Minimizing Transmission Energy in Sensor Networks via Trajectory Control

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Abstract—**Energy optimization is a significant component of Wireless Sensor Network (WSN) design. In this paper we consider transmission energy optimization in WSNs where messages are collected by a mobile receiver (collector). The collector is responsible for gathering data messages by choosing the *optimal path* that minimizes the total transmit energy at the sensors subject to a maximum travel delay constraint. We show, both analytically and through simulation, that letting the mobile collector come closer to sensors with more data to transmit leads to significant reduction in energy consumption. Using this intuition, we propose an algorithm for choosing both the transmission radii and the mobile collector's path that achieves over 50% improvement in energy consumption compared to schemes that use a fixed communication radius for every sensor, and a 95% improvement as compared to the case of a stationary base station. We extend our results to the case of stochastic arrivals to the sensors and propose an *Adaptive* Algorithm that dynamically adjusts the transmission radii of the sensors based on the sensors' current queue sizes. We show that it can achieve 80% transmission energy improvement with respect to a non-adaptive scheme that uses fixed radii computed using the average arrival rates.**

I. INTRODUCTION

Energy efficient communication in WSNs is of paramount importance since the devices have limited battery life [8], [19], [28]. In networks where sensors use multihop routing to relay their messages to a fixed base station at the center of the network, (e.g., [12]), the well known problem of "hot spots" occurs. Namely, sensors closer to the base station relay most of the messages and exhaust their batteries quickly, leaving the network disconnected [17], [19]. One approach to solve this problem is to utilize mobile nodes that physically move towards the sensor locations in order to decrease the number of communication hops, and in a sense, distribute the hot spots throughout the network [17], [19], [21]. In this paper, we consider the problem of optimizing the mobile collector's path as a function of the traffic density at the sensors in order to minimize the total transmission energy with constraints on total travel delay.

We propose an optimal trajectory planning algorithm in a network of sensors with arbitrary amounts of data at each sensor to be transmitted to the mobile collector. The collector's job is to choose the optimal trajectory in order to receive the data from the sensors using the minimum total transmission energy subject to a maximum travel delay constraint. Fig. 1

(left) shows an example trajectory for the collector. In this setting, there is a tradeoff between the transmission energies of the sensors and the travel delay of the mobile collector captured by the choice of communication ranges: as we use larger communication ranges, the travel delay is reduced at the expense of transmission energy.

It is well known that transmission energy increases super linearly with transmission distance [8], [10], [23]. We take this phenomena into account *by choosing trajectories that come closer to sensors with more data to transmit*. This approach, when subject to a constraint on the maximum travel delay, considerably reduces the transmission energy consumption. Note that we only consider the transmission energy consumption of the static sensor nodes; and not the energy consumed by the mobile element for travel. This assumption is justified by the fact that the static sensor nodes are more likely to be energy constrained [8]. Moreover, the energy consumption of the mobile collector can be explicitly captured by the constraint on the total travel delay if the travel energy is modeled proportional to the travel time [10], [23]. Multihop relaying can also be used to reduce the communication distances, however, in addition to creating hot spots in the network, due to the increased data transfer rate at each node, the total energy consumption in a multihop forwarding system can be greater than the transmission plus mobility energies combined in the data collector approach (see e.g., [23]).

Utilizing special mobile elements that can control their mobility to collect sensor data in WSNs has received considerable attention over the past decade (e.g., see [3], [9], [20], [25], [27]). Mobile trajectory control, (e.g., [3], [17], [27]), mobile collector scheduling (e.g., [25]), or sensor energy minimization (e.g., [9], [13], [17], [20], [22]) have been proposed to maximize network lifetime, to provide connectivity or to minimize delay. A detailed survey of the related work on using mobility to facilitate communication in WSNs can be found in [23], [27].

Our approach is novel in that we introduce and analyze the problem of optimal vehicle trajectories as a function of the amount of data at each sensor in order to minimize the total transmission energy within a given travel time limit. We formulate this as a transmission energy optimization problem with constraints on travel time and propose a novel algorithm that assigns communication radii to sensors as a function of their load. We show via both analysis and simulation that our algorithm, which utilizes shorter transmission ranges for

heavily loaded sensors and larger transmission ranges for *lightly loaded* sensors, is about 50% more energy efficient than schemes utilizing a fixed transmission radius for every sensor. We also show that our algorithm obtains a 95% improvement as compared to the case of a stationary base station.

Finally, we generalize our model to the case of stochastic arrivals to the sensors, i.e., the messages arrive to sensor nodes via stochastic processes of different rates. We study the resulting network as a queuing system, in particular, as a Polling model with queues (corresponding to sensors) distributed in space and a mobile server performing a cyclic service of the queues. We propose two algorithms to compute the sensors' transmission radii: A *Non-Adaptive* Algorithm that computes the radii based on the average arrival rate to the sensors and maintains this choice for all the cycles, and an *Adaptive* Algorithm that dynamically adjusts the radii based on sensors' available data at the beginning of each cycle. We show that choosing sensors' radii inversely proportional to the queue sizes at the sensors and applying the Adaptive Algorithm, can lead to 80% improvement as compared to the non-adaptive case if the arrival processes have high variance.

The paper is organized as follows. In Section II we describe the system model. In Section III we analytically characterize the simple case in which the mobile collector serves only two sensors. Then, we extend the analysis to the general network case with many sensors in Section IV. In Section V we consider the use of multiple collector nodes and extend our results to the case of stochastic arrivals in Section VI.

II. MODEL DESCRIPTION

We consider a square network region $\Re$ of area A consisting of N sensors at fixed locations and a mobile node (collector) that gathers messages from the sensors. The collector starts from a given location and serves the sensors in a cyclic fashion exhaustively by traveling on straight lines from one message reception point to the next until all messages have been served. The collector is capable of receiving a single transmission at a given time and it grants access to the sensor whose message it wants to receive next. We assume that the collector moves at a constant speed v and that it can receive messages only when it stops. An example of the collector's trajectory is displayed in Fig. 1 (left) where sensor i has transmission radius r_i and the collector's path is represented by dashed lines. We assume that the transmission range of each sensor is a disk around the sensor location and that the mobile collector has to come within this disk in order to serve this sensor.

We let D and $D_{\max}$ denote the total travel time in one cyclic tour and the maximum allowed travel time in one cyclic tour, respectively. We consider a sparse sensor network, therefore, the transmission times are assumed to be negligible compared to the mobile collector's travel time D. We denote by S_i the number of bits at sensor i that need to be transmitted to the mobile collector.

The transmitted signals are subject to attenuation that varies with distance. We assume that the transmission rate is a constant that is the same for every sensor. In such a system, it is
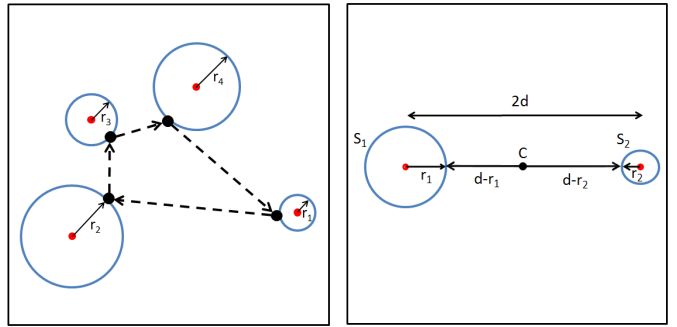

Fig. 1. An example collector's tour (left). Example of a two-sensor system with $S_1 < S_2$ (right): we choose $r_1 > r_2$ in order to minimize the total transmission energy.

reasonable to model the power required for communication at a distance r_i from the receiver as $P_T(r_i) = Kr_i^\alpha$ where K is a proportionality constant [8], [10], [20] and α is the power loss exponent (typically between 2 and 6). Since the transmissions are assumed to be at a constant rate, the transmission time is proportional to the number of bits transmitted. Therefore, the transmission energy required for transmitting S_i data bits across distance r_i is given by [8], [10], [20]:

$$E_T(r_i, S_i) = S_i P_T(r_i) = KS_i r_i^\alpha, \qquad (1)$$

where K is a constant (for instance, $K \cong 10^{-10} Jm^{-\alpha}/bit$ is reasonable for certain practical systems [10]).

A. Problem Statement

Under the above assumptions, we formulate the problem of finding the optimal trajectory for the mobile collector in order to minimize the total transmission energy per cycle with a constraint on the maximum travel delay. Denote by $\Im$ the set of all possible trajectories in $\Re$, then this optimization problem is given by,

$$\text{minimize}_{p \in \Im} \quad E_T(p) = K \sum_{i=1}^N S_i(r_i(p))^\alpha \qquad (2)$$

$$s.t. \quad D(p) \leq D_{max},$$

where $r_i \geq 0$, $\forall i \in \{1, 2, ..., N\}$. The radii $r_1, r_2, ..., r_N$ and the delay D are functions of the choice of the travel path p in $\Im$. Note that all the results in this paper can be applied to the case where both the transmission energies of the sensors and the mobility energy of the collector are taken into account. This is due to the fact that it is reasonable to model the energy spent on mobility proportional to the distance traveled [10], [23]. Thus, the constraint on travel time can be replaced by a constraint on travel energy.

III. EXAMPLE: THE CASE OF TWO SENSORS

We analytically solve the optimization problem (2) for the case of one collector serving two sensors as displayed in Fig. 1 (right). When there are only two sensors, optimizing the travel path over the space is reduced to an optimization over a line, which in turn reduces to finding the optimal values of the

transmission radii r_1 and r_2. Hence, the optimization problem in (2) can be written as

$$\text{minimize}_{r_1, r_2} \quad K(S_1 r_1^\alpha + S_2 r_2^\alpha) \qquad (3)$$

$$s.t. \quad \frac{2[(d-r_1)+(d-r_2)]}{v} \leq D_{max},$$

where $r_1, r_2 \geq 0$, and d is the distance between the starting position of the mobile node and each sensor[1]. Since (3) is a convex optimization problem, we can easily find the optimal solution given below using the method of Lagrange multipliers:

$$r_1^* = \frac{2d - v D_{max}/2}{1 + \left(\frac{S_1}{S_2}\right)^{\left(\frac{1}{\alpha-1}\right)}}; \qquad r_2^* = r_1^* \left(\frac{S_1}{S_2}\right)^{\left(\frac{1}{\alpha-1}\right)}. \quad (4)$$

This result suggests that, if $S_2 > S_1$, we have $r_2^* < r_1^*$. Hence, in order to minimize the transmission energy, the collector should be closer to the sensor that has more data to transmit. In Section IV we generalize this idea to have the controller arrange its travel path in order to stay closer to sensors with more data to transmit and show that with this controlled mobility policy, considerable energy savings can be achieved.

IV. THE GENERAL NETWORK CASE

We extend our analysis to the general case of a network with N sensors as given in (2). The network case entails a path optimization problem in order to minimize the total transmission energy with a constraint on the length of the path. Note that, if we were given the communication radii, the minimum length tour visiting the communication range of each sensor would be a special case of the Traveling Salesman Problem with Neighborhoods (TSPN), where the neighborhoods are the disks around each sensor [1], [5], [6]. However, in general the optimal radii are unknown and finding these radii is the central part of our optimization problem. Furthermore, the TSPN problem is known to be NP-hard for which several approximation algorithms have been proposed [1], [4], [6], [7]. These algorithms find a TSPN tour in polynomial time with tour length that is guaranteed to be within a constant factor of the optimal tour. We propose two approaches for the solution of (2). The first approach uses a lower bound on the TSPN tour length in the optimization problem (2), while the second approach is a heuristic iterative algorithm which uses an upper bound on TSPN.

A. Optimization Approach

In this section we consider networks in which the locations of the sensors are independent and uniformly distributed over the network region $\Re$. The solution of the optimization problem (2) is a TSPN tour with neighborhoods of optimal size. Therefore, in this section we are interested in finding the optimal set of sensor transmission radii subject to the

constraint that the optimal TSPN tour length is less than $v D_{max}$. Since the optimal TSPN tour problem is NP-Hard, we use a lower bound on the optimal tour length given by $TSPN \geq TSP - 2\sum_{i=1}^{N} r_i$ [3], where TSP denotes the length of the optimal Traveling Salesman Tour, namely, the length of the tour when all neighborhoods have zero radius. The intuition behind this lower bound is that the line segment joining two neighborhoods is at least as large as the distance between the centers of the two neighborhoods minus the sum of the radii. When the locations of the nodes are i.i.d. uniform over the region, the TSP tour length can be approximated by $\beta\sqrt{NA}$, where $\beta \cong 0.72$, which is exact as $N \to \infty$ [24]. Furthermore, if the region $\Re$ is a convex and compact set in the plane (e.g., a square), then the above formula provides a very close estimate of the optimal TSP tour length for values of N as low as 15 [15]. Putting it all together, we approximate the optimization problem in (2) by replacing the tour length with the lower bound on TSPN:

$$\text{minimize}_{\{r_1,...,r_N\}} \quad K \sum_{i=1}^{N} S_i r_i^\alpha \qquad (5)$$

$$s.t. \quad \frac{\beta\sqrt{NA} - 2\sum_{i=1}^{N} r_i}{v} \leq D_{max}, \qquad (6)$$

where $r_i \geq 0, \; \forall i \in \{1, 2, ..., N\}$. Again, this becomes a convex optimization problem, hence it can be solved easily to obtain[2],

$$r_i^* = \frac{(\beta\sqrt{NA} - v D_{max})/2}{1 + \sum_{k=2}^{N} \left(\frac{S_i}{S_k}\right)^{\left(\frac{1}{\alpha-1}\right)}} \qquad i = 1, ..., N. \qquad (7)$$

We again notice that the transmission radius is smaller for sensors that have more data to transmit and that it is proportional to $S_i^{-\left(\frac{1}{\alpha-1}\right)}$.

1) Numerical Example: We consider a network with $N = 50$ sensors deployed in an area of size (1000×1000)m. M_i represents the number of messages that sensor i transmits where $M_i \in \{1, 2, ..., 50\}$ and each message is of size 1500 Bytes (i.e., sensor i transmits $M_i * 1500$ bytes). We use $K = 10^{-10} Jm^{-\alpha}/bit$ for transmission energy constant as in [10]. Fig. 2 (top) shows the solution of the optimization problem in (5), for different values of the delay constraint D_{max}. As displayed in the figure, the optimum radius r_i^* decreases as M_i increases as expected. Furthermore, as the maximum allowed delay D_{max} increases, the optimal radii decrease since the collector has more time to travel and can come closer to sensors in order to minimize their transmission energy consumption.

We compare the energy efficiency of our model with that of a scheme in which a collector gathers data from sensors that have all equal communication radius, i.e. $r_i = r$, for $i = 1, ..., N$, independently of their loads. The solution to (5), subject to $r_i = r$, is given by $r^* = \frac{\beta\sqrt{NA} - v D_{max}}{2N}$. Fig. 2 (bottom) shows the transmission energy versus the maximum

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[1] We assume that $D_{max} < 4d/v$ because otherwise the collector can visit each sensor location (i.e., $r_1 = r_2 = 0$) and the corresponding transmission energy is 0.

[2] We assume that $D_{max} < \beta\sqrt{NA}/v$ because otherwise the collector can perform a TSP tour by visiting each sensor location (i.e., $r_i = 0$, $\forall i \in \{1, 2, ..., N\}$) and the corresponding transmission energy is 0.
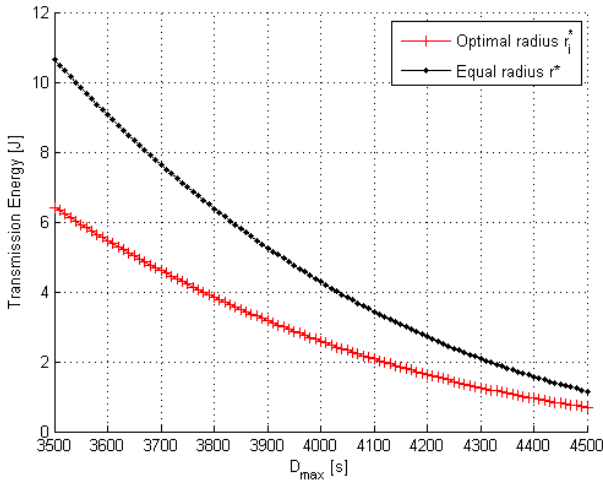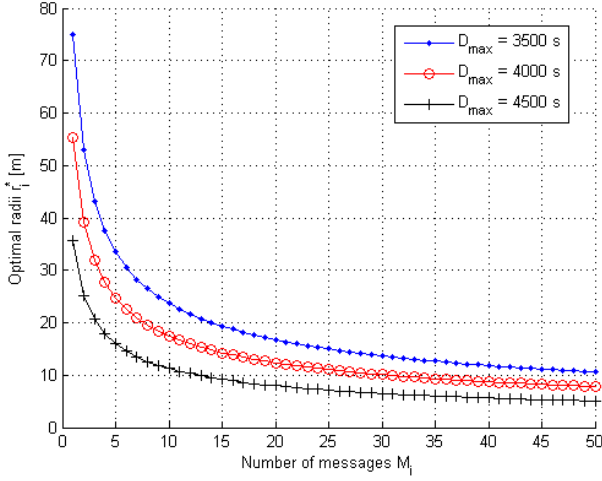
Fig. 2. Optimal radii r_i^* versus the number of messages M_i for each sensor i (top); transmission energy versus the maximum delay D_{max} both for optimal radii r_i^* and equal radii r^* (bottom).

Step 1. Choose $r_i = \Gamma \dfrac{C}{S_i^{\frac{1}{\alpha-1}}}$.

Step 2. Compute $TSPN$ using Elbassioni's algorithm.

Step 3. Compute $D = \dfrac{TSPN}{v}$:

    if $D < D_{max}$
      $\Gamma = \Gamma - \Delta$
      $r_i = \Gamma \dfrac{C}{S_i^{\frac{1}{\alpha-1}}}$ go to Step 2

    if $D > D_{max}$
      $\Gamma = \Gamma + \Delta$
      $r_i = \Gamma \dfrac{C}{S_i^{\frac{1}{\alpha-1}}}$ go to Step 2

    if $D == D_{max}$
      END

Fig. 3. Iterative algorithm for solving the optimization problem in (7).

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transmit.

A brief description of our iterative algorithm is shown in Fig. 3. The algorithm starts with a given set of radii $r_i = \Gamma C / S_i^{\frac{1}{\alpha-1}}$, where C is an appropriate constant and Γ is a parameter initially set to one. Step 2 computes the TSPN tour length using these set of radii, and Step 3 updates the radii depending on whether the tour delay is below or above the target delay value D_{max}: if the tour length is below D_{max}, the radii r_i are decreased by decreasing Γ by a small value, and if the tour length exceeds D_{max}, the radii are increased (by increasing Γ). The algorithm terminates when the TSPN tour delay is equal to the maximum allowed travel delay D_{max}.

At Step 2 of our algorithm, in order to compute the TSPN tour length given the current set of radii, we utilize the TSPN algorithm of Elbassioni et al. [6]. Note that Elbassioni's algorithm for TSPN assumes the neighborhoods around each point are disjoint, i.e., the sensors' disks cannot overlap. This algorithm works as follows: it sorts all N disk regions in ascending order based on their radii (the region of radius r_i around sensor i corresponds to a neighborhood in the TSPN algorithm which shall be visited by the collector). Then, the algorithm selects a random point on the boundary of the smallest region. Next, for every other region, the algorithm picks the point on the boundary of the region that is closest to the already chosen points, i.e., the point at the minimum Euclidean distance from the already selected points. Finally, it constructs the TSP tour based on the set of N chosen points. For this we use the well-known TSP solver *Concorde* [30][3]. In our experiments, the Elbassioni algorithm produced results that are within a factor of two of the TSPN lower bound.

1) Simulation Results: Now we present simulation results for the iterative algorithm. In order to be able to use the Elbassioni algorithm in the simulations, we have utilized a deterministic sensor topology, where neighborhoods do not overlap, instead of the uniform random topology used in the previous section. We study the case of overlapping regions in the next section.

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delay both for the case of optimum radii r_i^* and for the case of optimum constant radius r^* as obtained from (5). As expected, choosing the optimal radius r_i^* (i.e., as a function of the load) uses less energy as compared to the equal radius case. For example, for $D_{max} = 3500$ s, the energy saving is about 40%. It is interesting to observe that as D_{max} decreases, the absolute energy saving increases, however, the relative savings remain roughly constant.

B. Iterative Algorithm for Non-overlapping Regions

Next we propose an iterative algorithm to find the set of radii that minimize the transmission energy consumption. We exploit the intuitive results of the two sensors case in (4) and the result of the optimization problem in the network case in (7) in the design of the iterative algorithm. The intuition is that in order to minimize the transmission energy consumption, the choice of each sensor's radius should be set inversely proportional to the amount of data that the sensor needs to

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[3]Note that the distance between each pair of sensor nodes is rounded to the nearest integer in *Concorde*, thus, the lengths of the tours are always integer.
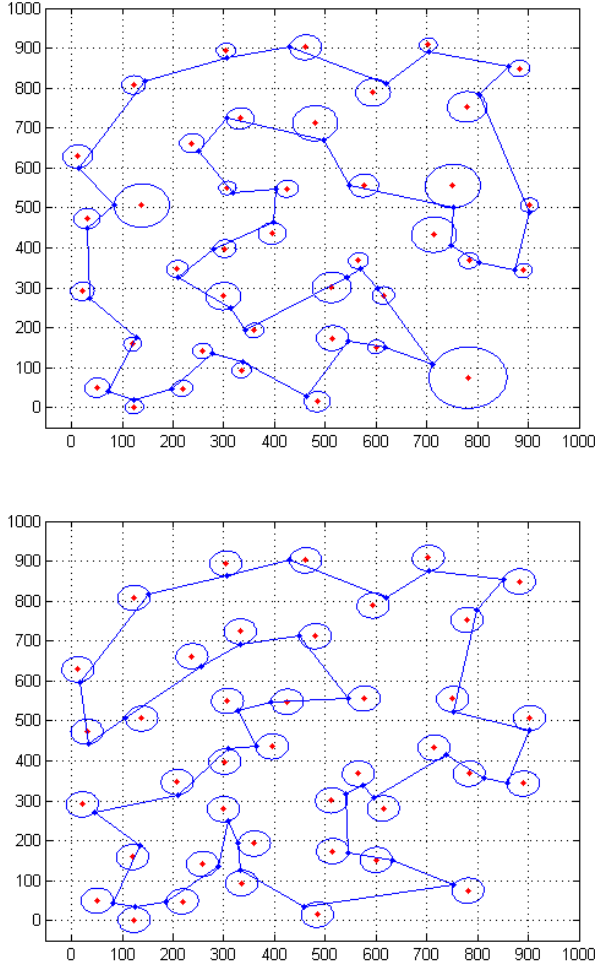
Fig. 4. TSPN tour with optimized sensor radii r_i^{*sim} (top) and equal sensor radii r^{*sim} (bottom).

We consider a network with 40 sensors deployed in a deterministic fashion in a $(1000\text{x}1000)$m square network region. Given the set of coordinates for the 40 sensors, we first compute the optimal TSP tour through these points using the *Concorde* program. Next, we set the maximum delay $D_{max} = 5200$ s, which is shorter than the TSP tour length of 5800 s (here the collector's speed v is set to 1 m/s). We assigned to each sensor a random number of messages $M_i \in \{1, ..., 20\}$ each of size 1500 Bytes. We compare our results with the case in which the sensors have the same transmission radii independent of their load. Let r_i^{*sim} be the optimal radii at the output of the iterative algorithm and let r^{*sim} be the optimal radii computed using a similar iterative algorithm for the case where each sensor has the same radius.

First, in order to calculate r_i^{*sim}, we use the optimal radius estimate of the optimization problem in (7) as an initial starting point in the iterative algorithm[4]. Then we iterate until we find the value of Γ such that the delay constraint is met. Fig. 4 (top) shows the TSPN tour using the set of radii found by our algorithm[5]. The transmission energy spent with this configuration is given by: $E_T^{r_i^{*sim}} = K \sum_{i=1}^{N} S_i r_i^{\alpha} \simeq 7.8$ J.

Next, in order to compute r^{*sim} using the iterative algorithm for the equal radius case, we again use the corresponding input from the optimization problem (5) as a starting point. We then run our algorithm to find the best value of the radius r^{*sim} so that the delay constraint is met. Fig. 4 (bottom) shows the TSPN tour when all sensors have the same radius r^{*sim}. The transmission energy spent using this set of constant radii is: $E_T^{r^{*sim}} = K r^{\alpha} \sum_{i=1}^{N} S_i \simeq 16.8$ J. The energy saving associated with optimizing the radii as a function of the load in the sensors is given by $\left(\frac{E_T^{r^{*sim}} - E_T^{r_i^{*sim}}}{E_T^{r^{*sim}}} \right) \cdot 100 \simeq 54\%$.

Finally, we compare our approach with a *static* approach in which every sensor in the network transmits directly to a fixed base station, located at the Euclidean one-center location. For this example, the transmission energy consumption is $E_T^{1-center} = 140.4$ J, corresponding to an energy saving of about 95%.

C. Overlapping Regions Case

In the previous section, we used the Elbassioni algorithm of [6] for non-overlapping regions to solve the TSPN problem. This algorithm has good performance in our experiments and produces results that are very close to the TSPN lower bound. Nevertheless, this algorithm requires the neighborhoods around each sensor to be disjoint. However, if the sensors' coordinates are i.i.d. over the region and if the delay constraint is arbitrarily tight, the sensors' neighborhoods may overlap. Therefore, in this section we utilize a TSPN algorithm that allows the neighborhoods around each sensor to overlap. The TSPN problem with overlapping neighborhoods of varying size is in general a more difficult problem than the case of disjoint neighborhoods and algorithms for TSPN with overlapping neighborhoods usually have worse approximation ratios [7], [18].

1) TSPN Algorithm for Overlapping Regions: There exist only a few TSPN algorithms that are applicable to the case of overlapping neighborhoods [5], [7], [18]. In [5], a constant-factor approximation algorithm for TSPN with unit disk neighborhoods was analyzed. Unfortunately, this algorithm cannot be applied to our case because the sensors have neighborhoods of different radii. Another approximation algorithm for connected polygonal regions was studied in [11] and [18]. In this section, we use the TSPN algorithm in [7] which allows different-size circular regions to intersect. The algorithm works as follows:

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[4]For the TSP tour length in (6) and (7), instead of $\beta\sqrt{NA}$, i.e., the TSP estimate for uniform node distribution, we utilize the output of the *Concorde* algorithm, i.e., 5800.

[5]Note that there are places in which the collector crosses over the sensors' disk, passing by the sensors from close distance. For these cases, we assume that the mobile collector starts communication with the sensor when it reaches the boundary of the sensor's communication range.

- Compute the minimum covering box B of the set of disks, i.e., the smallest box that hits each sensor's neighborhood at at least one point.
- Find the minimal hitting pointset inside B, i.e., the minimum number of points such that each region has exactly one hitting point inside B.
- Compute the TSP tour on the set of selected points.

In order to find the minimal hitting pointset, we first pick one point at the boundary of each disk, then, we eliminate the points that are redundant. Namely, we eliminate the points belonging to disks that contain at least one hitting point that was already selected on other disks. Note that the algorithm allows the hitting points on the regions to be chosen at random inside the box B. Nevertheless, in our context it is preferable to choose the points on the boundary of the disks in order to apply our iterative algorithm given in Fig. 3.

Note that utilizing the TSPN algorithm of [7] (via choosing points at random on the boundary of each disk and then removing redundant points) can lead to convergence issues in our iterative algorithm. We note that a better way to solve the problem is to choose the hitting points not at random, but rather via applying the basic idea of the algorithm used for the non-overlapping regions case. In particular, we start from a random point on the boundary of the smallest disk region and for every other region we select the point on the boundary that is closest to the already chosen points. Note that all these points have to lie inside the minimum bounding box B. Finally, we eliminate the redundant points in order to obtain the minimal hitting pointset, and compute the TSP tour over these points. We note that by using this heuristic approach to choose the hitting points, the algorithm converges in all the cases we have analyzed.

2) Simulation Results: We consider a square network region of size (1000x1000)m with $N = 50$ sensors distributed independently and uniformly over the region. The associated TSP tour length is 5877m. Each sensor is assigned a random number of messages $M_i \in \{1, ..., 50\}$. We utilize the TSPN algorithm in [7] in our iterative algorithm. Namely, we start with the radii proportional to the inverse of the sensors' data size as in Section IV-B. Then we iteratively increase the radii and find the corresponding TSPN tour and the corresponding transmission energy consumption for each set of radii. Note that we use (1) to compute the transmission energy, however, here r_i represents the distance between the center of the sensor i's disk and its corresponding hitting point given by the TSPN algorithm.

Fig. 5 shows the transmission energy expenditure for given maximum delay constraints both for the equal radii and optimized radii cases. We observe that the trend of the transmission energy consumption versus the maximum delay is similar to the one at the bottom of Fig. 2 obtained analytically. Furthermore, in Fig. 5 the optimized radii case always yields less energy consumption compared to the equal radii case confirming our analytical results for the non-overlapping regions case in Section IV-A. Fig. 6 (a) shows the resulting TSPN tour corresponding to the maximum travel delay value
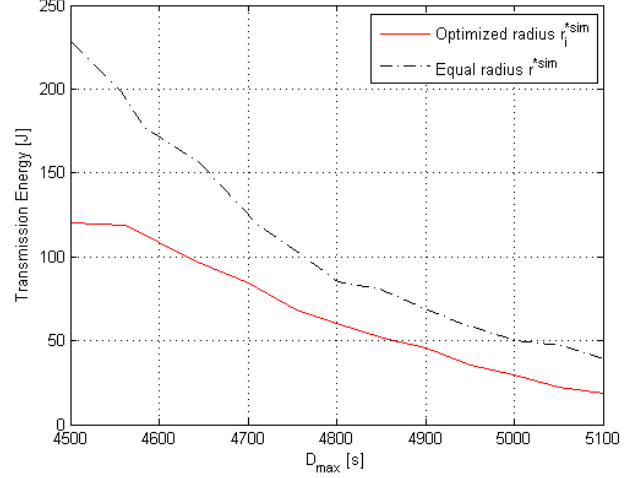

Fig. 5. Transmission energy versus the maximum delay D_{max} both for optimized radii r_i^{*sim} and equal radii r^{*sim}.

$D_{max} = 5050$ s (shorter than the TSP tour length of 5877m). The transmission energy consumption for this configuration is $E_T^{r_i^{*sim}} = 22.3$ J. Next, we run our iterative algorithm for the case of equal radii for the sensor neighborhoods and obtain $E_T^{r^{*sim}} = 47.0$ J. The energy savings associated with optimizing the radii as a function of the sensor data sizes is about 52.5%; similar to the results in the previous section.

V. MULTIPLE COLLECTORS

Now we extend our analysis to the case where sensor data gathering is performed by multiple collectors in the network. This model can be useful in large scale sensor networks in which the sensor nodes are far away from each other and the delay associated with collecting the sensors' data with a single collector is excessive. Here we note that, when the sensors are uniformly distributed over the network region, partitioning the region into equal area subregions and assigning a collector to each sub-region yields close to optimal performance [2], [14].

We show that the use of multiple collectors can substantially decrease the transmission energy consumption in cases where the delay requirement is too stringent. In particular, we consider the network configuration of the previous example and assume a very stringent delay constraint. For instance, imposing $D_{max} = 2820$ s (TSP tour is 5877 m) and running the single collector algorithm, the resulting transmission energy consumption is very high: $E_T^{r_i^{*sim}} = 5925.3$ J. In order to reduce energy consumption, one can utilize multiple collectors. For example, for the case of two collectors, we divide the region into two equal subregions and assign a collector to each subregion. We impose the same travel delay constraint for each collector ($D_{max} = 2820$ s) and we find that the total transmission energy consumption reduces to: $E_T^{r_i^{*sim}} = 7.4$ J, significantly less than the energy consumption of the single collector case.
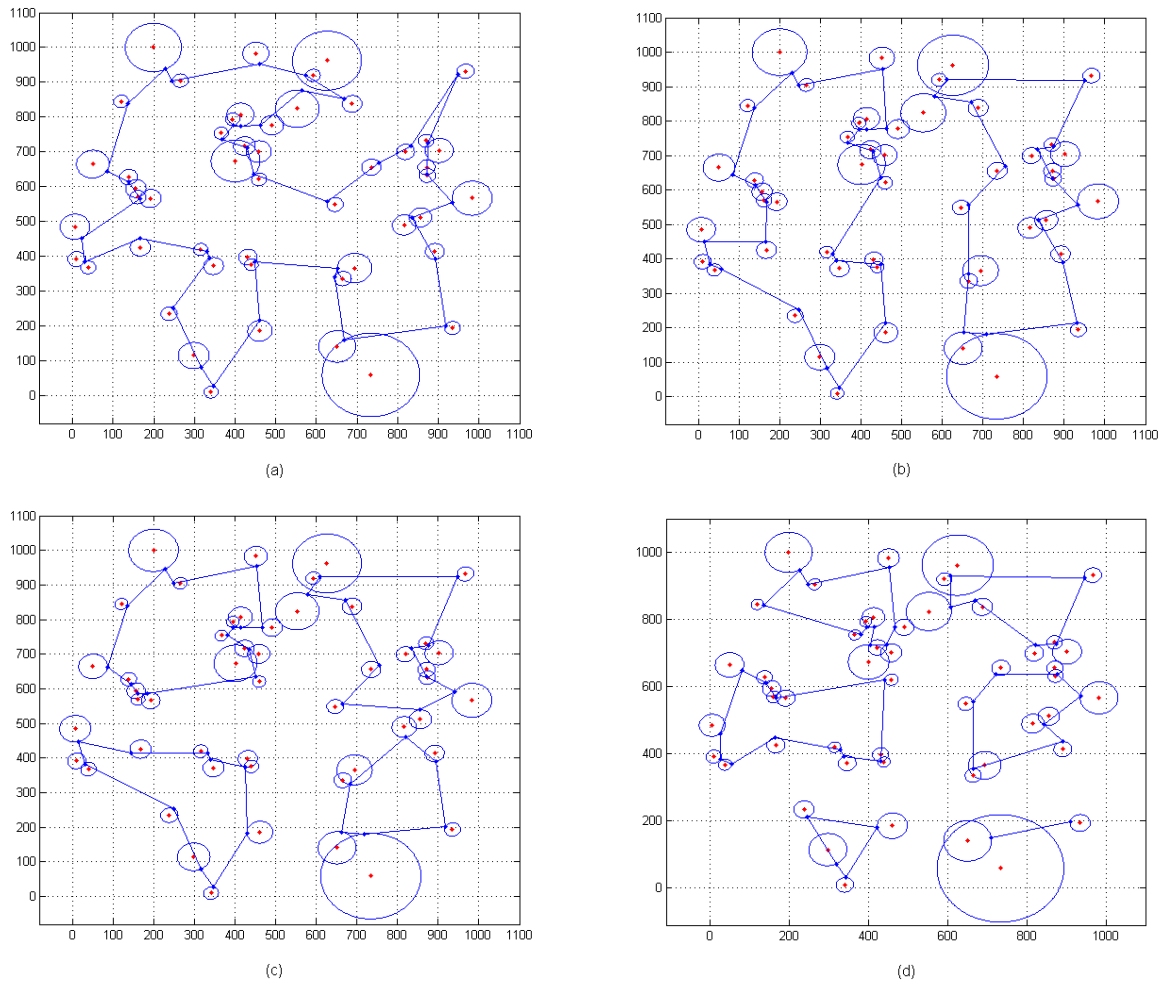
Fig. 6. Example of m-collectors TSPN tours: $m = 1$ (a), $m = 2$ (b), $m = 4$ (c), $m = 6$ (d).

Next, for the same sensor locations and data configuration as in the previous examples, we demonstrate our results for multiple collectors. Since the sensors' coordinates are i.i.d. and uniformly distributed, we partition the region into equal subregions, and assign each collector to a subregion. Then, for each subregion, we compute the total travel delay (TSPN path) such that the total transmission energy consumption in each subregion is equal to the transmission energy consumption of the one collector case divided by the number m of subregions. This ensures that the summation of the transmission energy consumption of the m-collectors case equals the transmission energy consumption of the single collector case. Fig. 6 (b) shows an example in which the total area is divided into two subregions each of which is assigned to one of the two collectors. The transmission energy for each subregion is $E_T^{r_i^{*sim}} = 22.3/2 = 11.15$ J. In Fig. 6 (c), four subregions are assigned to four collectors where the transmission energy consumption for each subregion is $E_T^{r_i^{*sim}} = 22.3/4 = 5.57$ J. Six collectors are used in the example shown in Fig. 6 (d), where the transmission energy is $E_T^{r_i^{*sim}} = 22.3/6 = 3.72$ J

per subregion. Table I reports, for different values of m, the maximum TSPN, i.e., the largest tour length among all the subregions, and the average TSPN tour of the different subregions. As expected, the increase of the number of collectors reduces the tour length drastically, demonstrating the benefit in delay associated with utilizing multiple collectors in the network.

TABLE I
MAXIMUM AND AVERAGE TSPN LENGTH USING m-COLLECTORS.

m	Max TSPN [s]	Average TSPN [s]
1	5050	5050
2	2756	2352.0
4	1562	1307.8
6	1366	883.5

VI. STOCHASTIC ARRIVALS CASE

In this section we generalize our model by assuming that the messages arrive to sensor nodes via stochastic processes of different arrival rates. Our aim is to characterize a relationship

between the message arrival rates and the transmission radii of the sensors in order to reduce the transmission energy consumption in the network. Similar to Section III, we first analytically show (for the case of two nodes) that the transmission radius has to be inversely proportional to the message arrival rate to the sensors in order to reduce the total transmission energy consumption. Then we apply this intuition to the network case and analyze an iterative algorithm similar to the one in Section IV-B but this time using arrival rates as inputs instead of the data sizes. We refer to this algorithm as the *Non-Adaptive* Algorithm. We also propose a dynamic *Adaptive* Algorithm that does not require the knowledge of the arrival rates and chooses the radii as a function of the current data size at each sensor. We show that the Adaptive Algorithm performs significantly better than the Non-Adaptive Algorithm if the variance of the message arrival process is high and performs similarly if the variance is low.

A. Queueing Model

Consider a network of two nodes with Poisson arrival processes of rates λ_1 and λ_2 bits per unit time. The sensor nodes are at a distance $2d$ apart and there is a collector gathering messages from these sensors. This system can be modeled as a *Polling System* with a mobile server performing a cyclic service of the two queues. For polling systems with cyclic service order, the Exhaustive policy, in which the server does not leave the current queue until the queue empties, is shown to be both throughput and delay optimal [16], [26]. Namely, the Exhaustive policy stabilizes the system (yields bounded expected queue sizes) whenever the arrival rates are stabilizable, and gives minimum total expected number of messages in the system. Therefore, we utilize the Exhaustive policy specifying the system as a *polling system with exhaustive and cyclic service*. Suppose each sensor transmits at a constant rate R when the collector is serving it. Then, $\rho_i = \frac{\lambda_i}{R}$ is the load at sensor i and $\rho = \rho_1 + \rho_2$ is the system load, where the condition $\rho < 1$ is necessary and sufficient for the stability of the system [26]. Furthermore, the average cycle time in such a system is given by $E[C] = \frac{2[(d-r_1)+(d-r_2)]}{v(1-\rho)}$, and the fraction of time that the collector spends serving sensor i is given by $\rho_i E[C]$ [26], [29]. Hence the average transmit energy of sensor i is given by the product of $\rho_i E[C]$ and the transmit power P_{T_i} leading to the following optimization problem[6]:

$$\min_{r_1, r_2} \quad K \frac{2\left((d-r_1)+(d-r_2)\right)\left(\rho_1 r_1^\alpha + \rho_2 r_2^\alpha\right)}{v(1-\rho)} \tag{8}$$
$$s.t. \quad 2[(d-r_1)+(d-r_2)]/v \le D_{max},$$

where $r_1, r_2 \ge 0$. This optimization problem can be solved in a similar manner to (3) to obtain,

$$r_2^* = r_1^* \left(\frac{\lambda_1}{\lambda_2}\right)^{\left(\frac{1}{\alpha-1}\right)}, \tag{9}$$

[6]Note that we still assume that the travel time dominates the transmission time, hence the constraint of the optimization problem is still given by the TSPN tour length.

where we assume that $D_{max} < 4d/v$ similar to Section III. We observe that the intuition we obtained from (4) extends to this stochastic model. Namely, it is more energy efficient to choose routes that come closer to sensors with higher arrival rate.

B. Stochastic Algorithms

Here we propose two algorithms that apply the above intuition to the general network case when there are stochastic arrivals to sensors. We assume that the data messages arrive to each sensor i according to a Poisson process with parameter λ_i [bit/s]. The two stochastic algorithms are described in the following:

- *Non-Adaptive* Algorithm: The collector's path is computed in the first TSPN cycle based on the average arrival rates λ_i. Namely, the sensors' radii are chosen inversely proportional to the arrival rates to the sensors as suggested in (9). This TSPN path remains the same for all the TSPN cycles.
- *Adaptive* Algorithm: A new path is computed for the collector at each TSPN cycle based on the total number of bits that are present at each sensor at the beginning of the corresponding cycle. Note that this algorithm does not require the knowledge of the average arrival rates λ_i.

For both of the above algorithms we adopt the ideas of Section IV. Namely, for the Non-Adaptive Algorithm we compute the optimal radii for the sensors and the collector's path by applying the iterative algorithm given in Fig. 3: we use the average arrival rates λ_i as input to the algorithm. For the Adaptive Algorithm, we again use our iterative algorithm, but this time the radii are chosen inversely proportional to the sensors' queue sizes at each TSPN cycle.

1) Simulation results: Now we compare the performances of the proposed algorithms in terms of transmission energy expenditure. We consider the same configuration as in Fig. 6 (a). We first assume that the data messages arrive to each sensor i according to a Poisson process with parameter λ_i. In particular, we choose the average arrival rates for sensors randomly in the range $\lambda_i \in \{3, ..., 119\}$ [bit/s] in order to obtain results comparable to the case of Fig. 6 (a).

We run the two stochastic algorithms for several TSPN cycles, and compute the average of the transmission energy consumption by averaging over the total number of TSPN cycles. Fig. 7 (top) shows the average transmission energy versus the delay constraint D_{max} for both algorithms. We observe that the performances of the two algorithms are very similar, meaning that the Adaptive Algorithm performs well without the knowledge of the average arrival rates.

Next we investigate the performance of the two algorithms under an arrival process with a higher variance than the Poisson process. In particular, we consider the Bernoulli arrival process for each sensor, i.e., the amount of bits per cycle arriving to a sensor can take only two values, A_1 and A_2, with probabilities p_1 and p_2, respectively. In this model, the average arrival rate per cycle is given as: $\lambda_i = A_1 p_1 + A_2 p_2$ for all i.
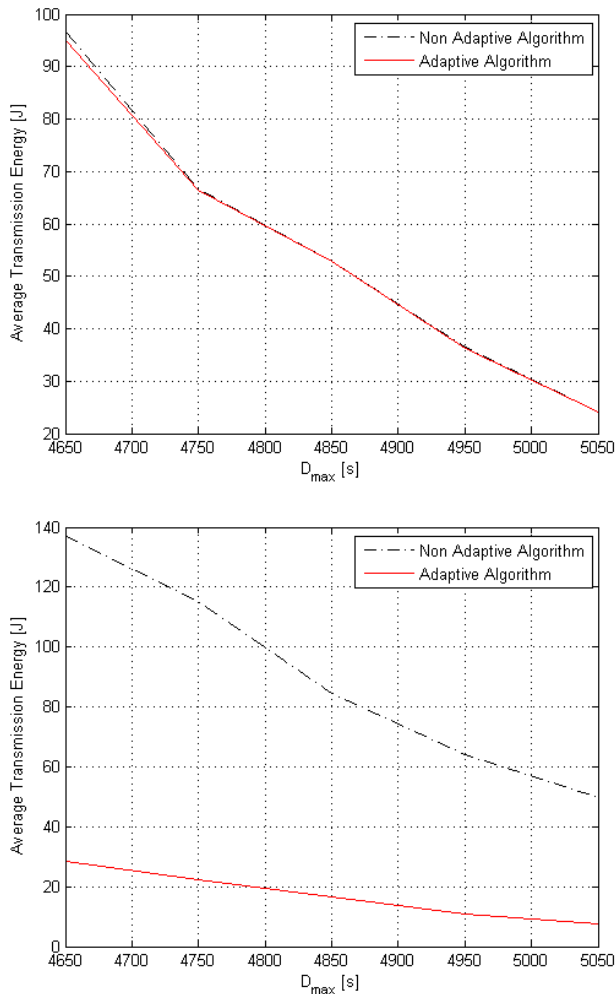
Fig. 7. Average transmission energy versus the delay constraint D_{max} for the Poisson arrival case (top) and for the Bernoulli case (bottom), considering both the stochastic algorithms.

For the following numerical example we consider $A_1 = 15$ Kbits and $A_2 = 60$ Kbits and $p_1 = p_2 = 0.5$. This ensures a scenario with a high variance for the amount of data present at each sensor for every cycle. Note that with this choice of the parameters, the radii are equal for all sensors for the Non-Adaptive Algorithm. We compute the average transmission energy consumption under the two algorithms for different delay constraints. The results are shown in Fig. 7 (bottom). We observe that the Adaptive Algorithm significatively outperforms the Non-Adaptive Algorithm, with energy savings of about 80%. This confirms our intuition that dynamically choosing sensors' radii inversely proportional to the amount of data at the sensors can lead to very significant transmission energy savings.

VII. Conclusion

In this paper we propose a novel use of mobile elements with controlled mobility for data collection in WSNs in order to improve the energy efficiency of such networks. We analytically show that given a travel delay constraint, choosing paths that come closer to sensors with more data to transmit yields significantly lower transmission energy expenditure than the energy expenditure of the approaches that use the same transmit distance for all the sensors. We demonstrated this idea in various different settings. In particular, we proposed a heuristic algorithm that exploits the results of the optimization problem in order to find the best set of sensor radii that minimize the transmission energy expenditure. When the sensors have arbitrary amount of data to transmit to the mobile collectors, our results show that energy savings of over 50% are achievable. Furthermore, we generalized our model to the case of stochastic arrivals to the sensors. In particular, we proposed two algorithms to compute the sensors' radii in case of stochastic arrivals. A *Non-Adaptive* Algorithm that chooses sensors' radii based on the average arrival rates, and an *Adaptive* Algorithm that computes the radii based on sensors' current queue sizes. We show that the Adaptive Algorithm, that does not require the knowledge of arrival rates, performs as good as the Non-Adaptive Algorithm in all the cases we considered and outperforms the Non-Adaptive Algorithm by achieving energy savings of about 80% when the arrival process has high variance.

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