probability density function (PDF) is then:

$$\begin{aligned} f(\mathcal{N}, \gamma_{th}) &= \sum_{n \in \mathcal{N}} \left(\sigma_z^2 e^{-\frac{\gamma_{th} \sigma_z^2}{P_n}} \alpha_n \sum_{k \notin \mathcal{N}} \frac{\beta_k}{P_k \gamma_{th} + P_n} \right. \\ &\quad \left. + e^{-\frac{\gamma_{th} \sigma_z^2}{P_n}} \alpha_n P_n \sum_{k \notin \mathcal{N}} \frac{\beta_k P_k}{(P_k \gamma_{th} + P_n)^2} \right) \end{aligned} \quad (11)$$

B. Successfully received throughput

We use the *goodput* as capacity metric G (bit/s/Hz). The goodput is the successfully received capacity, taking into account the error probability of the transmission rate. For a given user with a set \mathcal{N} , the goodput is obtained by optimising the transmission rate R so that the goodput is maximum.

$$G(\mathcal{N}_i) = \max_R R(1 - P^{out}(\mathcal{N}, R)) \quad (12)$$

To fairly compare performance of mobiles attached to one or several BSs, we need to take into account the cost of resource it induces: Consider a cellular network in which the mobiles can cooperate with two BS. If we decide to attach the mobile to two BSs instead of one, this mobile will consumes twice the resource it receives (one for each cooperating station).

Thus, interested in spectral usage, we need to look at a per-BS spectral efficiency.

User's spectral efficiency C (bit/s/Hz/BS) is:

$$C(\mathcal{N}) = \frac{G(\mathcal{N})}{\text{card}(\mathcal{N})} \quad (13)$$

To optimise user's spectral efficiency, we then just have to select the best set of antennas \mathcal{N}^* that maximises (13), such that $C^{opt} = C(\mathcal{N}^*)$.

IV. NUMERICAL RESULTS

To illustrate, we use a numerical implementation to simulate received powers at mobile. We use a 2D area where BS are uniformly distributed with a density of 100 BS/km² (which is similar to a hexagonal scheme with 124m inter-cell distance). A path loss model is that of [10] $g_{dB}(d) = 34.53 + 38 \log_{10}(d) + \delta$, where δ represents the log-normal shadowing.

Fig.1 illustrates the CDF of $G(\mathcal{N}_N)$ for $N = \{1, 2, 3, 4\}$, over a large number of users. We can obviously see that the throughput increases with the number of cooperative stations (through macro-diversity, increase of useful signal and reduction of interference level), and so the effective risk of capacity (and so service) outage is strongly decreased. But this does not take into account the cost in resource required to perform this cooperation.

Looking at spectral efficiency, we are now interested in finding the conditions in which users benefit of CoMP. Considering users i and j with a given spectral efficiency $C_i(\mathcal{N}_{i,1}) = C_j(\mathcal{N}_{j,1})$ we can not say that their spectral efficiency with N antennas will be equal as well. This depends on the received powers from neighbour cells.

We sorted users according to their spectral efficiency without cooperation $C(\mathcal{N}_1)$ and looked at the gain they obtain with $N = \{2, 3, 4\}$ and \mathcal{N}^* . Fig. 2 shows the relative gain $C(\mathcal{N}_N)/C(\mathcal{N}_1)$ of the 5th, 50th, 75th and 95th percentile of users that have $C(\mathcal{N}_1) = C_1$. For example, we can see that using $N = 2$ BSs, 95% of the users with $C(\mathcal{N}_1) = 1$ will have a relative gain lower than 1.6. The relative gain using N BSs tends to reach $1/N$ for good SINR users, since we use N distributed resource without significant gain in goodput.

As expected, the major gains appear when initial conditions $C(\mathcal{N}_1)$ is low, such as cell-edge users. Spectral efficiency can even be improved when using more than two cooperating stations for very low SINR since this condition is certainly due to more than only one very close interfering station. By optimising the number of cooperating stations for each user, we ensure the optimal spectral usage. Looking at Fig.1 and Fig.2, CoMP gains benefits to a large proportion of users.

V. CONCLUSION

This paper provides a theoretical formulation of capacity distribution for simple CoMP under Rayleigh fading. With only statistical CSIT, we used this expression to determine the optimal set of BS association for CoMP and the optimal transmission rate. We shown the gain of using CoMP in a small cell environment using a fully distributed algorithm, which is one of the objective of SONs.

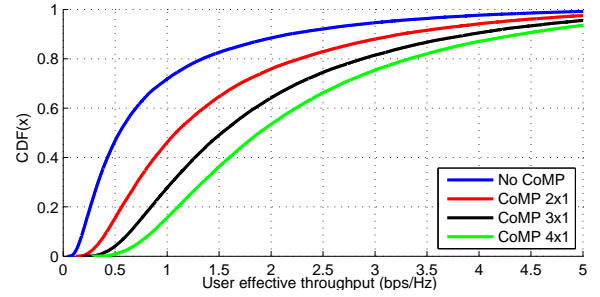


Fig. 1: CDF of users throughput $G(\mathcal{N}_N)$, for $N = \{1, 2, 3, 4\}$.

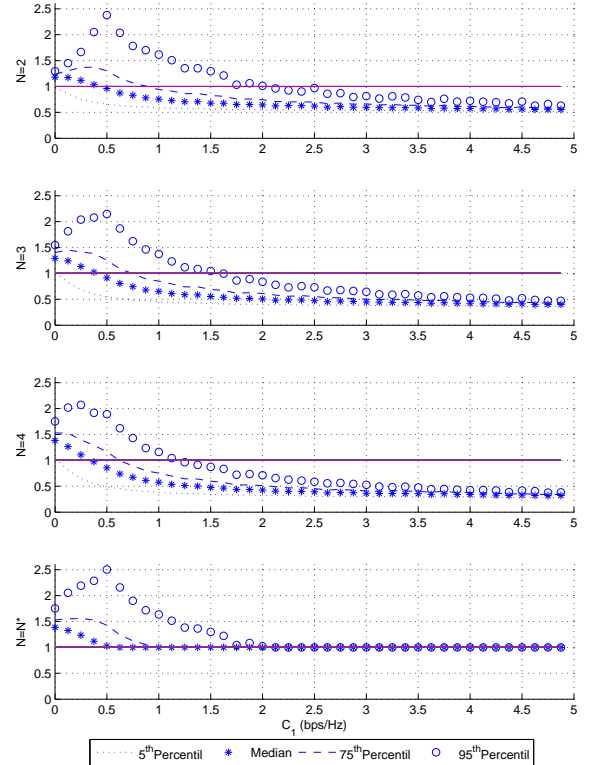


Fig. 2: Repartition of the gain for users using CoMP. Users are grouped by their value of their $C(\mathcal{N}_1)$ on the X axis. Graphs, from top to bottom, correspond to $N = \{2, 3, 4\}$ and \mathcal{N}^* .

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