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Benchmarking the (1,4)-CMA-ES With Mirrored Sampling and Sequential Selection on the Noisy BBOB-2010 Testbed

[Black-Box Optimization Benchmarking Workshop]

Anne Auger, Dimo Brockhoff, and Nikolaus Hansen
Projet TAO, INRIA Saclay—Ile-de-France
LRI, Bât 490, Univ. Paris-Sud
91405 Orsay Cedex, France
firstname.lastname@inria.fr

ABSTRACT

The Covariance-Matrix-Adaptation Evolution-Strategy (CMA-ES) is a robust stochastic search algorithm for optimizing functions defined on a continuous search space \mathbb{R}^D . Recently, mirrored samples and sequential selection have been introduced within CMA-ES to improve its local search performances. In this paper, we benchmark the $(1,4_m^s)$ -CMA-ES which implements mirrored samples and sequential selection on the BBOB-2010 noisy testbed. Independent restarts are conducted until a maximal number of $10^4 D$ function evaluations is reached.

Although the tested $(1,4_m^s)$ -CMA-ES is only a local search strategy, it solves 8 of the noisy BBOB-2010 functions in 20D and 9 of them in 5D for a target of 10^{-8} . There is also one additional function in 20D and 5 additional functions in 5D where a successful run for at least one of the 15 instances can be reported. Moreover, on 7 of the 8 functions that are solved by the $(1,4_m^s)$ -CMA-ES in 20D, we see a large improvement over the best algorithm of the BBOB-2009 benchmarking for the corresponding functions—ranging from an 37% improvement on the sphere with moderate Cauchy noise to a speed-up by a factor of about 3 on the Gallagher function with Cauchy noise.

Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization

1. INTRODUCTION

Evolution Strategies (ESs) are stochastic search algorithms designed to minimize¹ objective functions, f , mapping a continuous search space \mathbb{R}^D into \mathbb{R} . Among ESs, the Covariance-Matrix-Adaptation Evolution-Strategy (CMA-ES) is now a well recognized algorithm. In the standard $(\mu/\mu_w, \lambda)$ -CMA-ES [18, 25], at each iteration step n , a set of λ candidate solutions is created by sampling random vectors distributed according to a multivariate normal distribution with mean vector zero and covariance matrix \mathbf{C}_n . Those λ random vectors denoted $(\mathcal{N}_i(\mathbf{0}, \mathbf{C}_n))_{1 \leq i \leq \lambda}$ are multiplied by a strictly positive factor, the step-size σ_n , and added to the current solution \mathbf{X}_n to constitute the offspring $\mathbf{X}_n^i = \mathbf{X}_n + \sigma_n \mathcal{N}_i(\mathbf{0}, \mathbf{C}_n)$. After evaluation of the λ offspring, the μ best, i.e., the ones having the smallest objective function values, are selected. The current solution is updated to the average value of the μ best solutions: $\mathbf{X}_{n+1} = \sum_{i=1}^{\mu} w_i \mathbf{X}_n^{i:\lambda}$, where $w_1 \geq \dots \geq w_\mu$ and $\sum_{i=1}^{\mu} w_i = 1$ and $\mathbf{X}_n^{i:\lambda}$ denotes the i -th best offspring. Covariance matrix and step-size are then updated using solely the information given by the ranking of the offspring. Though originally designed to be a robust local search [26], the $(\mu/\mu_w, \lambda)$ -CMA-ES turns out to be also effective for multi-modal functions provided a large enough population size $\mu = \lambda/2$ is chosen [25]. An automatic way to increase the probability to converge on multi-modal functions consists in applying restarts with a successively increasing population size. The strategy is then called IPOP-CMA-ES [13]. However, deceptive functions were constructed for the IPOP-CMA-ES [27, 23]. The BBOB function f_{24} presents, on the larger scale an attraction region for the global optimum which is smaller than the one for the local optimum. For that reason, the BIPOP-CMA-ES, combining restarts with increasing population size as well as with a fixed small population size, was proposed [19]. For the large budgets that are needed for most multi-modal problems, the BIPOP-CMA-ES performed overall best for the BBOB-2009 workshop [22].

While BIPOP-CMA-ES was benchmarked, the local search $(1+1)$ -CMA-ES was as well tested [14, 15]. Surprisingly, the $(1+1)$ -variant of CMA-ES could outperform the BIPOP-CMA-ES algorithm by a significant factor on the Gallagher functions f_{21} and f_{22} [11]. On f_{21} , the $(1+1)$ -CMA-ES is 8.2 times (resp. 68.7 times) faster than the BIPOP-CMA-

¹We assume without loss of generality minimization since maximizing f amounts to minimize $-f$.

ES in dimension 20 (resp. 40); for f_{22} , the (1+1)-CMA-ES is 37 times faster than the BIPOP-CMA-ES in 20D and is able to solve the problem in 40D which the BIPOP-CMA-ES does not allow. However, one major drawback of elitist selection, used in the (1+1)-CMA-ES, is the complete lack of robustness in presence of noise [15].

Motivated by the surprisingly large improvement over the BIPOP-CMA, new *non-elitist* local search ESs have been proposed [6]. Those $(1, \lambda)$ -ESs combine a derandomization technique by means of *mirrored samples* with a *sequential selection* scheme. Mirrored samples replace the independent random vectors used for the offspring. Instead of the λ independent random vectors, only $\lambda/2$ (assuming λ is even) independent samples are generated $(\mathcal{N}_{2i-1}(\mathbf{0}, \mathbf{C}_n))_{1 \leq i \leq \lambda/2}$. The other $\lambda/2$ samples are replaced by the already generated samples multiplied by -1 , i.e., $\mathcal{N}_{2i}(\mathbf{0}, \mathbf{C}_n) = -\mathcal{N}_{2i-1}(\mathbf{0}, \mathbf{C}_n)$ for all $1 \leq i \leq \lambda/2$. The resulting offspring are two by two symmetrical or *mirrored* with respect to \mathbf{X}_n . Sequential selection consists in performing the evaluations of the λ offspring sequentially and comparing after each evaluation the offspring solution \mathbf{X}_n^i with the current solution \mathbf{X}_n . If $f(\mathbf{X}_n^i) \leq f(\mathbf{X}_n)$, the sequence of evaluations is stopped and $\mathbf{X}_{n+1} = \mathbf{X}_n^i$, saving thus the remaining offspring evaluations.

The impact of mirrored samples and sequential selection has been investigated on the BBOB-2010 for the (1,2)-CMA-ES [2, 3, 7, 8] and for the (1,4)-CMA-ES [4, 5, 9, 10]. The purpose of this paper is to present the results of one of those strategies tested, namely the (1,4)-CMA-ES with mirrored samples and sequential selection on the BBOB-2010 noisy testbed. Since the algorithm tested is a local search strategy, we do not expect that it will perform well on the whole testbed but rather want to see whether the strategy can bring some improvements over last year's results on *certain* functions.

The tested algorithm $(1,4_m^s)$ -CMA-ES as well as the CPU timing experiments are described in a complementing paper in the same proceedings [1].

2. RESULTS AND DISCUSSION

Results from experiments according to [21] on the benchmark functions given in [16, 24] are presented in Figures 1, 2 and 3 and in Tables 1, 2 and 3.

Although the tested $(1,4_m^s)$ -CMA-ES is only a local search strategy, it solves 8 of the noisy BBOB-2010 functions in 20D and 9 of them in 5D for a target of 10^{-8} . In addition, there is one function in 20D and 5 functions in 5D where a successful run for at least one of the 15 instances can be reported. In the light of this result, it is worth to mention that the noisy test functions in the BBOB-2009 testbed² have not been solved as successfully as the noiseless ones: 9 out of the 30 functions could not be solved by any algorithm or solely by the BIPOP-CMA-ES of [20], see [12] for details about the BBOB-2009 results. Moreover, on 7 of the 8 functions that are solved by $(1,4_m^s)$ -CMA-ES in 20D, we see even an improvement over the function-wise best algorithm of the BBOB-2009 benchmarking which we detail below.

On the sphere function with moderate noise (f_{103}), the $(1,4_m^s)$ -CMA-ES is about 35% better than the best algo-

rithm for this function in 20D and for a target of 10^{-7} . For all other targets, the $(1,4_m^s)$ -CMA-ES also outperforms the best algorithm for this function. The best algorithm of the BBOB-2009 benchmarking on this function turns out to be the IPOP-SEP-CMA-ES [28] for small, i.e., difficult targets.

On the Rosenbrock function with moderate Cauchy noise (f_{106}), the $(1,4_m^s)$ -CMA-ES outperforms the best BBOB-2009 algorithm for this function in all dimensions (2D, 3D, 5D, 10D, and 20D) and for all small targets which also here is the IPOP-SEP-CMA-ES [28]. The expected running time of the $(1,4_m^s)$ -CMA-ES is thereby about 40% smaller than for the IPOP-SEP-CMA-ES for a target value of 10^{-7} in 20D.

Also the sphere function with Cauchy noise (f_{109}) is solved faster by the $(1,4_m^s)$ -CMA-ES than the best algorithm of the BBOB-2009 benchmarking on this function in 5D, 10D, and 20D where the improvement is about 50% in 20D and where both the IPOP-SEP-CMA-ES and the BIPOP-CMA-ES [20] are the best algorithms of BBOB-2009 on this function.

A 50% improvement can also be seen on the Rosenbrock function with Cauchy noise (f_{112}) in 20D and for small targets. Better results than the IPOP-SEP-CMA-ES, the best algorithm for this function in BBOB-2009, are also obtained in 2D, 3D, 5D and 10D here.

The improvement over the best algorithm of BBOB-2009 on the ellipsoid function with Cauchy noise (f_{118}) is about 40% in 20D (for all small targets). The best algorithms in BBOB-2009 on this function are the IPOP-SEP-CMA-ES (for a target value of 10^{-7}) as well as the algorithm VNC [17] (for all target values). Better results on f_{118} than the best algorithm of BBOB-2009 are also obtained in 2D, 3D, 5D, and 10D.

On the sum of different powers function comprising Cauchy noise (f_{121}), the $(1,4_m^s)$ -CMA-ES shows expected running times that are at least 48% lower than the best algorithm of BBOB-2009 on this function in 20D and for small target values. Better results than the best BBOB-2009 algorithm on this function in 2D, 3D, 5D, and 10D can also be reported.

Last, also on the Gallagher function with Cauchy noise (f_{130}), an improvement over the best algorithm of BBOB-2009 on this function can be seen in the results. Here, we see the largest impact of the $(1,4_m^s)$ -CMA-ES, where the expected running times are, in 20D and for several small targets, by a factor of about 3 smaller than the ones of the best algorithm of BBOB-2009 on this function. Also the results in 2D, 3D, 5D, and 10D are better for the $(1,4_m^s)$ -CMA-ES where the improvement factor only slightly differs in 3D, 5D, and 10D. At least for small targets and dimension ≥ 5 , the IPOP-SEP-CMA-ES is here also the best algorithm of BBOB-2009.

Note that all functions, where an improvement over the best algorithm of the BBOB-2009 benchmarking can be reported, comprise a Cauchy noise. Cauchy noise is only sampled 20% of the time, such that it is enough to be robust to positive and negative outliers for solving those functions. For the other noise types, most probably a larger population size or another method to cope with the noise is needed. Furthermore, the maximum number of function evaluations was chosen quite small for solving the more difficult noise types up to the final target value.

²These are the same functions than in the BBOB-2010 testbed with the only difference that instead of 15 instances per function, three independent runs were performed on 5 different instances within BBOB-2009.

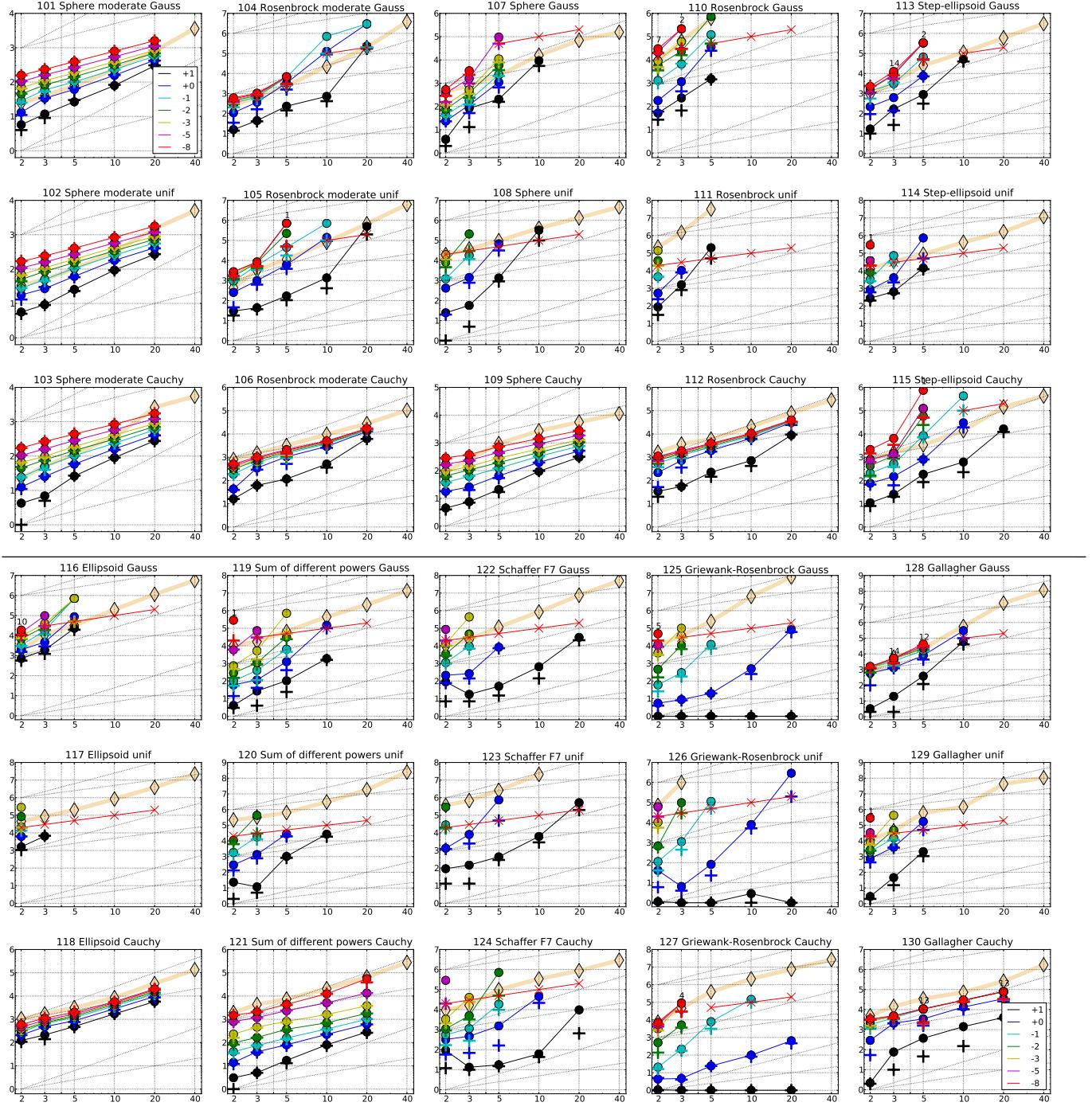


Figure 1: Expected Running Time (ERT, ●) to reach $f_{\text{opt}} + \Delta f$ and median number of f -evaluations from successful trials (+), for $\Delta f = 10^{\{+1,0,-1,-2,-3,-5,-8\}}$ (the exponent is given in the legend of f_{101} and f_{130}) versus dimension in log-log presentation. For each function and dimension, $\text{ERT}(\Delta f)$ equals to $\#\text{FEs}(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\text{opt}} + \Delta f$ was surpassed. The $\#\text{FEs}(\Delta f)$ are the total number (sum) of f -evaluations while $f_{\text{opt}} + \Delta f$ was not surpassed in the trial, from all (successful and unsuccessful) trials, and f_{opt} is the optimal function value. Crosses (×) indicate the total number of f -evaluations, $\#\text{FEs}(-\infty)$, divided by the number of trials. Numbers above ERT-symbols indicate the number of successful trials. Y-axis annotations are decimal logarithms. The thick light line with diamonds shows the single best results from BBOB-2009 for $\Delta f = 10^{-8}$. Additional grid lines show linear and quadratic scaling.

Δf	f101 in 5-D, N=15, mFE=450				f101 in 20-D, N=15, mFE=1823				Δf	f102 in 5-D, N=15, mFE=524				f102 in 20-D, N=15, mFE=2063							
	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}						
10	15	2.7e1	3.0e0	4.6e1	2.7e1	15	3.0e2	2.5e2	3.5e2	3.0e2	10	15	2.5e1	7.0e0	4.8e1	2.5e1	15	2.7e2	2.2e2	3.2e2	2.7e2
1	15	6.1e1	2.7e1	8.2e1	6.1e1	15	4.3e2	3.7e2	5.0e2	4.3e2	1	15	6.1e1	3.3e1	8.0e1	6.1e1	15	4.2e2	3.6e2	4.7e2	4.2e2
1e-1	15	1.0e2	7.2e1	1.2e2	1.0e2	15	5.6e2	4.9e2	6.6e2	5.6e2	1e-1	15	1.1e2	8.2e1	1.3e2	1.1e2	15	5.7e2	4.7e2	6.8e2	5.7e2
le-3	15	1.8e2	1.5e2	2.0e2	1.8e2	15	8.4e2	7.7e2	9.3e2	8.4e2	le-3	15	1.9e2	1.5e2	2.1e2	1.9e2	15	8.9e2	7.8e2	1.0e3	8.9e2
le-5	15	2.7e2	2.1e2	3.1e2	2.7e2	15	1.1e3	1.1e3	1.2e3	1.1e3	le-5	15	2.7e2	2.3e2	3.0e2	2.7e2	15	1.2e3	1.1e3	1.3e3	1.2e3
le-8	15	3.9e2	3.7e2	4.5e2	3.9e2	15	1.6e3	1.4e3	1.7e3	1.6e3	le-8	15	4.1e2	3.5e2	5.0e2	4.1e2	15	1.8e3	1.6e3	1.9e3	1.8e3
	f103 in 5-D, N=15, mFE=521				f103 in 20-D, N=15, mFE=1917					f104 in 5-D, N=15, mFE=25439				f104 in 20-D, N=15, mFE=200004							
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}						
10	15	2.6e1	2.0e0	4.5e1	2.6e1	15	2.8e2	2.0e2	3.8e2	2.8e2	10	15	2.3e2	7.3e1	8.7e2	2.3e2	9	2.3e5	1.5e4	4.4e5	9.8e4
1	15	5.8e1	3.1e1	8.9e1	5.8e1	15	4.1e2	3.5e2	5.0e2	4.1e2	1	15	3.3e3	5.0e2	8.3e3	3.3e3	1	3.0e6	4.7e5	5.2e6	1.7e5
1e-1	15	1.0e2	8.3e1	1.3e2	1.0e2	15	5.8e2	4.8e2	6.8e2	5.8e2	1e-1	15	6.4e3	8.6e2	1.7e4	6.4e3	1	3.0e6	3.7e5	6.0e6	1.7e5
le-3	15	1.9e2	1.4e2	2.4e2	1.9e2	15	8.8e2	7.9e2	9.8e2	8.8e2	le-3	15	6.7e3	1.2e2	2.0e4	6.7e3	0	9le-1	19e-1	12e+0	1.1e5
le-5	15	2.8e2	2.5e2	3.2e2	2.8e2	15	1.2e3	1.0e3	1.4e3	1.2e3	le-5	15	6.8e3	1.8e2	2.0e4	6.8e3
le-8	15	4.4e2	3.7e2	4.7e2	4.4e2	15	1.7e3	1.6e3	1.9e3	1.7e3	le-8	15	7.0e3	1.5e3	2.0e4	7.0e3
	f105 in 5-D, N=15, mFE=50004				f105 in 20-D, N=15, mFE=200004					f106 in 5-D, N=15, mFE=4228				f106 in 20-D, N=15, mFE=34780							
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}						
10	15	1.7e2	8.3e1	3.3e2	1.7e2	15	5.0e5	7.5e4	1.2e6	1.0e5	10	15	1.2e2	7.6e1	1.9e2	1.2e2	15	6.3e3	3.6e3	8.5e3	6.3e3
1	15	6.2e3	2.1e2	1.2e4	6.2e3	0	13e+0	9le-1	16e+0	8.5e4	1	15	1.0e3	3.3e2	2.8e3	1.0e3	15	1.3e4	4.2e3	1.7e4	1.3e4
1e-1	15	4.5e4	3.4e3	1.1e5	1.2e4	1e-1	15	1.5e3	7.4e2	3.1e3	1.5e3	15	1.4e4	6.6e3	1.9e4	1.4e4
le-3	1	7.2e5	6.7e4	1.4e6	1.7e4	le-3	15	1.8e3	1.0e3	3.5e3	1.8e3	15	1.6e4	8.0e3	2.0e4	1.6e4
le-5	1	7.2e5	6.7e4	1.4e6	1.7e4	le-5	15	2.0e3	1.2e3	3.6e3	2.0e3	15	1.6e4	8.5e3	2.1e4	1.6e4
le-8	1	7.2e5	6.7e4	1.5e6	1.7e4	le-8	15	2.1e3	1.4e3	3.7e3	2.1e3	15	1.7e4	9.1e3	2.1e4	1.7e4
	f107 in 5-D, N=15, mFE=50004				f107 in 20-D, N=15, mFE=200004					f108 in 5-D, N=15, mFE=50004				f108 in 20-D, N=15, mFE=200004							
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}						
10	15	2.0e2	1.1e2	5.1e2	2.0e2	0	56e+0	40e+0	76e+0	1.5e5	10	15	1.3e3	1.1e1	3.2e3	1.3e3	0	95e+0	75e+0	11e+1	1.4e5
1	15	1.1e3	1.5e2	3.2e3	1.1e3	1	8	6.8e4	2.4e4	1.7e5	2.4e4
1e-1	15	2.3e3	3.1e2	5.5e3	2.3e3	1e-1	0	94e-2	34e-2	21e-1	2.4e4
le-3	15	1.1e4	6.8e2	2.5e4	1.1e4	le-3	
le-5	6	9.6e4	1.8e4	2.4e5	2.1e4	le-5	
le-8	0	15e-6	59e-8	30e-5	2.0e4	le-8	
	f109 in 5-D, N=15, mFE=975				f109 in 20-D, N=15, mFE=3250					f110 in 5-D, N=15, mFE=50004				f110 in 20-D, N=15, mFE=200004							
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}						
10	15	2.1e1	6.0e0	3.0e1	2.1e1	15	3.0e2	2.4e2	3.8e2	3.0e2	10	15	1.5e3	9.3e1	3.1e3	1.5e3	0	55e+3	44e+3	11e+4	8.1e4
1	15	6.5e1	3.7e1	9.2e1	6.5e1	15	5.1e2	4.1e2	6.1e2	5.1e2	1	11	3.7e4	3.0e3	8.4e4	1.8e4
1e-1	15	1.2e2	7.3e1	1.8e2	1.2e2	15	7.6e2	6.0e2	8.6e2	7.6e2	1e-1	5	1.2e5	3.0e3	2.8e5	2.1e4
le-3	15	2.8e2	2.3e2	3.4e2	2.8e2	15	1.3e3	1.2e3	1.4e3	1.3e3	le-3	0	28e-2	30e-3	16e-1	3.0e4
le-5	15	4.6e2	3.8e2	5.9e2	4.6e2	15	1.9e3	1.7e3	2.3e3	1.9e3	le-5	
le-8	15	7.2e2	5.6e2	8.6e2	7.2e2	15	2.7e3	2.5e3	2.9e3	2.7e3	le-8	15	4.0e3	2.4e3	5.8e3	4.0e3	15	3.8e4	2.5e4	5.7e4	3.8e4
	f111 in 5-D, N=15, mFE=50004				f111 in 20-D, N=15, mFE=200004					f112 in 5-D, N=15, mFE=6622				f112 in 20-D, N=15, mFE=64245							
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}						
10	3	2.0e5	3.9e3	5.0e5	3.3e3	0	70e+3	56e+3	11e+4	8.9e4	10	15	2.3e3	9.1e1	3.0e2	2.3e2	15	8.9e3	3.9e3	1.3e4	8.9e3
1	0	22e+0	80e-1	95e+0	2.3e4	1	15	1.9e5	5.5e4	4.0e3	1.9e3	15	3.1e4	1.4e4	5.1e4	3.1e4
1e-1	1e-1	15	2.8e3	1.1e3	4.6e3	2.8e3	15	3.5e4	2.1e3	5.3e4	3.5e4
le-3	le-3	0	3.4e-2	30e-3	16e-1	3.0e4	15	3.7e4	2.3e4	5.6e4	3.7e4
le-5	le-5	5	3.7e3	2.2e3	5.4e3	3.7e3	15	3.8e4	2.5e4	5.7e4	3.8e4
le-8	le-8	15	4.0e3	2.4e3	5.8e3	4.0e3	15	4.0e4	2.2e4	5.8e4	4.0e4
	f113 in 5-D, N=15, mFE=50004				f113 in 20-D, N=15, mFE=200004					f114 in 5-D, N=15, mFE=50003				f114 in 20-D, N=15, mFE=200004							
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}						
10	15	8.9e2	7.8e1	1.8e3	8.9e2	0	18e+1	12e+1	23e+1	1.4e5	10	13	2.3e4	8.5e2	5.3e4	1.5e4	0	23e+3	19e+3	33e+3	5.0e4
1	15	7.5e3	5.4e2	1.4e4	7.5e3	1	6	8.7e4	4.3e3	1.6e5	1.2e4
1e-1	8	6.6e4	7.7e3	2.0e5	2.2e4	1e-1	1	7.2e5	1.2e5	1.9e6	1.5e4
le-3	2	3.4e5	5.8e4	8.6e5	1.1e4	le-3	5	2.2e5	2.4e4	5.6e4	3.4e3
le-5	5	3.4e5	6.4e4	8.4e5	1.1e4	le-5	5	3.7e3	2.2e3	5.4e3	3.7e3
le-8	2	3.4e5	1.5e4	8.4e5	1.4e4	le-8	15	4.0e3	2.4e3	5.8e3	4.0e3	15	4.0e4	2.2e4	5.8e4	4.0e4
	f115 in 5-D, N=15, mFE=50004				f115 in 20-D, N=15, mFE=200004					f116 in 5-D, N=15, mFE=50004				f116 in 20-D, N=15, mFE=200004							
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}						
10	15	1.8e2	4.7e1	4.1e2	1.8e2	15	1.7e4	4.6e3	4.9e4	1.7e4	10	13	2.3e4	8.5e2	5.3e4	1.5e4	0	23e+3	19e+3	33e+3	

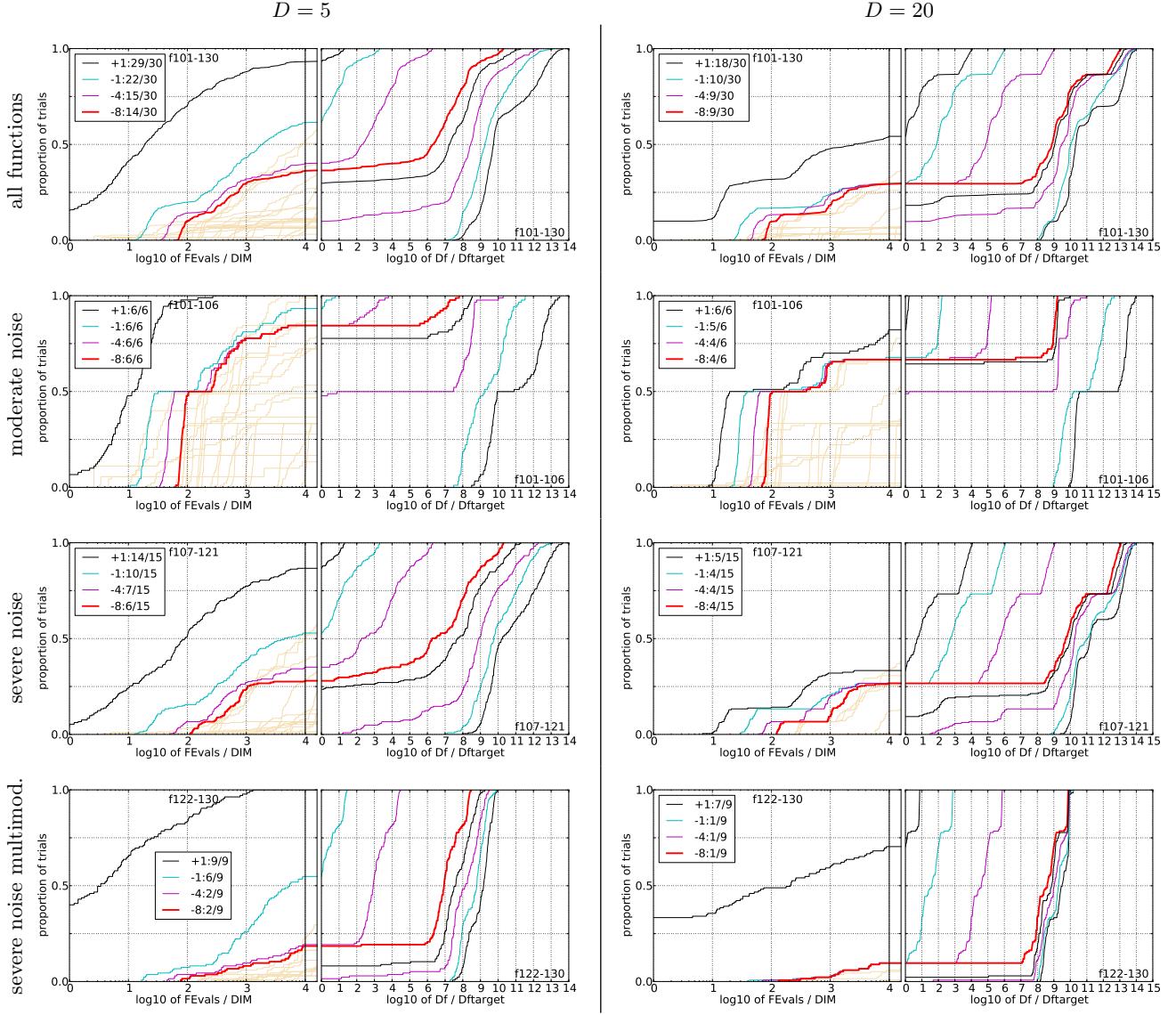


Figure 2: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus Δf (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D , to fall below $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10D, 100D \dots$ function evaluations (from right to left cycling black-cyan-magenta). The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and **DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value. Light brown lines in the background show ECDFs for target value 10^{-8} of all algorithms benchmarked during BBOB-2009.**

Δf	f121 in 5-D, N=15, mFE=7754	f121 in 20-D, N=15, mFE=185451	f122 in 5-D, N=15, mFE=50004	f122 in 20-D, N=15, mFE=200004
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 1.7e1 3.0e0 3.3e1 1.7e1	15 2.8e2 2.0e2 3.8e2 2.8e2	15 5.0e1 7.0e0 4.8e1 5.0e1	15 3.0e4 2.2e3 7.7e4 3.0e4
1	15 8.0e1 2.6e1 1.0e2 8.0e1	15 6.1e2 4.9e2 7.3e2 6.1e2	15 8.3e3 1.7e3 1.9e4 8.3e3	0 75e-1 63e-1 85e-1 8.2e4
1e-1	15 1.6e2 1.0e2 2.1e2 1.6e2	15 1.0e3 8.1e2 1.1e3 1.0e3	0 35e-2 18e-2 44e-2 3.8e4	.
1e-3	15 8.1e2 5.6e2 1.2e3 8.1e2	15 3.8e3 3.2e3 4.5e3 3.8e3	.	.
1e-5	15 2.3e3 1.7e3 3.0e3 2.3e3	15 1.3e4 1.2e4 1.6e4 1.3e4	.	.
1e-8	15 4.4e3 3.2e3 5.2e3 4.4e3	15 5.5e4 3.4e4 1.2e5 5.5e4	.	.
	f123 in 5-D, N=15, mFE=50004	f123 in 20-D, N=15, mFE=200004	f124 in 5-D, N=15, mFE=50004	f124 in 20-D, N=15, mFE=200004
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 4.3e2 2.0e0 7.4e2 4.3e2	5 5.2e5 9.6e4 1.3e6 1.2e5	10 1.7e1 1.0e0 3.3e1 1.7e1	15 9.7e3 1.7e2 5.1e4 9.7e3
1	1 7.4e5 9.4e4 1.8e6 4.4e4	0 1e+0 92e-1 14e+0 9.7e4	1 1.5e3 8.0e1 6.7e3 1.5e3	0 40e-1 24e-1 49e-1 1.3e5
1e-1	0 18e-1 16e-1 23e-1 2.6e4	.	.	.
1e-3
1e-5
1e-8
	f125 in 5-D, N=15, mFE=50004	f125 in 20-D, N=15, mFE=200004	f126 in 5-D, N=15, mFE=50004	f126 in 20-D, N=15, mFE=200003
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 1.0e0 1.0e0 1.0e0 1.0e0	15 1.0e0 1.0e0 1.0e0 1.0e0	10 1.0e0 1.0e0 1.0e0 1.0e0	15 1.0e0 1.0e0 1.0e0 1.0e0
1	15 2.0e1 4.5e0 3.9e1 2.0e1	14 8.2e4 2.8e3 1.4e5 6.8e4	1 8.2e1 1.0e0 4.4e2 8.2e1	1 2.9e6 3.2e5 7.2e6 1.2e5
1e-1	14 1.2e4 5.5e2 2.8e4 8.5e3	0 95e-2 82e-2 98e-2 9.8e4	1 1.1e5 6.8e3 2.4e5 1.4e4	0 13e-1 11e-1 15e-1 9.5e4
1e-3	0 54e-3 29e-3 94e-3 1.4e4	.	.	.
1e-5
1e-8
	f127 in 5-D, N=15, mFE=50004	f127 in 20-D, N=15, mFE=200004	f128 in 5-D, N=15, mFE=50001	f128 in 20-D, N=15, mFE=200004
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 1.0e0 1.0e0 1.0e0 1.0e0	15 1.0e0 1.0e0 1.0e0 1.0e0	10 3.8e2 2.8e1 8.6e2 3.8e2	0 65e+0 43e+0 69e+0 9.2e4
1	15 2.4e1 3.0e0 4.7e1 2.4e1	15 6.5e2 3.2e2 1.3e3 6.5e2	1 7.9e3 2.0e2 2.0e4 7.9e3	.
1e-1	14 7.8e3 1.6e3 1.1e4 4.2e3	0 23e-2 13e-2 53e-2 1.4e5	1 1.7e4 8.3e1 3.8e4 1.7e4	.
1e-3	0 42e-3 24e-3 90e-3 1.6e4	.	.	.
1e-5
1e-8
	f129 in 5-D, N=15, mFE=50004	f129 in 20-D, N=15, mFE=200004	f130 in 5-D, N=15, mFE=50003	f130 in 20-D, N=15, mFE=200003
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.1e3 3.8e1 5.7e3 2.1e3	0 72e+0 68e+0 74e+0 8.9e4	10 3.7e2 1.7e1 1.6e3 3.7e2	15 4.1e3 4.4e2 1.2e4 4.1e3
1	4 1.7e5 4.1e4 3.9e5 3.4e4	.	.	15 3.4e3 8.3e1 1.6e4 3.4e3
1e-1	0 19e-1 39e-2 25e-1 2.3e4	.	.	15 3.5e4 5.1e2 8.9e4 3.5e4
1e-3	.	.	.	13 1.1e4 1.0e2 5.1e4 2.9e3
1e-5	.	.	.	13 7.8e4 4.6e3 1.3e5 4.7e4
1e-8	.	.	.	13 7.8e4 5.1e3 1.4e5 4.7e4
	f131 in 5-D, N=15, mFE=50003	f131 in 20-D, N=15, mFE=200003	f132 in 5-D, N=15, mFE=50003	f132 in 20-D, N=15, mFE=200003
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.1e3 3.8e1 5.7e3 2.1e3	0 72e+0 68e+0 74e+0 8.9e4	10 3.7e2 1.7e1 1.6e3 3.7e2	15 4.1e3 4.4e2 1.2e4 4.1e3
1	4 1.7e5 4.1e4 3.9e5 3.4e4	.	.	15 3.4e3 8.3e1 1.6e4 3.4e3
1e-1	0 19e-1 39e-2 25e-1 2.3e4	.	.	15 3.5e4 5.1e2 8.9e4 3.5e4
1e-3	.	.	.	13 1.1e4 1.0e2 5.1e4 2.9e3
1e-5	.	.	.	13 7.8e4 4.6e3 1.3e5 4.7e4
1e-8	.	.	.	13 7.8e4 5.1e3 1.4e5 4.7e4
	f133 in 5-D, N=15, mFE=50003	f133 in 20-D, N=15, mFE=200003	f134 in 5-D, N=15, mFE=50003	f134 in 20-D, N=15, mFE=200003
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.1e3 3.8e1 5.7e3 2.1e3	0 72e+0 68e+0 74e+0 8.9e4	10 3.7e2 1.7e1 1.6e3 3.7e2	15 4.1e3 4.4e2 1.2e4 4.1e3
1	4 1.7e5 4.1e4 3.9e5 3.4e4	.	.	15 3.4e3 8.3e1 1.6e4 3.4e3
1e-1	0 19e-1 39e-2 25e-1 2.3e4	.	.	15 3.5e4 5.1e2 8.9e4 3.5e4
1e-3	.	.	.	13 1.1e4 1.0e2 5.1e4 2.9e3
1e-5	.	.	.	13 7.8e4 4.6e3 1.3e5 4.7e4
1e-8	.	.	.	13 7.8e4 5.1e3 1.4e5 4.7e4
	f135 in 5-D, N=15, mFE=50003	f135 in 20-D, N=15, mFE=200003	f136 in 5-D, N=15, mFE=50003	f136 in 20-D, N=15, mFE=200003
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.1e3 3.8e1 5.7e3 2.1e3	0 72e+0 68e+0 74e+0 8.9e4	10 3.7e2 1.7e1 1.6e3 3.7e2	15 4.1e3 4.4e2 1.2e4 4.1e3
1	4 1.7e5 4.1e4 3.9e5 3.4e4	.	.	15 3.4e3 8.3e1 1.6e4 3.4e3
1e-1	0 19e-1 39e-2 25e-1 2.3e4	.	.	15 3.5e4 5.1e2 8.9e4 3.5e4
1e-3	.	.	.	13 1.1e4 1.0e2 5.1e4 2.9e3
1e-5	.	.	.	13 7.8e4 4.6e3 1.3e5 4.7e4
1e-8	.	.	.	13 7.8e4 5.1e3 1.4e5 4.7e4
	f137 in 5-D, N=15, mFE=50003	f137 in 20-D, N=15, mFE=200003	f138 in 5-D, N=15, mFE=50003	f138 in 20-D, N=15, mFE=200003
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.1e3 3.8e1 5.7e3 2.1e3	0 72e+0 68e+0 74e+0 8.9e4	10 3.7e2 1.7e1 1.6e3 3.7e2	15 4.1e3 4.4e2 1.2e4 4.1e3
1	4 1.7e5 4.1e4 3.9e5 3.4e4	.	.	15 3.4e3 8.3e1 1.6e4 3.4e3
1e-1	0 19e-1 39e-2 25e-1 2.3e4	.	.	15 3.5e4 5.1e2 8.9e4 3.5e4
1e-3	.	.	.	13 1.1e4 1.0e2 5.1e4 2.9e3
1e-5	.	.	.	13 7.8e4 4.6e3 1.3e5 4.7e4
1e-8	.	.	.	13 7.8e4 5.1e3 1.4e5 4.7e4
	f139 in 5-D, N=15, mFE=50003	f139 in 20-D, N=15, mFE=200003	f140 in 5-D, N=15, mFE=50003	f140 in 20-D, N=15, mFE=200003
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.1e3 3.8e1 5.7e3 2.1e3	0 72e+0 68e+0 74e+0 8.9e4	10 3.7e2 1.7e1 1.6e3 3.7e2	15 4.1e3 4.4e2 1.2e4 4.1e3
1	4 1.7e5 4.1e4 3.9e5 3.4e4	.	.	15 3.4e3 8.3e1 1.6e4 3.4e3
1e-1	0 19e-1 39e-2 25e-1 2.3e4	.	.	15 3.5e4 5.1e2 8.9e4 3.5e4
1e-3	.	.	.	13 1.1e4 1.0e2 5.1e4 2.9e3
1e-5	.	.	.	13 7.8e4 4.6e3 1.3e5 4.7e4
1e-8	.	.	.	13 7.8e4 5.1e3 1.4e5 4.7e4
	f141 in 5-D, N=15, mFE=50003	f141 in 20-D, N=15, mFE=200003	f142 in 5-D, N=15, mFE=50003	f142 in 20-D, N=15, mFE=200003
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.1e3 3.8e1 5.7e3 2.1e3	0 72e+0 68e+0 74e+0 8.9e4	10 3.7e2 1.7e1 1.6e3 3.7e2	15 4.1e3 4.4e2 1.2e4 4.1e3
1	4 1.7e5 4.1e4 3.9e5 3.4e4	.	.	15 3.4e3 8.3e1 1.6e4 3.4e3
1e-1	0 19e-1 39e-2 25e-1 2.3e4	.	.	15 3.5e4 5.1e2 8.9e4 3.5e4
1e-3	.	.	.	13 1.1e4 1.0e2 5.1e4 2.9e3
1e-5	.	.	.	13 7.8e4 4.6e3 1.3e5 4.7e4
1e-8	.	.	.	13 7.8e4 5.1e3 1.4e5 4.7e4
	f143 in 5-D, N=15, mFE=50003	f143 in 20-D, N=15, mFE=200003	f144 in 5-D, N=15, mFE=50003	f144 in 20-D, N=15, mFE=200003
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.1e3 3.8e1 5.7e3 2.1e3	0 72e+0 68e+0 74e+0 8.9e4	10 3.7e2 1.7e1 1.6e3 3.7e2	15 4.1e3 4.4e2 1.2e4 4.1e3
1	4 1.7e5 4.1e4 3.9e5 3.4e4	.	.	15 3.4e3 8.3e1 1.6e4 3.4e3
1e-1	0 19e-1 39e-2 25e-1 2.3e4	.	.	15 3.5e4 5.1e2 8.9e4 3.5e4
1e-3	.	.	.	13 1.1e4 1.0e2 5.1e4 2.9e3
1e-5	.	.	.	13 7.8e4 4.6e3 1.3e5 4.7e4
1e-8	.	.	.	13 7.8e4 5.1e3 1.4e5 4.7e4
	f145 in 5-D, N=15, mFE=50003	f145 in 20-D, N=15, mFE=200003	f146 in 5-D, N=15, mFE=50003	f146 in 20-D, N=15, mFE=200003
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.1e3 3.8e1 5.7e3 2.1e3	0 72e+0 68e+0 74e+0 8.9e4	10 3.7e2 1.7e1 1.6e3 3.7e2	15 4.1e3 4.4e2 1.2e4 4.1e3
1	4 1.7e5 4.1e4 3.9e5 3.4e4	.	.	15 3.4e3 8.3e1 1.6e4 3.4e3
1e-1	0 19e-1 39e-2 25e-1 2.3e4	.	.	15 3.5e4 5.1e2 8.9e4 3.5e4
1e-3	.	.	.	13 1.1e4 1.0e2 5.1e4 2.9e3
1e-5	.	.	.	13 7.8e4 4.6e3 1.3e5 4.7e4
1e-8	.	.	.	13 7.8e4 5.1e3 1.4e5 4.7e4
	f147 in 5-D, N=15, mFE=50003	f147 in 20-D, N=15, mFE=200003	f148 in 5-D, N=15, mFE=50003	f148 in 20-D, N=15, mFE=200003
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.1e3 3.8e1 5.7e3 2.1e3	0 72e+0 68e+0 74e+0 8.9e4	10 3.7e2 1.7e1 1.6e3 3.7e2	15 4.1e3 4.4e2 1.2e4 4.1e3
1	4 1.7e5 4.1e4 3.9e5 3.4e4	.	.	15 3.4e3 8.3e1 1.6e4 3.4e3
1e-1	0 19e-1 39e-2 25e-1 2.3e4	.	.	15 3.5e4 5.1e2 8.9e4 3.5e4
1e-3	.	.	.	13 1.1e4 1.0e2 5.1e4 2.9e3
1e-5	.	.	.	13 7.8e4 4.6e3 1.3e5 4.7e4
1e-8	.	.	.	13 7.8e4 5.1e3 1.4e5 4.7e4
	f149 in 5-D, N=15, mFE=50003	f149 in 20-D, N=15, mFE=200003	f150 in 5-D, N=15, mFE=50003	f150 in 20-D, N=15, mFE=200003
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.1e3 3.8e1 5.7e3 2.1e3	0 72e+0 68e+0 74e+0 8.9e4	10 3.7e2 1.7e1 1.6e3 3.7e2	15 4.1e3 4.4e2 1.2e4 4.1e3
1	4 1.7e5 4.1e4 3.9e5 3.4e4	.	.	15 3.4e3 8.3e1 1.6e4 3.4e3
1e-1	0 19e-1 39e-2 25e-1 2.3e4	.	.	15 3.5e4 5.1e2 8.9e4 3.5e4
1e-3	.	.	.	13 1.1e4 1.0e2 5.1e4 2.9e3
1e-5	.	.	.	13 7.8e4 4.6e3 1.3e5 4.7e4
1e-8	.	.	.	13 7.8e4 5.1e3 1.4e5 4.7e4
	f151 in 5-D, N=15, mFE=50003	f151 in 20-D, N=15, mFE=200003	f152 in 5-D, N=15, mFE=50003	f152 in 20-D, N=15, mFE=200003
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.1e3 3.8e1 5.7e3 2.1e3	0 72e+0 68e+0 74e+0 8.9e4		

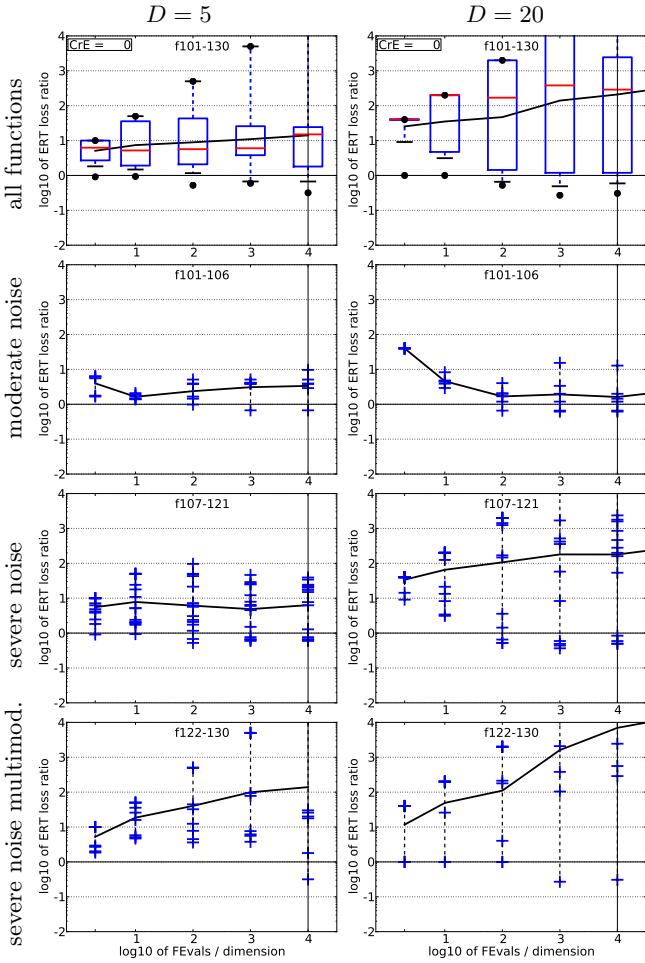


Figure 3: ERT loss ratio versus given budget FEvals. The target value f_t for ERT (see Figure 1) is the smallest (best) recorded function value such that $\text{ERT}(f_t) \leq \text{FEvals}$ for the presented algorithm. Shown is FEvals divided by the respective best $\text{ERT}(f_t)$ from BBOB-2009 for functions $f_{101}-f_{130}$ in 5-D and 20-D. Each ERT is multiplied by $\exp(\text{CrE})$ correcting for the parameter crafting effort. Line: geometric mean. Box-Whisker error bar: 25-75%-ile with median (box), 10-90%-ile (caps), and minimum and maximum ERT loss ratio (points). The vertical line gives the maximal number of function evaluations in this function subset.

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