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Session : Optimisation combinatoire

## The weak limit of Boltzmann random matchings on diluted graphs

par Charles Bordenave, Marc Lelarge et **Justin Salez**

A matching on a finite graph  $G = (V, E)$  is a collection of pairwise non-adjacent edges  $M \subseteq E$ . The Boltzmann random matching at temperature  $z > 0$  on  $G$  is distributed as follows : for any matching  $M$  on  $G$ ,

$$\mathbb{P}(\mathcal{M}_G^z = M) = \frac{z^{|V|-2|M|}}{P_G(z)}, \text{ with } P_G(z) = \sum_M z^{|V|-2|M|}.$$

We are interested in the asymptotic behavior of  $\mathcal{M}_G^z$  as  $|G| \rightarrow \infty$ . Specifically, we establish that for any graph sequence  $(G_n)_{n \geq 1}$  converging to an infinite tree  $\mathcal{T}$  with finite Hausdorff dimension,  $\mathcal{M}_{G_n}^z$  converges in distribution to a properly defined random matching  $\mathcal{M}_{\mathcal{T}}^z$  on  $\mathcal{T}$  with determinantal marginals. Moreover, the zero-temperature limit  $\mathcal{M}_{\mathcal{T}}^0 = \lim_{z \rightarrow 0} \mathcal{M}_{\mathcal{T}}^z$  exists in some sense, and under an extra condition on  $\mathcal{T}$  it is precisely the weak limit of a the uniform maximum matching on  $G_n$ . When the  $(G_n)_{n \geq 1}$  are random and converge weakly to a Galton-Watson tree, the limit turns out to be characterized by a recursive distributional equation, which we solve. We thus obtain an explicit formula for the asymptotic size of a maximum matching on  $G_n$ , generalizing that of Karp and Sipser for Erdős-Rényi graphs.

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