

# Disparity and Normal Estimation through Alternating Maximization

Ramya Narasimha<sup>1</sup>, Elise Arnaud<sup>1,2</sup>, Florence Forbes<sup>1</sup>, Radu Horaud<sup>1</sup>

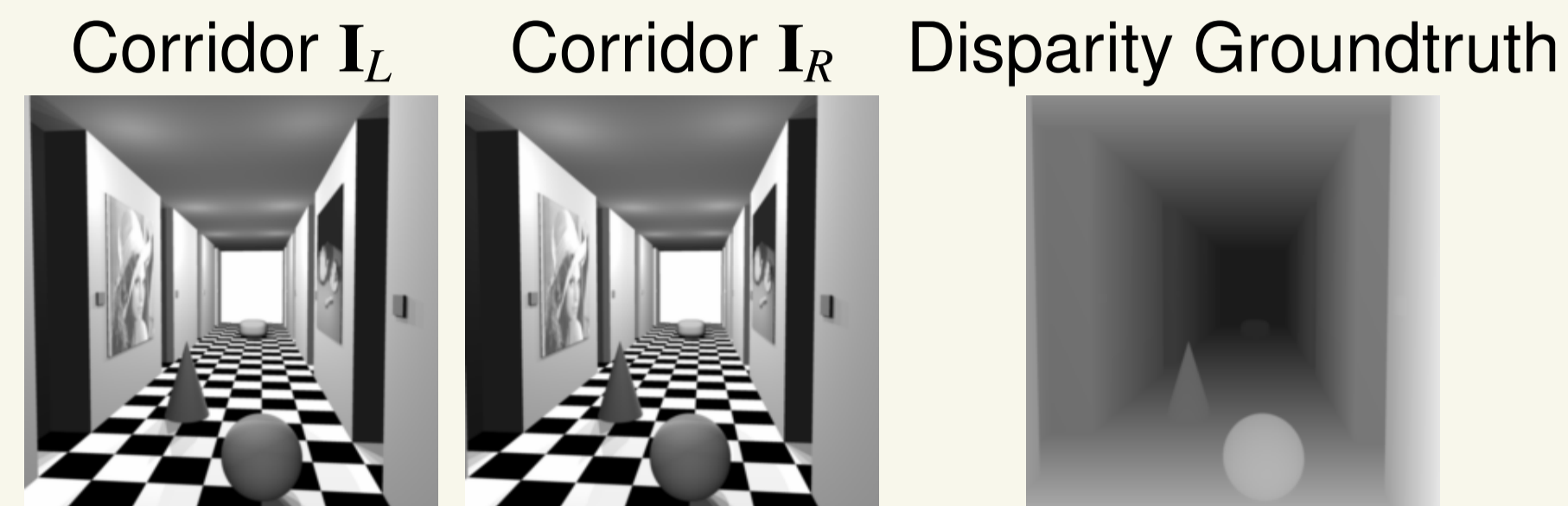
<sup>1</sup> INRIA Rhône-Alpes <sup>2</sup> Université Joseph Fourier, Laboratoire Jean Kuntzmann, Grenoble, France



## Stereo Matching

**Stereo Matching:** Involves determining **disparities** indicating difference in locations of corresponding pixels in left ( $I_L$ ) and right ( $I_R$ ) images.

**Application:** 3D scene reconstruction, using disparity and depth inverse relation and known camera geometry.



## Bayesian-MRF model for Stereo Matching

**Disparity Markov Random Field (MRF) model:** With **disparity**  $\mathbf{d}$  and observation  $\mathbf{I} = (I_L, I_R)$ .

**Posterior distribution**  $p(\mathbf{d}|\mathbf{I})$ :

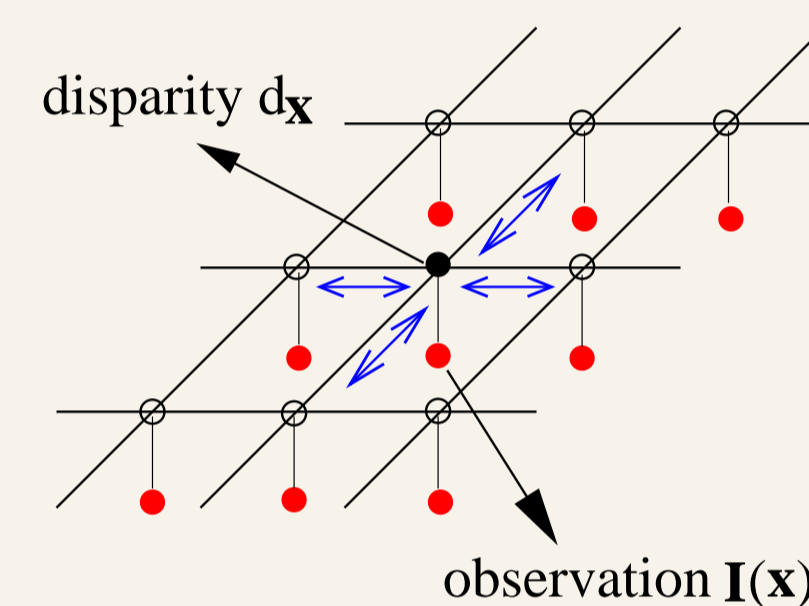
$$p(\mathbf{d}|\mathbf{I}) \propto \Phi(\mathbf{I}|\mathbf{d}) \Psi(\mathbf{d})$$

**Likelihood term**  $\Phi(\mathbf{I}|\mathbf{d})$ : measures observation agrees with the disparity.

**Prior term**  $\Psi(\mathbf{d})$ : encodes **interaction** between the neighbouring disparities.

**Interaction:** **First-order differences** (blue arrows in the figure).

**Approximate inference:** Belief Propagation, Mean Field or Graph-cuts.



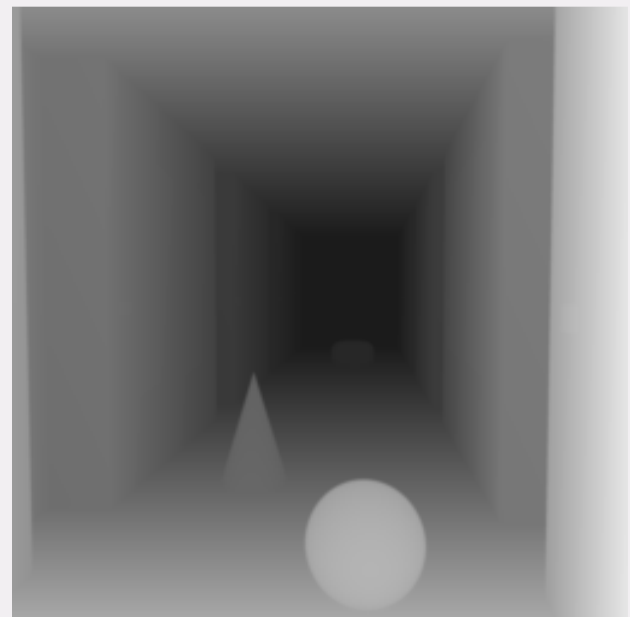
## Limitation

- Using first-order differences involves an inherent **fronto-parallel assumption**.
- Fronto-parallel assumption:** Supposes that the scene under consideration can be approximated by patches of constant disparities.
- This biases the results towards **staircase** solutions.

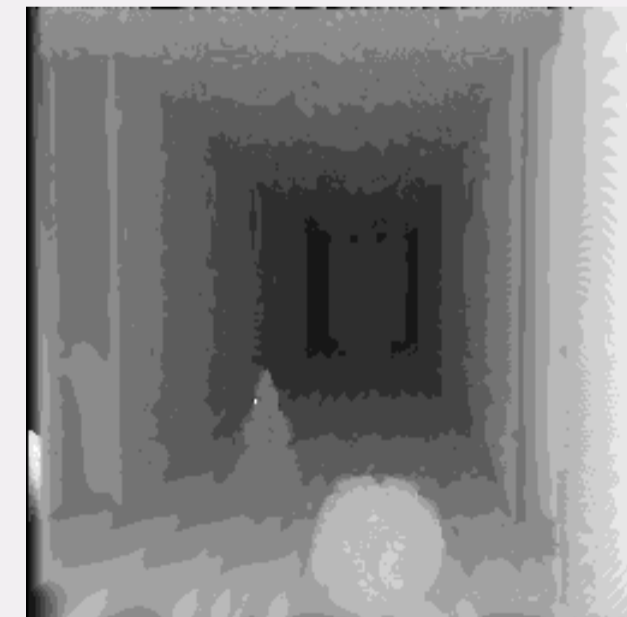
**Possible Solution:** Post-processing the disparities using plane-fitting on segmented regions.

**Limitation:** Cannot handle curved surfaces.

Groundtruth disparity



Staircase solution



## Related Work

Solution Proposed by Li and Zucker CVPR 2006

Explicitly takes into account the both **disparities/depth** and scene-structure in terms of **surface normals** in an MRF-based framework.

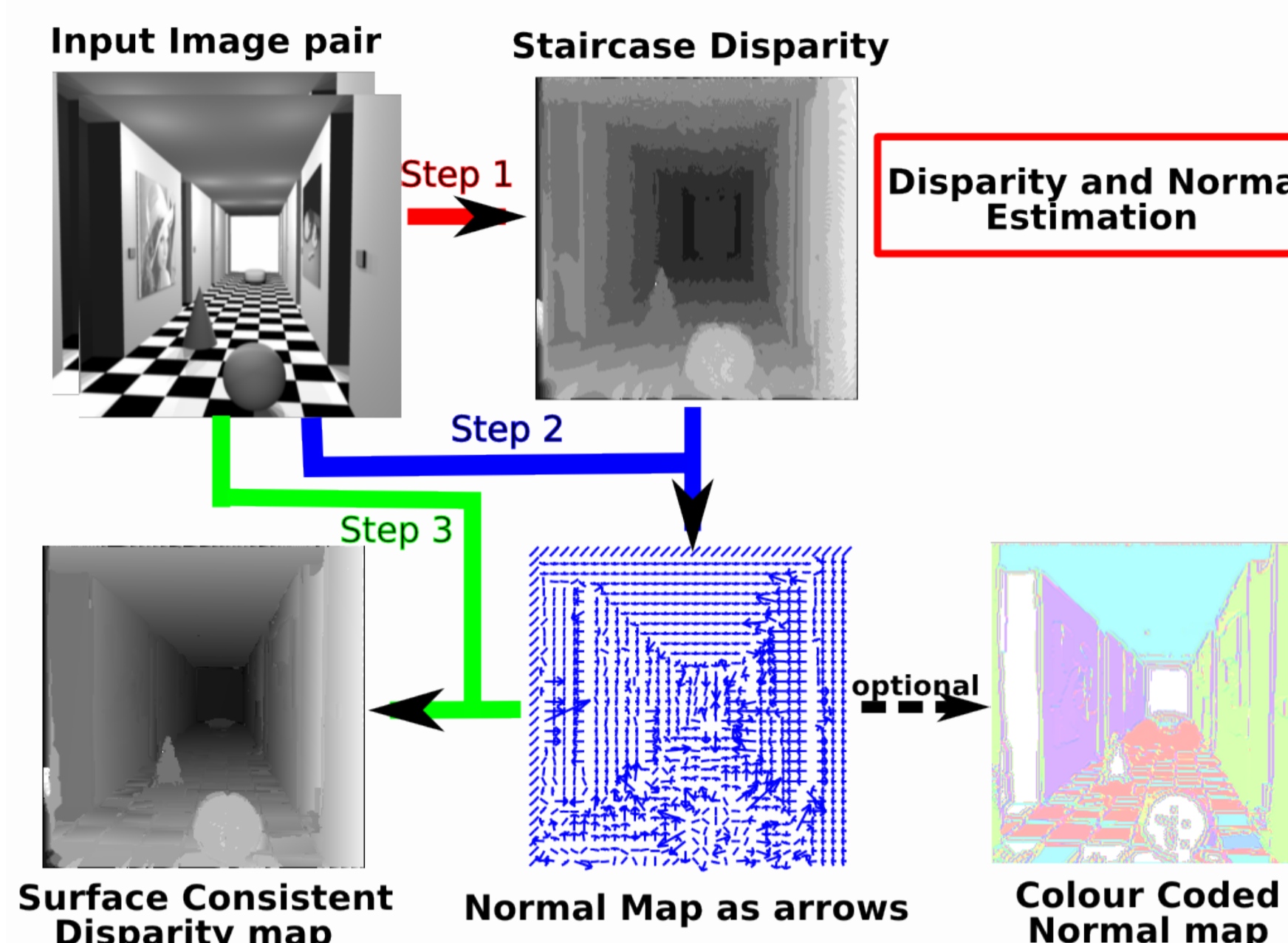
**Geometric consistency:** An additional geometric constraint to ensure both disparity/depth and normals are consistent with the surface.

Limitations:

- Requires knowledge of the internal camera parameters.**
- Precomputes the local surface normals.**

## Proposed Solution

**Cooperatively estimate both disparity and normal using:**



**Disparity-MRF,  $\mathbf{d}$ :** incorporates differential geometric constraints.

**Normal Conditional Random Field (CRF),  $\mathbf{n}$ :** assumes a piecewise smooth scene surface.

**Alternating Maximization:** Iteratively maximizes the posterior probability alternately in  $\mathbf{d}$  and  $\mathbf{n}$ .

$$\mathbf{d}^{(i+1)} = \arg \max_{\mathbf{d}} p(\mathbf{d}|\mathbf{n}^{(i)}, \mathbf{I})$$

$$\mathbf{n}^{(i+1)} = \arg \max_{\mathbf{n}} p(\mathbf{n}|\mathbf{d}^{(i+1)}, \mathbf{I})$$

## Disparity-MRF model

Distribution of **disparity**  $\mathbf{d}$  conditional to the **normal field**  $\mathbf{n}$  and the **observation**  $\mathbf{I}$ :

$$p(\mathbf{d}|\mathbf{n}, \mathbf{I}) \propto \Phi_d(\mathbf{d}, \mathbf{I}) \Psi(\mathbf{d}, \mathbf{n})$$

**Data term**  $\Phi_d(\mathbf{d}, \mathbf{I})$

$$\Phi_d(\mathbf{d}, \mathbf{I}) = \exp \left( - \sum_x \lambda \min(\phi(I_L, I_R, \mathbf{d}), 2T) \right)$$

$\phi(I_L, I_R, \mathbf{d})$ : Cost at location  $\mathbf{x}$  based on **weighted window matching metric**.

Takes into account:

- Both the color and the proximity of the pixels within the window.
- Weighted windows in both left and right images.

**Interaction term**  $\Psi(\mathbf{d}, \mathbf{n})$

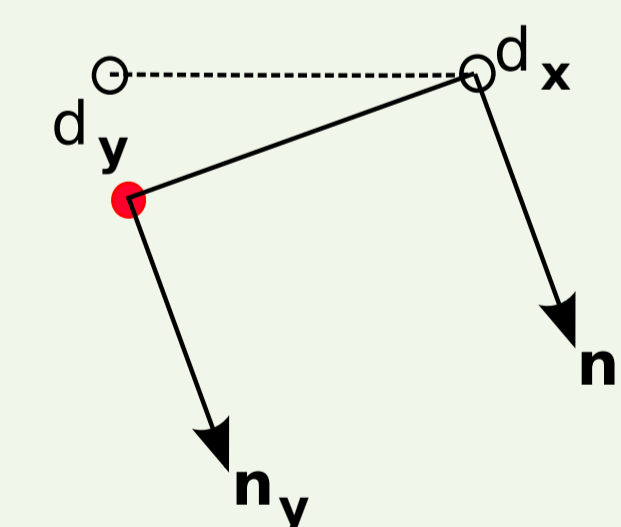
$$\Psi(\mathbf{d}, \mathbf{n}) = \prod_{(\mathbf{x}, \mathbf{y})} \psi(d_x, d_y, \mathbf{n})$$

$\psi(d_x, d_y, \mathbf{n})$  incorporates a general surface model:

$$\exp \left( - |d_y - d_x - \frac{\partial d_x}{\partial u} \Delta u - \frac{\partial d_x}{\partial v} \Delta v| - |d_x - d_y - \frac{\partial d_y}{\partial u} \Delta u - \frac{\partial d_y}{\partial v} \Delta v| \right)$$

With  $\mathbf{x} = (u, v)$ ,  $\mathbf{n}_{\mathbf{x}} = (n_u, n_v, n_d)$   $\frac{\partial d_x}{\partial u} = -\frac{n_u}{n_d}$  and  $\frac{\partial d_x}{\partial v} = -\frac{n_v}{n_d}$ , similarly for  $\mathbf{n}_{\mathbf{y}}$ .

The disparity  $d_x$  imposes  $d_y$  to lie on the same plane based on the normals  $\mathbf{n}_{\mathbf{x}}$  and  $\mathbf{n}_{\mathbf{y}}$  respectively.



## Normal-CRF model

**Normal Field**  $\mathbf{n}$  conditioned on **disparity**  $\mathbf{d}$  and **observation**  $\mathbf{I}$  expressed as a Gaussian distribution:

$$p(\mathbf{n}|\mathbf{d}, \mathbf{I}) \propto \prod_{\mathbf{x} \sim \mathbf{y}} \exp \left( - \frac{1}{2\sigma^2} \|\mathbf{n}_{\mathbf{x}} - w_{\mathbf{xy}}(\mathbf{d}, \mathbf{I}) \vec{N}_{\mathbf{xy}}\|^2 \right)$$

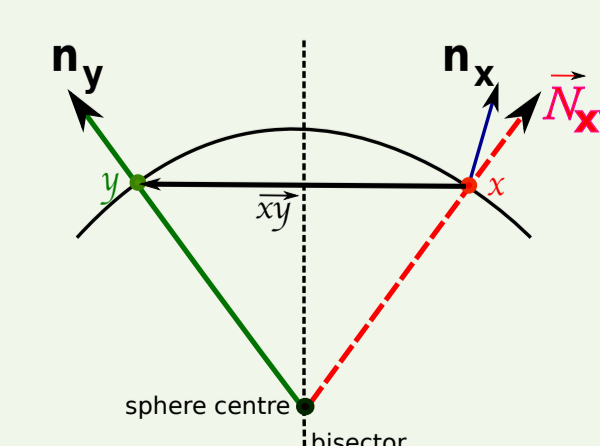
Error between  $\vec{x}\vec{y}$  and the plane described by  $\mathbf{n}_{\mathbf{y}}$ .

$$\vec{N}_{\mathbf{xy}} = \mathbf{n}_{\mathbf{y}} - 2 \cos(\theta_{\mathbf{xy}}(\mathcal{S}, \mathbf{d})) (\vec{x}\vec{y} / \|\vec{x}\vec{y}\|)$$

$$w_{\mathbf{xy}}(\mathbf{d}, \mathbf{I}) = \exp \left( - |d_x - d_y| + |\nabla \mathbf{I}(\mathbf{x})| \right)$$

$\theta_{\mathbf{xy}}$ : angle between  $\mathbf{n}_{\mathbf{y}}$  and the vector  $\vec{x}\vec{y}$ , from  $\mathbf{x} = (x, d_x)$  to  $\mathbf{y} = (y, d_y)$

The normal  $\mathbf{n}_{\mathbf{y}}$  is reflected at the bisector of  $\vec{x}\vec{y}$  to obtain  $\vec{N}_{\mathbf{xy}}$



## Alternating Maximization

**Initialization:** Normal map values are set to  $(0, 0, 1)$ . Disparity map is obtained using standard Mean Field.

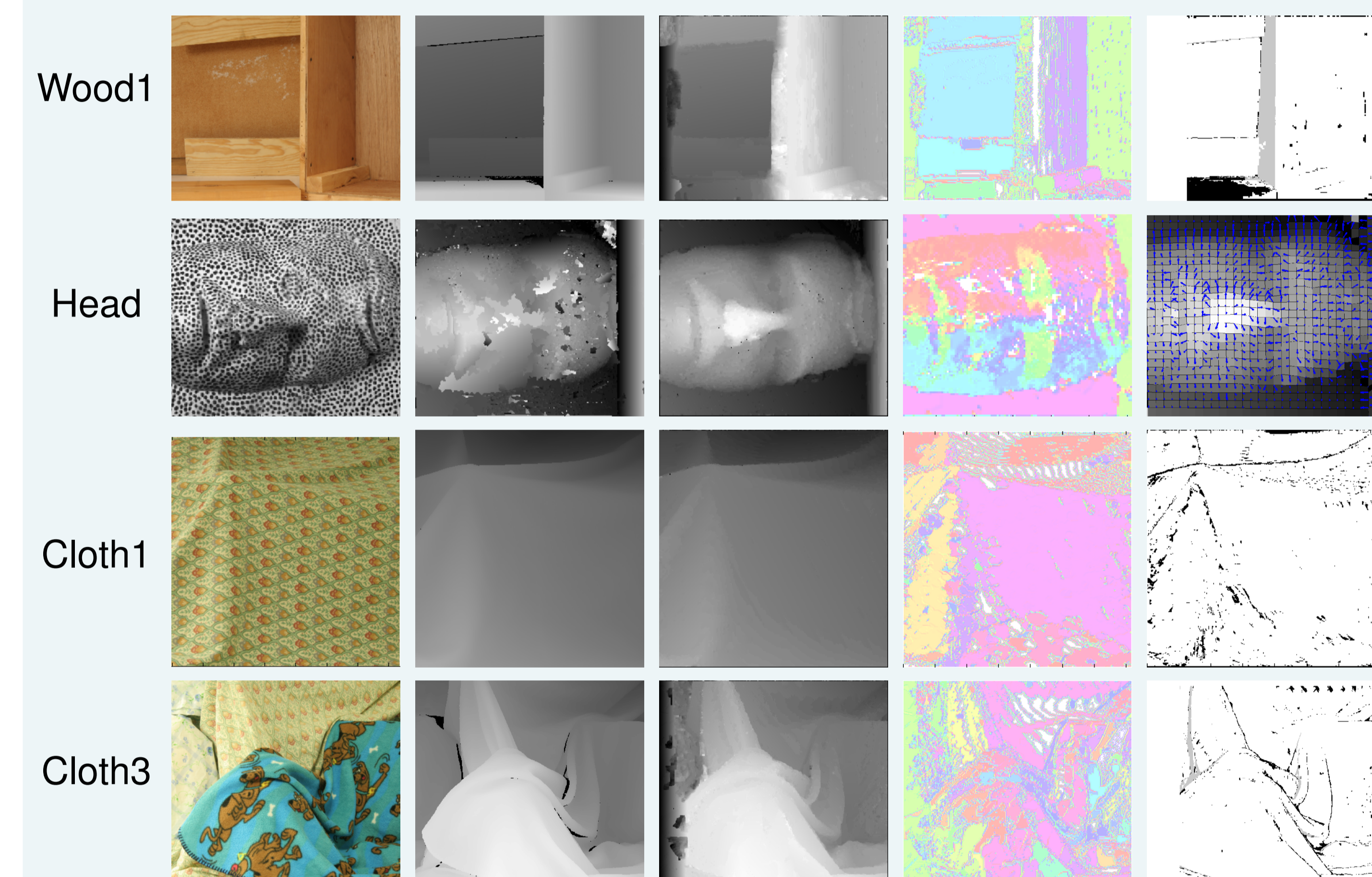
**Alternation:** **Update normal field** by optimizing the proposed normal model using Iterated Conditional Mode (ICM).

**Update disparity field by:**

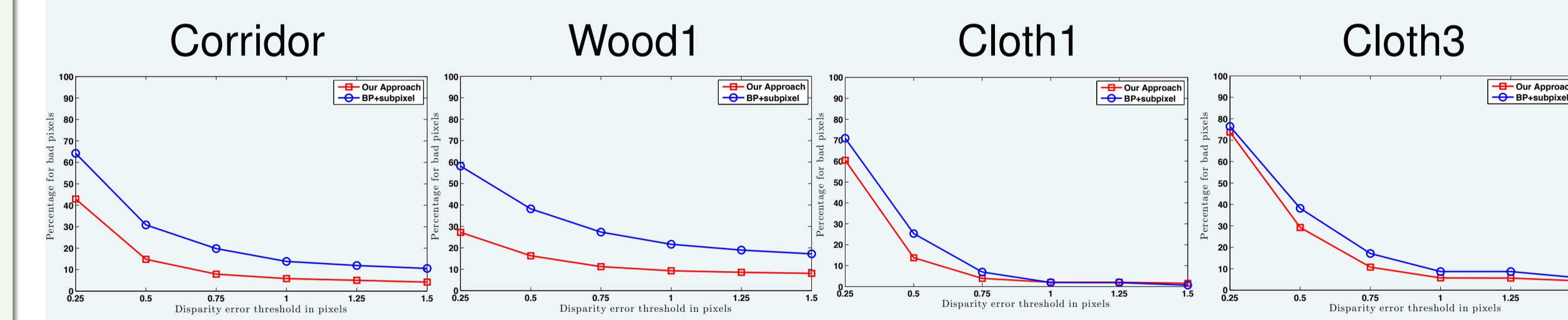
- Computing the first order disparity derivatives using  $\mathbf{n}$  and
- updating disparity estimates into  $\mathbf{d}$  with Mean Field applied to the proposed disparity model.

## Results

### Estimated Disparity and Normals



### Error Plots



## Contributions

- Estimation of surface consistent disparity solutions.**
- Embedding of the surface properties in the model** rather than refining the results using post-processing.
- Uses a separate random field to estimate the normals based on the disparity and vice-versa.
  - Does not involve precomputing normals.**
- Alternating maximization procedure results in **mutual improvement of both disparities and normals**.
- The two conditional models allow for more dependence or independence according to the information to be incorporated