



**HAL**  
open science

## Mesh Parameterization for an Open Connected Surface without Partition

Yu-Shen Liu, Pi-Qiang Yu, Ming-Cui Du, Jun-Hai Yong, Hui Zhang,  
Jean-Claude Paul

► **To cite this version:**

Yu-Shen Liu, Pi-Qiang Yu, Ming-Cui Du, Jun-Hai Yong, Hui Zhang, et al.. Mesh Parameterization for an Open Connected Surface without Partition. Proceedings of the Ninth International Conference on Computer Aided Design and Computer Graphics(CAD/CG 2005), Dec 2005, HongKong, China. 10.1109/CAD-CG.2005.55 . inria-00518334

**HAL Id: inria-00518334**

**<https://inria.hal.science/inria-00518334>**

Submitted on 17 Sep 2010

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Mesh Parameterization for an Open Connected Surface without Partition

Yu-Shen Liu

Tsinghua University, Beijing, P. R. China  
liuyushen00@gmail.com

Pi-Qiang Yu

Beijing Jiaotong University, P. R. China  
yupiqiang@tsinghua.org.cn

Ming-Cui Du

Tsinghua University, Beijing, P. R. China  
dmc02@mails.tsinghua.edu.cn

Jun-Hai Yong

Tsinghua University, Beijing, P. R. China  
yongjh@mail.tsinghua.edu.cn

Hui Zhang

Tsinghua University, Beijing, P. R. China  
huizhang@mail.tsinghua.edu.cn

Jean-Claude Paul

Tsinghua University, Beijing, P. R. China  
paul@tsinghua.edu.cn

## Abstract

*A novel mesh parametrization method for an open connected surface is presented. The parametrization method is based on Hessian-based Locally Linear Embedding (HLLÉ). Our method operates directly on the surface without using any partition technique and can preserve the local and global structure, while partition-based methods often produce high distortion and discontinuity nearby partition boundaries. In addition, some examples about texture mapping show the efficiency of our method*

## 1. Introduction

Surface parametrization is a fundamental problem in computer graphics. It is essential for some operations such as texture mapping, mesh resampling, and mesh compression. A parameterization defines a mapping between regions on the 2D plane and the surface embedded in the 3D space [6]. This operation is called surface parameterization or surface flattening and this planar representation is called embedding. There have been many surface parameterization methods proposed (e.g. Refs [2, 5, 4, 6, 8, 11]). A review of surface parameterization methods is beyond the scope of this paper. The reader may consult the recent survey [5] for detailed expositions. In general, the topology of 3D surfaces is always the same as a disk [2, 4]. For complex surfaces, a popular technique is to partition those surfaces into a set of charts corresponding to the simply shapes (e.g. Refs [8, 11]). In especial, if an open connected surface has more than one boundary (i.e. multiple holes), a general solving

method is to partition the surface into a set of charts. Texture mapping, which is widely used to enhance the reality of 3D models in computer graphics, is one of important applications on surface parameterization.

However, there are two limitations on texture mapping on complex open connected surfaces based on the partition techniques.

- It is a non-trivial task to build a reasonable partition for texture mapping, and it is highly complicated when dealing with large and complex surfaces.
- Partition often produces high distortion and discontinuity nearby partition boundaries (see Fig. 2(c)).

One possible solution is to fill holes (e.g. [7, 9]) and then use the common parameterization techniques to compute the mapping of surfaces. The method requires the expenditure of large amounts of time and space for large and complex surfaces. Furthermore, it is also an additional computation for parameterizing and changing the topology of original surfaces, so it is not a good choice for surface parameterization.

To solve those problems, we apply a form of Hessian-based Locally Linear Embedding (HLLÉ) [3]. HLLÉ is a dimensionality reduction technique that finds a low-dimensional embedding of a set of points lying on an open connected manifold embedded in high-dimensional input space. There have been some studies that extend nonlinear dimensionality reduction methods [10, 12] to surface parameterization in the last few years. Zigelman et al. [14] combine multidimensional scaling (MDS) with the fast marching method that computes a geodesic distance from a given surface point. In essence, their method is an ex-

tension of Isomap [12] to surface parameterization. Zhou et al. give a similar application using the Isomap method [13]. However, those dimensionality reduction techniques require surfaces to be convexity [3], so they are not suitable for an open connected surface with multiple holes. Our method may be regarded as an extension of the HLLC method. Moreover, most of the previous parameterization methods require fixed boundary conditions (e.g. [4]). In the proposed method, boundary conditions are not required for a valid solution, so our method is also a free-boundary parameterization method [2]. In computer graphics and geometric modeling, shapes are commonly represented by triangular meshes, and surfaces referred to in this paper are triangular meshes.

The remainder of this paper is organized as follows. In Section 2, we briefly review dimensionality reduction techniques. In Section 3, the parameterization based on HLLC is described. In Section 4, the application on texture mapping for an open connected surface is presented. In Section 5, some experimental results are introduced, and conclusions are given in Section 6.

## 2. Dimensionality reduction

Dimensionality reduction, whose goal is to find a low-dimensional embedding of high-dimensional input data, is an important process in many areas, such as artificial intelligence and data mining [1, 10, 12]. Nonlinear dimensionality reduction, including Isomap [12], locally linear embedding (LLE) [10], and Laplacian Eigenmaps (LEM) [1], currently is an active area of research. Isomap performs dimensionality reduction by applying MDS on the geodesic distance matrix. LLE computes low-dimensional and neighborhood-preserving embedding of high-dimensional inputs. LEM attempts to maintain ordering between points in a local embedding. However, most of those methods assume that the high-dimensional data lies on a convex manifold. Donoho et al. propose the HLLC method [3], which is suitable for an open connected manifold. The HLLC method may be viewed as a modification of LLE. The LLE method solves a linear least-square problem to estimate the local weights [10], while the HLLC method calculates a singular value decomposition and uses Gram-Schmidt orthonormalization to estimate each local Hessian [3]. The major feature is that, HLLC is a global embedding and is locally isometric to an open, connected manifold.

For the HLLC method, the input of dimensionality reduction is a set of high-dimensional points lying on an open connected manifold. In particular, we consider a manifold as a 3D surface. Most previous dimensionality reduction methods [10, 12] are suitable for convex surfaces. However, the assumption is not suitable for surfaces with multiple holes. Here, we use an example, which is provided

in Ref. [3], to show the difference of embedding results generated by three methods: Isomap, LLE, and HLLC, as illustrated in Fig. 1. The original data contains a set of 600 points sampled on a 3D surface with one hole. The LLE result is generated using the code published by Roweis et al. [10], and the Isomap embedding is produced using the code published by Tenenbaum et al. [12]. The correct parameter space generated from the original data is a square with a central square removed, while the embedding obtained by the HLLC method is perfect. The detailed expositions of the HLLC algorithm can be found in Donoho et al. [3].

## 3. Parameterization based on HLLC

The first step of our procedure is to find an appropriate parameterization method without any partition. Our algorithm builds upon the dimensionality reduction method HLLC [3]. Given an open connected surface  $S$  with  $n$  vertices  $\{\mathbf{x}_i \in \mathbb{R}^3 | i = 1, \dots, n\}$ , we first apply the HLLC method to the 3D mesh surface  $S$ . The parameterization method based on HLLC can be described as follows.

1. The first step in HLLC determines each data point's neighborhood with  $k$ -neighbors in Euclidean distance. Since  $S$  is a 3D mesh surface and its connectivity is well-known, we choose the 1-ring neighborhood of a vertex instead of  $k$ -neighbors. For each vertex  $\mathbf{x}_i$ , let  $N_i$  be a set of adjacent vertex indices of  $\mathbf{x}_i$ . For each neighborhood  $N_i, i = 1, \dots, n$ , form a matrix  $M^i$  whose rows consist of the points  $\mathbf{x}_j - \mathbf{x}_i, j \in N_i$ .
2. Perform a singular value decomposition for each  $M^i$  and obtain local coordinates. Use the local coordinates and Gram-Schmidt orthonormalization to estimate each local Hessian.
3. Build Hessian matrix with estimating the local Hessian over neighborhood.
4. Perform an eigenanalysis of Hessian matrix, and compute the three eigenvectors corresponding to three smallest eigenvalues. Discard the eigenvector which is corresponding to zero eigenvalue. The remaining two eigenvectors form the  $n \times 2$  matrix denoted by  $V$ . Define the  $n \times n$  matrix  $R = VV^T$ , and the  $2 \times n$  matrix of embedding coordinates is obtained from  $W = V^T R^{-1/2}$ , where  $V^T$  denotes the transpose matrix of  $V$ .

The main difference between parameterization and HLLC is in the first and second step. The last two steps are similar to the HLLC algorithm.

## 4. Application on texture mapping

The parameterization method presented in this paper provides a powerful toolbox. In this section we consider its

application on texture mapping.

After applying the HLLC based parameterization to the surface points, we get a 2D mapping of the surface. Thus, the mapping of every point on the surface to its corresponding 2D flattened point is also obtained. Given a flat texture image, we can easily map each point from the 2D mapping to a point on the texture image. Then we will map the texture back to the surface. The technique is straightforward.

The HLLC method is computationally expensive because of the matrix eigenanalysis [3]. In practice, we select a subset of the vertices and apply the HLLC parameterization on this subset. After flattening the subset, we interpolate the local coordinates to find the local mapping of the rest of the vertices. The strategy is also used by Zigelman et al. [14].

## 5. Experiment results

We demonstrate our results in Figs. 2 and 3. The execution time is reported on a Pentium IV 1.70GHz processor with 256M RAM excluding that of loading meshes. Figs. 2(d) and 3(c) are obtained in about 22 seconds and 7 seconds, respectively. In Fig. 2, we compare our result to the result based on partition of a face model with one hole. Fig. 2(b) shows the new model partitioned by a curve. Observe that highly distorted and discontinuous texture appears nearby the partition boundary in Fig. 2(c), and the texture is smooth by our method in Fig. 2(d). In Fig. 3, we also show a more complex surface with seven holes. As can be seen, the proposed HLLC based parameterization reduces the deformations and better preserves the local and global structure of the texture.

## 6. Conclusions

In this paper, we have presented a novel mesh parameterization method for an open connected surface based on the HLLC method, and gave its application on texture mapping. The proposed approach has several advantages: 1) Boundary conditions are not required for a surface parameterization. 2) Our method does not use any partition technique and can preserve the local and global structure of the texture.

The major drawback in our current implementation is that the HLLC mapping method may contain local overlapping triangles in the planar embedding. In order to produce unfolding mapping, a possible solution is to apply the free-boundary method introduced by Desbrun et al. [2], which is an iterative method, on the resulting embedding. In the future, it is an interesting topic to improve the HLLC method for minimizing geometric stretch in some ways.

## Acknowledgments

We would like to thank Jiang Qian and Dong-Ming Yan for many helpful discussions. The research was supported by the Chinese 973 Program (2004CB719400) and the National Science Foundation of China (60403047). The second author was supported by a Foundation for the Author of National Excellent Doctoral Dissertation of PR China (200342).

## References

- [1] M. Belkin and P. Niyogi. Laplacian eigenmaps for dimensionality reduction and data representation. *Neural Computation*, 15(6):1373–1396, 2003.
- [2] M. Desbrun, M. Meyer, and P. Alliez. Intrinsic parameterizations of surface meshes. *Computer Graphics Forum*, 21(3):209–218, 2002.
- [3] D. Donoho and C. Grimes. Hessian eigenmaps: new locally linear embedding techniques for high-dimensional data. Technical report, 2003-08, Department of Statistics, Stanford University, 2003.
- [4] M. Floater. Parameterization and smooth approximation of surface triangulations. *Computer Aided Geometric Design*, 14(3):231–250, 1997.
- [5] M. Floater and K. Hormann. Surface parameterization: a tutorial and survey. In *Advances in Multiresolution analysis of Geometric Modelling*, pages 259–284, 2004.
- [6] X. Gu. *Parameterization for surfaces with arbitrary topologies*. Phd thesis, Computer Science, Harvard University, 2002.
- [7] T. Ju. Robust repair of polygonal models. In *Proceedings of SIGGRAPH'04*, pages 888–895, 2004.
- [8] B. Lévy, S. Petitjean, N. Ray, and J.-L. Maillot. Least squares conformal maps for automatic texture atlas generation. In *Proceedings of SIGGRAPH'02*, pages 362–371, 2002.
- [9] P. Liepa. Filling holes in meshes. In *Proceedings of the Eurographics/ACM SIGGRAPH symposium on Geometry processing*, pages 200–205, 2003.
- [10] S. Roweis and L. Saul. Nonlinear dimensionality reduction by locally linear embedding. *Science*, 290:2323–2326, 2000.
- [11] P. Sander, J. Snyder, S. Gortler, and H. Hoppe. Texture mapping progressive meshes. In *Proceedings of SIGGRAPH'01*, pages 409–416, 2001.
- [12] J. Tenenbaum, V. Silva, and J. Langford. A global geometric framework for nonlinear dimensionality reduction. *Science*, 290:2319–2323, 2000.
- [13] K. Zhou, J. Snyder, B. Guo, and H. Shum. Iso-charts: stretch-driven mesh parameterization using spectral analysis. In *Proceedings of the Eurographics/ACM SIGGRAPH Symposium on Geometry Processing*, pages 47–56, 2004.
- [14] G. Zigelman, R. Kimmel, and N. Kiryati. Texture mapping using surface flattening via multidimensional scaling. *IEEE Transactions on Visualization and Computer Graphics*, 8(2):198–207, 2002.

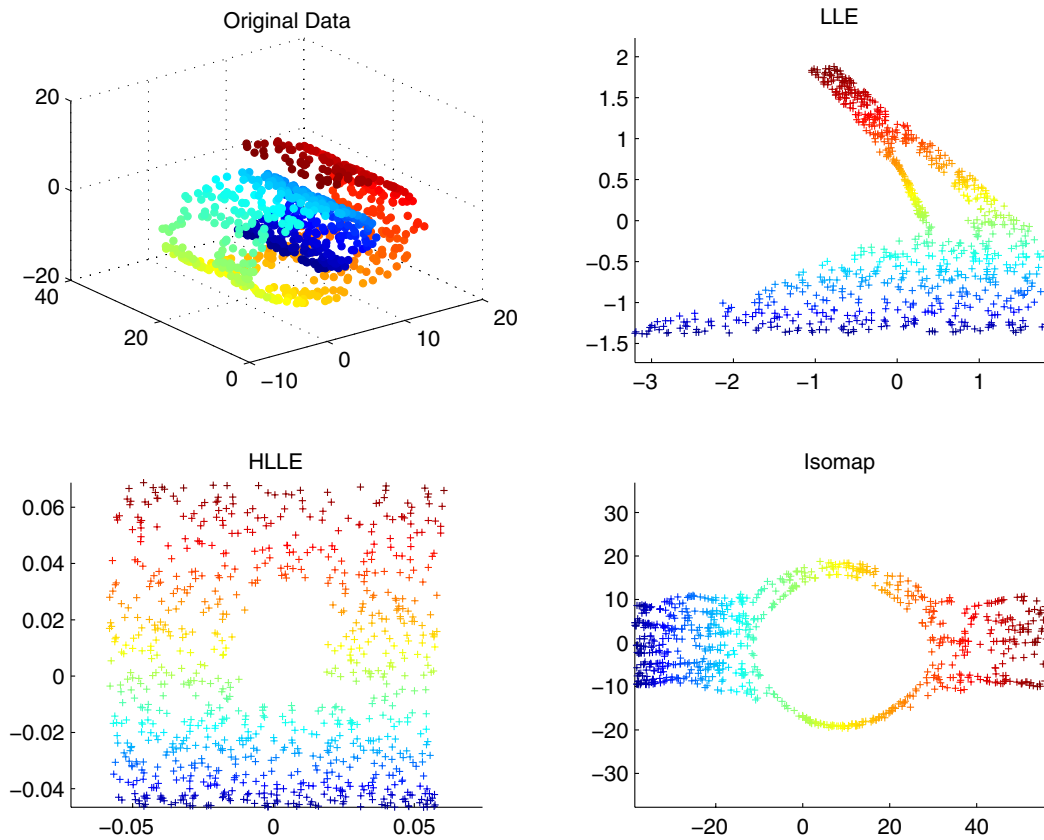


Figure 1: A comparison of embedding results.

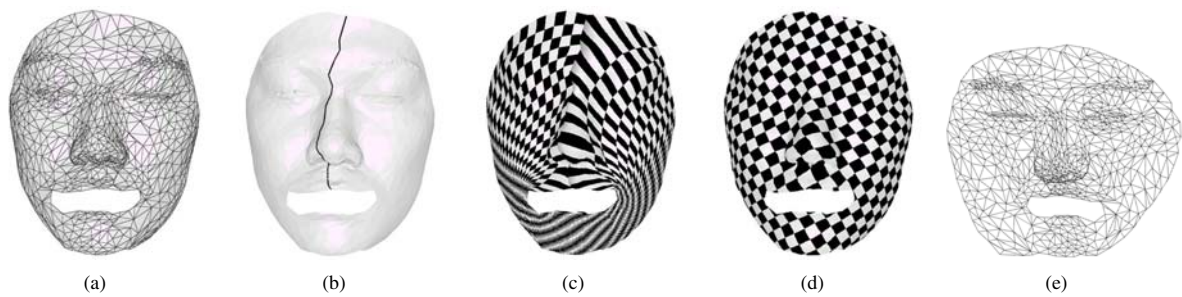


Figure 2: A comparison between partition-based mapping and the HLLC mapping on a face model. (a) The original mesh with 824 vertices. (b) Partition. (c) A checker texture mapped to (b) using Floater's method [4]. (d) The same checker texture mapped to (a) using the HLLC mapping. (e) The corresponding parameterization to (a) using the HLLC mapping.

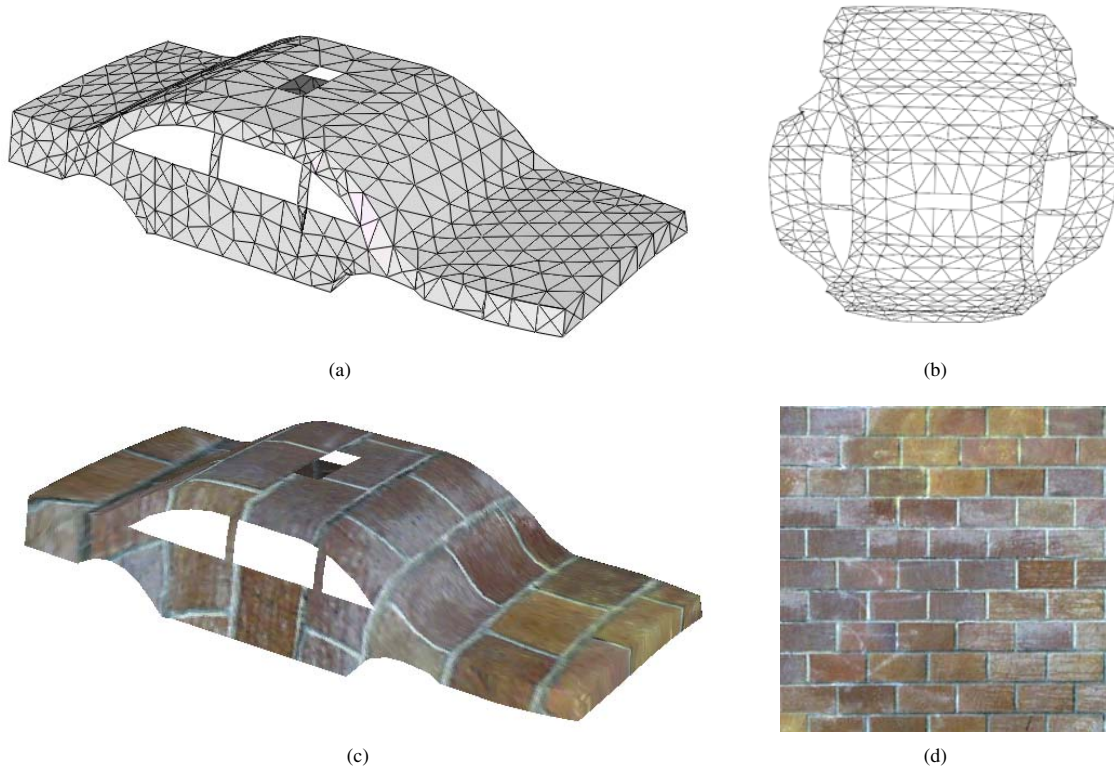


Figure 3: An example of texture mapping on a complex car model. (a) The original mesh with 543 vertices. (b) The corresponding parameterization using the HLL mapping. (c) Texture mapped to (a). (d) The corresponding texture image.