



QP-Collide: A New Approach to Collision Treatment

Laks Raghupathi

François Faure

Co-encadre par

Marie-Paule CANI

EVASION/GRAVIR

INRIA Rhône-Alpes, Grenoble



Teaser Video



Classical Physical Simulation

- Advance time-step (solve ODE for Δv)

$$\mathbf{v}(t + \Delta t) \leftarrow \mathbf{v}(t) + \nabla \Lambda$$

$$\mathbf{x}(t + \Delta t) \leftarrow \mathbf{x}(t) + \Delta t \cdot \mathbf{v}(t + \Delta t)$$

- Detect collisions at $\mathbf{x}(t + dt)$
- Make corrections to $\mathbf{v}(t + dt)$ and/or $\mathbf{x}(t + dt)$



Problem at Hand

- Detect and handle *multiple* collisions
 - Discrete methods miss thin, fast objects
 - Displacement correction results in large deformation for stiff objects
- Avoid *looping* problems in the case of multiple collisions
 - Responding one-by-one will provoke further collisions



Our Strategy

- Solve mechanics implicit (stable) or explicit form
- Formulate collisions as linear inequality constraints
 - Detect both at discrete and continuous time
- Solve mechanic equations + constraint inequalities
 - collision correction via *quadratic programming* (QP)



Previous Techniques

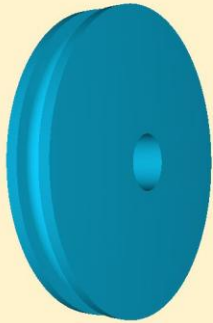
- [Baraff94] – contact force computation of rigid bodies using LCP/QP
- Requires inertial matrix inversion \mathbf{M}^{-1}
- Not always possible to compute \mathbf{K}^{-1} for deformable objects
- Others use iterative correction [Volino00] and [Bridson02]



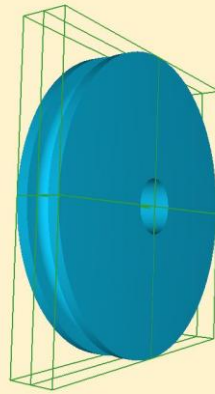
Overall Collision Approach

- i. Perform broad-phase rejection tests
 - E.g. Box-Box test between Cable and Object
- ii. If i. is true, enter narrow-phase
 - Recursively test Cable Primitives and Object-Children (e.g. ray-box)
 - Primitive tests between Cable and Object primitives (e.g. ray-triangle)
 - Apply Response for each valid collision

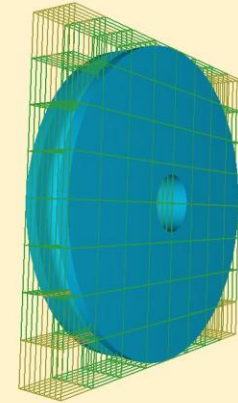
Broad Phase Detection



Polygonal model



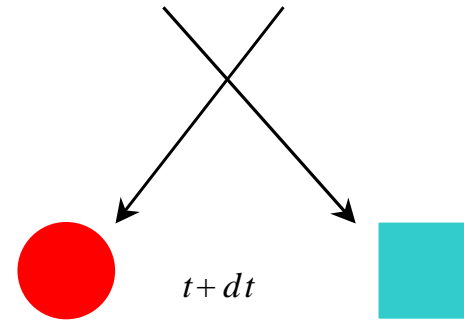
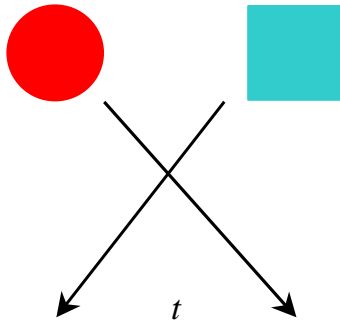
1-subdivision



3-subdivisions



Why Continuous?



No interpenetration at t and $t + \Delta t \Rightarrow$ Undetected by static methods



Case for Continuous

- Static collision detection missed if:

$$\text{Relative Velocity} \times \Delta t > \text{ObjectSize}$$

- Robust detection for thin, fast moving objects
- Permits large time step simulations
- Continuous methods detect first contact during collision detection

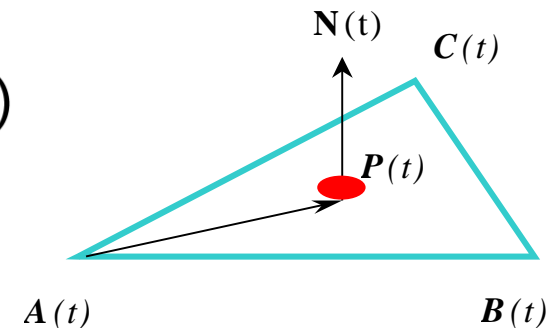


Narrow Phase – Vertex-Triangle

- Point $\mathbf{P}(t)$ colliding with triangle ($\mathbf{A}(t)$, $\mathbf{B}(t)$, $\mathbf{C}(t)$) and normal $\mathbf{N}(t)$

$$\mathbf{AP}(t_c) = u \cdot \mathbf{AB}(t_c) + w \cdot \mathbf{AC}(t_c)$$

$$\mathbf{AP}(t_c) \cdot \mathbf{N}(t_c) = 0$$



- Cubic equation in t_c
- Find valid $t_c \in [t_0, t_0 + dt]$ and $u, w \in [0, 1]$, $u + w \leq 1$



Quadratic Form (1)

- Mechanical equation $\mathbf{K} \nabla \Lambda = \mathbf{p}$
 - Implicit (Baraff98)

$$\left(\mathbf{M} - dt^2 \frac{\partial \mathbf{f}}{\partial \mathbf{x}} - dt \frac{\partial \mathbf{f}}{\partial \mathbf{v}} \right) \Delta \mathbf{v} = dt \left(\mathbf{f} + dt \frac{\partial \mathbf{f}}{\partial \mathbf{v}} \mathbf{v} \right)$$

- Quadratic form with linear constrains
 - Minimize: $d(\nabla \Lambda) \equiv \int \nabla \Lambda_{\perp} \mathbf{K} \nabla \Lambda - \mathbf{p}_{\perp} \nabla \Lambda$
 - Subject to: $\mathbf{J} \Delta \mathbf{v} \leq \mathbf{c}$



Quadratic Form (2)

- Lagrange form

- Such that
$$\begin{bmatrix} -\mathbf{1} & \mathbf{0} \\ \mathbf{K} & -\mathbf{1}_{\mathcal{A}} \end{bmatrix} \begin{pmatrix} \gamma \\ \nabla \Lambda \end{pmatrix} = \begin{pmatrix} -\mathbf{c} \\ \mathbf{p} \end{pmatrix}$$

$$\lambda_i \geq 0, i \in \mathcal{A}$$

$$\mathbf{J}_k \Delta \mathbf{v}_k \leq \mathbf{c}_k, k \notin \mathcal{A}$$

- \mathcal{A} - set of “active” constraints
- \mathcal{A}' - set of “non-active” constraints



Active Set Method (1)

- Find ϕv in $K \nabla \Lambda = p$
- Detect Collisions (populate \mathbf{J} and \mathbf{c} matrices)
- Classify constraints

for all constraint $J_i \in J[1 \dots m]$ do

if $J_i \phi v_i > c_i$ then

 Add i^{th} constraint to A

else

 Add i^{th} constraint to A'

- Find new ϕv and λ



Active Set Method (2)

- Verify Lagrange multiplier sign

for all $\lambda_q(k)$, $q \in A$ do

if $\lambda_q(k) > 0$ then

Move q from A to A' (active to non-active)

endloop = false

- Verify if solution satisfies constraints

for all c_p , $p \in A$ do

if $J_p v(k) > c_p$ then

Move p from A' to A (non-active to active)

endloop = false



On Convergence

- Using iterative method such as conjugate gradient
 - Should converge within $3n + m$ steps
 - ‘ n ’ number vertices, ‘ m ’ number of constraints
- Watch out for numerical errors
- Toggling Constraints



Collisions as Linear Constraints

- Vertex-Triangle condition for non-penetration:

$$(\mathbf{v}_{ij} + \Delta \mathbf{v}_{ij}) \cdot \hat{\mathbf{n}} \, dt \leq g - (r_i + r_j)$$

$$(w_0 \Delta \mathbf{v}_{j0} + w_1 \Delta \mathbf{v}_{j1} + w_2 \Delta \mathbf{v}_{j2} - \Delta \mathbf{v}_i) \cdot \hat{\mathbf{n}} \leq c$$

- Left-hand side of \mathbf{J} matrix:

$$\begin{pmatrix} \cdots & j_0 & j_1 & j_2 & \cdots & i & \cdots \\ \cdots & w_0 \hat{\mathbf{n}} & w_1 \hat{\mathbf{n}} & w_2 \hat{\mathbf{n}} & \cdots 0 \cdots & \hat{\mathbf{n}} & 0 \cdots \end{pmatrix}$$

- Similar for edge-edge and other constraints



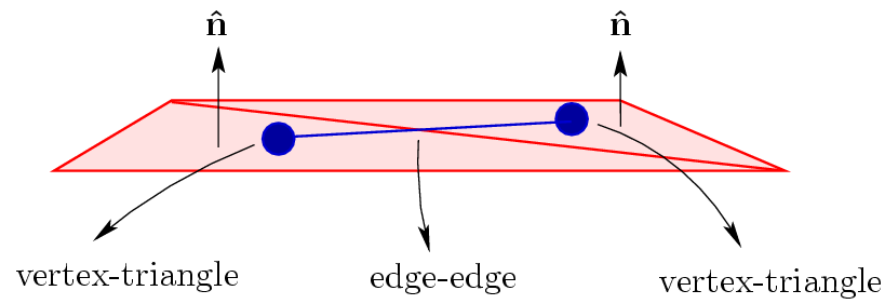
Demos



Known Problems

- Degradation of matrix conditioning
 - Susceptible to numerical errors
 - Disadvantage vis-à-vis penalty approaches
- Linearly dependent constraints
 - High condition number
 - Simple fix in lieu of QR decomposition
- Looping
 - Constraints toggle between “active” and “non-active”
 - Suppress them – not theoretically justified

Linearly dependent constraints



$$\begin{pmatrix} \cdots & 0 & n_x & n_y & n_z & 0 & 0 & 0 & 0 \cdots \\ \cdots & 0 & 0 & 0 & 0 & n_x & n_y & n_z & 0 \cdots \\ \cdots & 0 & a_1 n_x & a_1 n_y & a_1 n_z & a_2 n_x & a_2 n_y & a_2 n_z & 0 \cdots \end{pmatrix}$$

- Add minor “perturbation” to avoid singularity:

$$\begin{bmatrix} \mathbf{K} & -\mathbf{J}^T \\ -\mathbf{J} & \mathbf{L} \end{bmatrix} \begin{pmatrix} \Delta \mathbf{v} \\ \lambda \end{pmatrix} = \begin{pmatrix} \mathbf{b} \\ -\mathbf{c} \end{pmatrix}$$

\mathbf{L} = diagonal matrix (l_1, \dots, l_m) , $l_i \sim 10^{-3}$



Summary

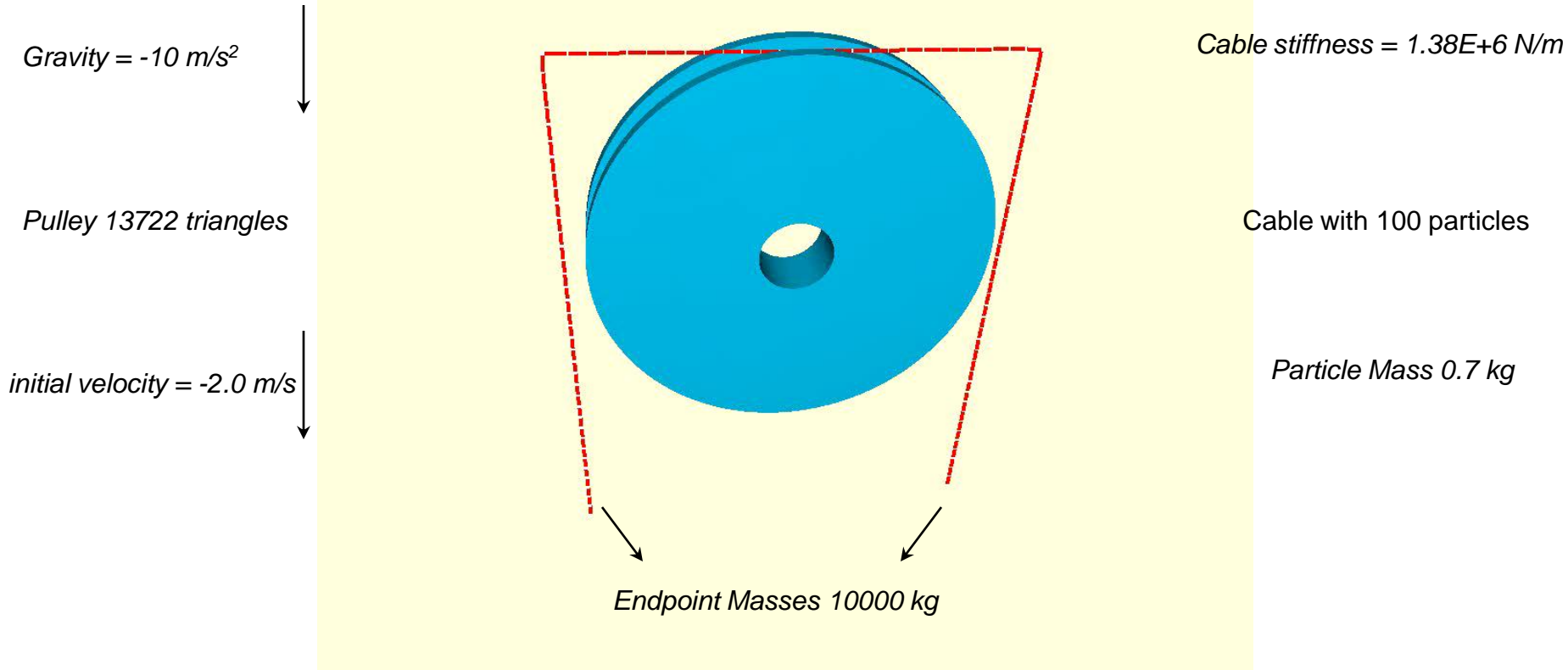
- New approach to handling multiple collisions and other linear constraints
- Works well for moderately difficult cases
- Further theoretical work needed for better numerical precision



Questions/Comments?

Et bien sur, allez les Bleus!

Demo: Mechanical Simulation (1)



Mechanics: Implicit Euler resolved by conjugate gradient

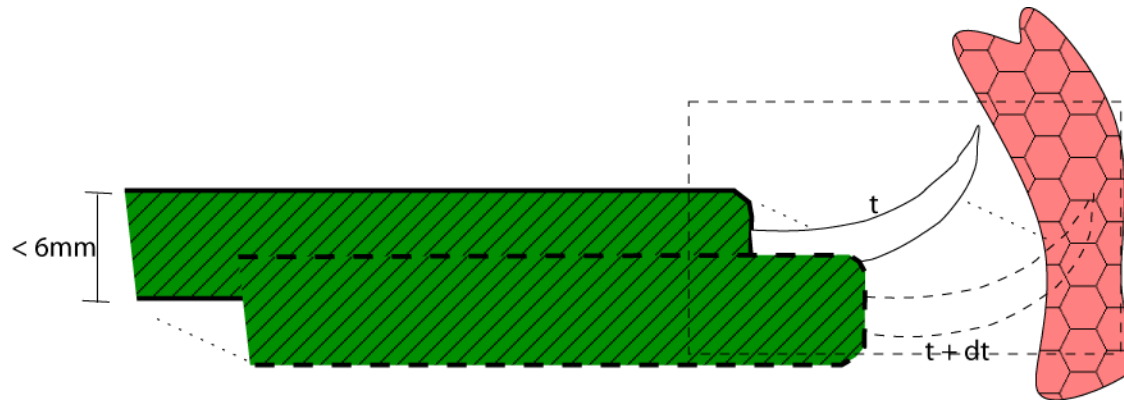
Run-time: 40 Hz in Pentium 4 3.0 GHz, 1GB RAM, GeForce 3



Primary Accomplishments

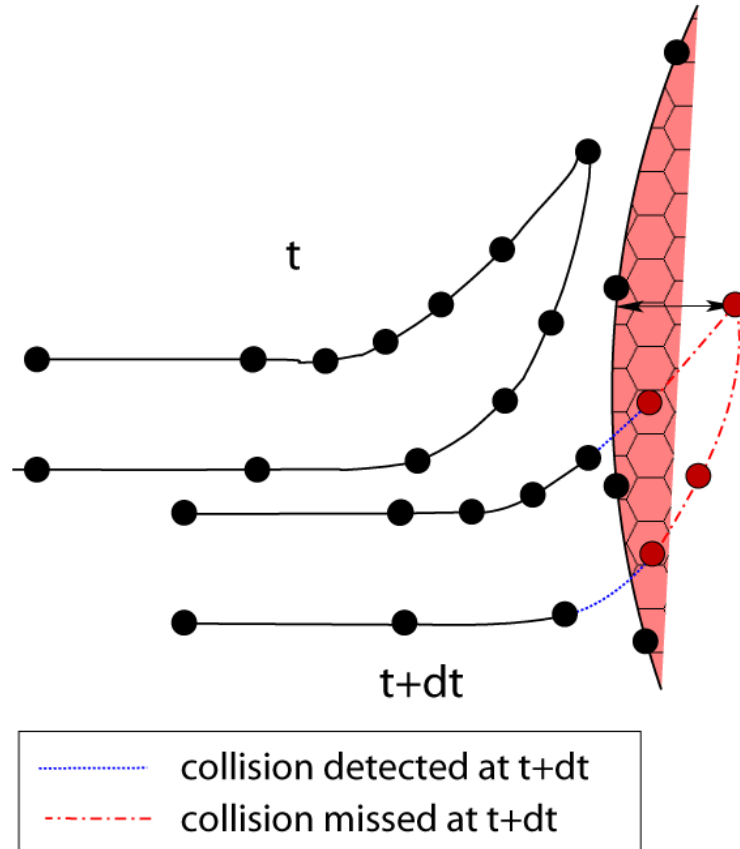
- Fast octree-based method for collision detection (rigid & deformable)
- Continuous collision detection
- Numerically stable ODE solver
- Demo: Simulation of interaction cable and rigid mechanical parts
- Relevance to medical applications

Relevant apps: thin tissue cutting

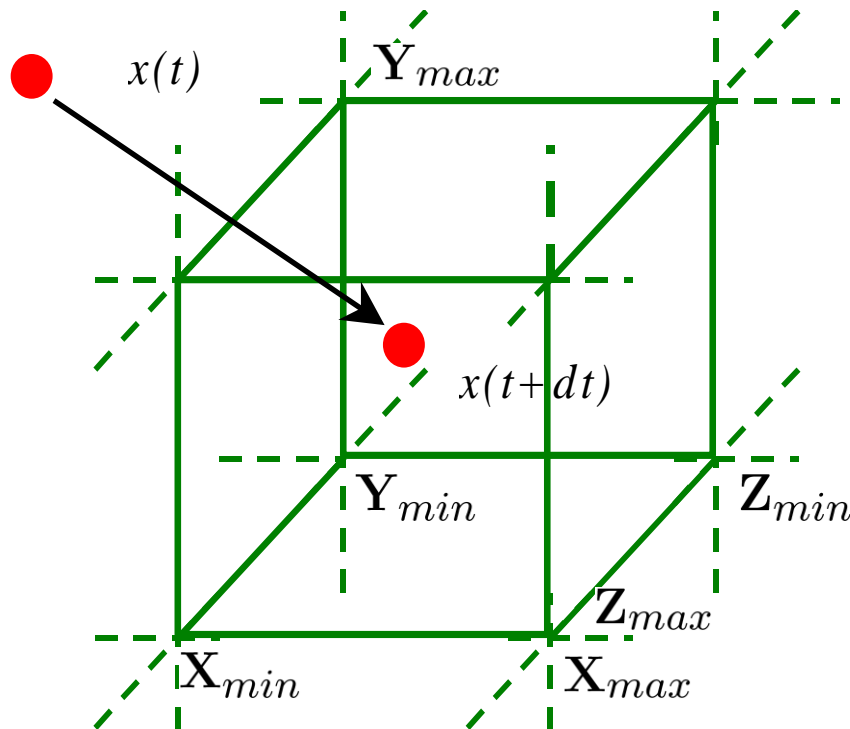


Scalpel cutting a thin tissue

Thin tissue cutting (2)



Bounding box tests



Ray-Box Test

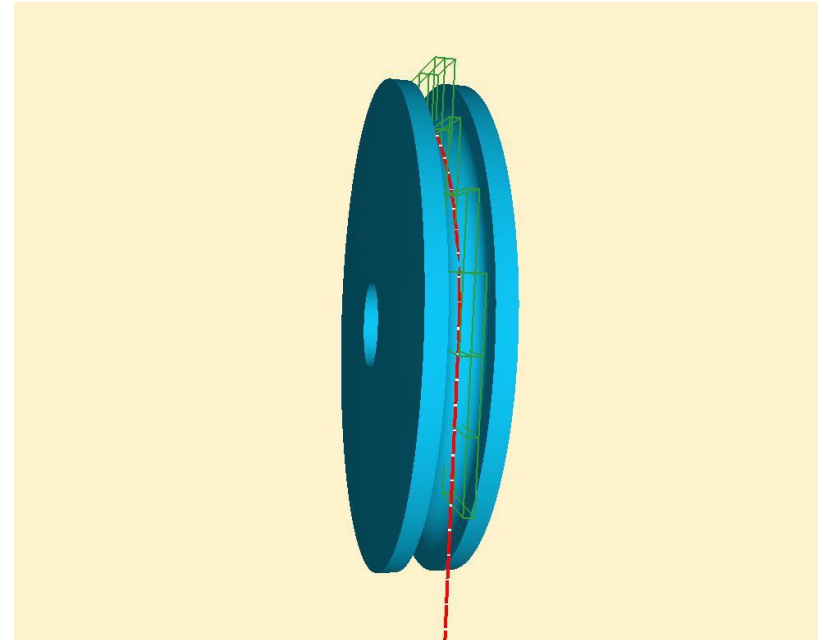
\Rightarrow 6 Ray-Plane Tests

Why ray?

\Rightarrow Ray is trajectory of positions between $x(t)$ and $x(t+dt)$

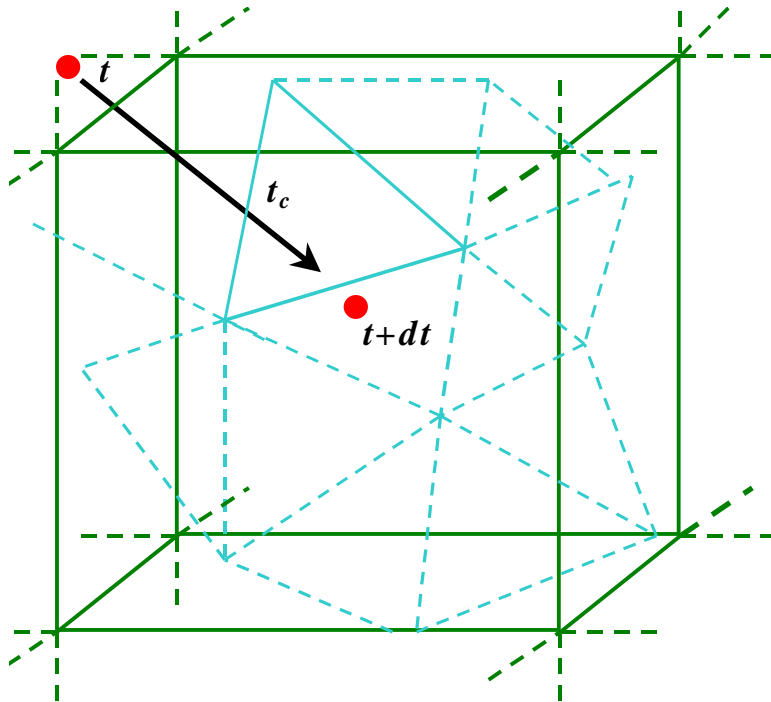
Recursive BB test

- If Ray collides with Box, check with all the 8-children of the boxes recursively till the end



Colliding AABBs

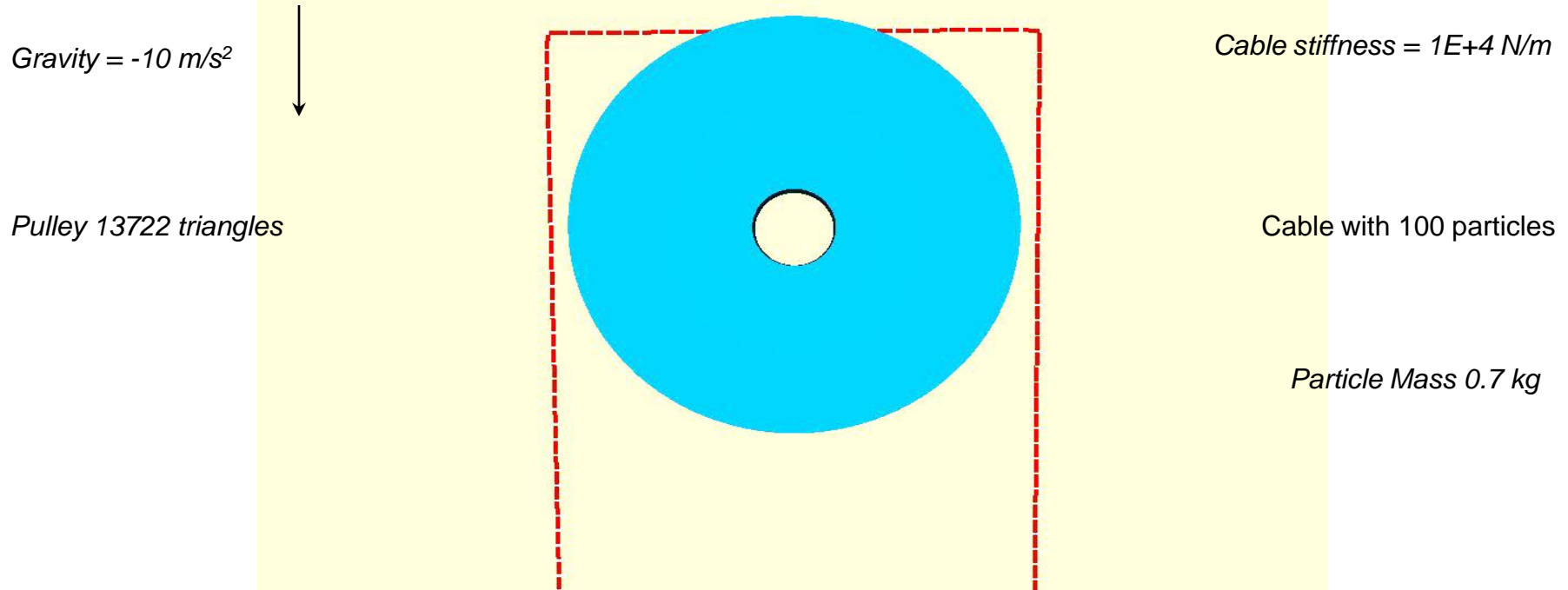
Primitive Tests



Ray-Triangle Tests

- Test primitives within the smallest box
- Continuous test:
 - Cubic equation for two objects moving
 - Quadratic equation for one fixed mobile

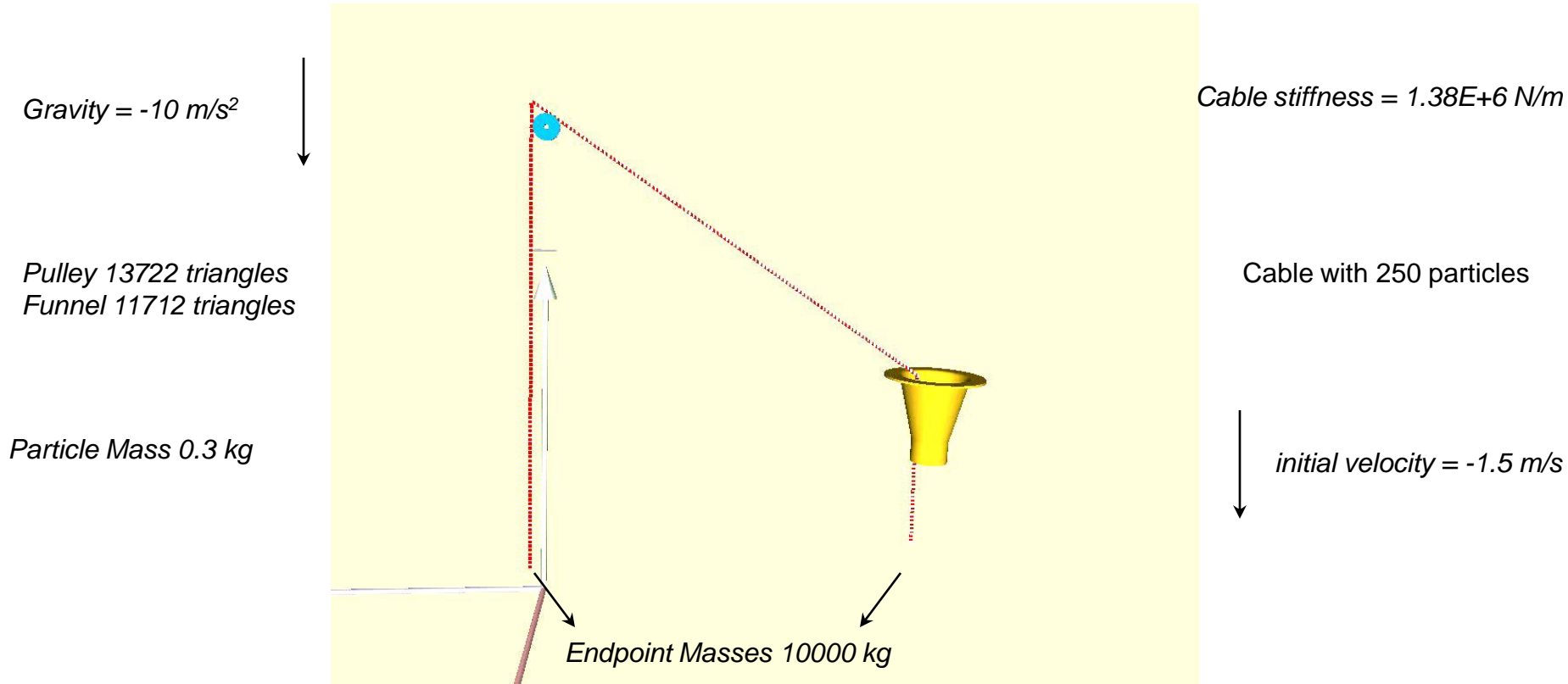
Demo: Mechanical Simulation (2)



Mechanics: Implicit Euler resolved by conjugate gradient

Run-time: 35-40 Hz in Pentium 4 3.0 GHz, 1GB RAM, GeForce 3

Demo: Mechanical Simulation (3)



Mechanics: Implicit Euler resolved by conjugate gradient

Run-time: 35-40 Hz in Pentium 4 3.0 GHz, 1GB RAM, GeForce 3



Work in Progress

- So far: Proof of concept
 - Approach works for Rigid Object + Cable
 - Now, extend it to deformable objects
- Extensions:
 - Deformable Octree for organ (trivial)
 - Cubic continuous test (just need a bit more CPU power)
 - Unified treatment of mechanics, collisions and other constraints (present challenge)

Coming Soon!

- Deformable organs being disconnected by cutting-off the interstitial tissues

