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Mirrored Variants of the (1,2)-CMA-ES Compared on the Noisy BBOB-2010 Testbed

[Black-Box Optimization Benchmarking Workshop]

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ABSTRACT

Derandomization by means of mirrored samples has been recently introduced to enhance the performances of $(1, \lambda)$ and $(1 + 2)$ Evolution-Strategies (ESs) with the aim of designing fast local search stochastic algorithms. In this paper, we investigate the impact of mirrored samples for noisy optimization. Since elitist selection is detrimental for noisy optimization, we investigate non-elitist ESs only here. We compare on the BBOB-2010 noisy benchmark testbed two variants of the (1,2)-CMA-ES where mirrored samples are implemented with the baseline (1,2)-CMA-ES. Each algorithm implements a restart mechanism. A total budget of $10^4 D$ function evaluations per trial has been used, where D is the dimension of the search space.

The experiments clearly show a ranking among the three algorithms: both mirroring variants have lower expected running times than the (1,2)-CMA-ES by at least 50% on 5 functions and they solve three additional functions in 20D that the (1,2)-CMA-ES cannot solve (or only with small probability). The comparison between the two mirroring variants is in favor of the algorithm employing a sequential selection in addition—outperforming the algorithm with only mirrored samples on five functions by at least 17% whereas no statistically significant worsening can be observed. Both algorithms using mirrored samples also outperform the function-wise best algorithm of the BBOB-2009 benchmarking on three (respectively four) functions comprising Cauchy noise by up to 65%.

Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization

1. INTRODUCTION

Evolution Strategies (ESs) are robust stochastic search algorithms for black-box optimization where the function to be minimized, f , maps the continuous search space \mathbb{R}^D into \mathbb{R} . ESs evolve a population of candidate solutions that are created by sampling λ independent random vectors following a multivariate normal distribution. Recently, a new derandomization technique replacing the independent sampling of new solutions (or offspring) by mirrored samples has been introduced to enhance the performances of ESs [1]. With *mirrored sampling*, a single sample \mathcal{N} of a multivariate normal distribution is used for two offspring of the same iteration. Denoting X the current solution, the two offspring will equal $X + \mathcal{N}$ and $X - \mathcal{N}$ respectively. The resulting offspring are thus symmetric or *mirrored* with respect to X and are thus *negatively correlated*. Mirrored samples have been implemented in the Covariance-Matrix-Adaptation Evolution-Strategy (CMA-ES), an ES whose characteristic is to adapt the full covariance matrix of the multivariate normal search distribution [6]. Another new concept called sequential selection was introduced together with mirrored samples [1]. *Sequential selection*, consists in performing sequential evaluations of the offspring and breaking the evaluation loop as soon as an offspring is better than the current solution X and thus saving the remaining fitness evaluations.

In this paper, we assess quantitatively the improvement that can be brought by mirrored samples and by mirrored samples coupled with sequential selection. We compare on the BBOB-2010 noisy testbed the (1,2)-CMA-ES with two variants: first the $(1, 2_m)$ -CMA-ES where mirrored samples are used, and second the $(1, 2_m^s)$ -CMA-ES that in addition to the mirrored samples uses sequential selection. The algorithms and the CPU timing experiments are described in a complementing paper in the same proceeding [2].

2. RESULTS

2.1 Comparing (1,2)- and (1,2_m)-CMA-ES

Results from experiments comparing (1,2)-CMA-ES and (1,2_m)-CMA-ES according to [4] on the benchmark functions given in [3, 5] are presented in Figures 1 and 2 and in Table 1. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value, $f_t = f_{\text{opt}} + \Delta f_t$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t , summed over all trials and divided by the number of trials that actually reached f_t [4, 7]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

According to the experiments, the (1,2_m)-CMA-ES clearly outperforms the baseline algorithm (1,2)-CMA-ES. Both in 5D and in 20D, the (1,2_m)-CMA-ES is on no function and for no target ≤ 1 worse than the (1,2)-CMA-ES. Moreover, statistically significant improvements can be reported on 8 and 11 functions in 20D and 5D respectively. The improvement factors are ranging from about 1.8 for f_{101} and f_{103} over 2–3 (f_{106} and f_{109}) to a factor of more than 5 for the sphere function with moderate uniform noise (f_{102} , all in 20D). In 20D, three functions can be solved for all 15 instances by the (1,2_m)-CMA-ES whereas the (1,2)-CMA-ES has a lower (on f_{106}) or much lower success probability (only 1 out of 15 instances were solved by the (1,2)-CMA-ES on f_{112} and 0 instances were solved for f_{118}). Similar results hold for 5D.

On f_{109} (in both 5D and 20D and for several difficult targets), on f_{130} (in 20D and for a target of 10^{-7}), as well as on medium targets on f_{121} , the (1,2_m)-CMA-ES outperforms the function-wise best algorithm of the BBOB-2009 benchmarking. The largest improvement is achieved for the Gallagher function with Cauchy noise (f_{130}) where the expected running time of the (1,2_m)-CMA-ES is about 40% lower than for the best algorithm of BBOB-2009 for that function of last year’s benchmarking.

2.2 (1,2_m)- and (1,2_m^s)-CMA-ES

The results of this comparison are shown in Fig. 3 and 4 and in Table 2.

In 5D, no statistically significant differences (except for f_{121} where the (1,2_m^s)-CMA-ES is outperforming the (1,2_m)-CMA-ES) can be observed. In 20D, however, the (1,2_m^s)-CMA-ES clearly shows a better (statistically significant) performance than the (1,2_m)-CMA-ES on 6 functions: on f_{101} , the improvement is 17%, on f_{102} and f_{103} about 20%, on f_{109} 25%, on f_{112} about 60%, and on f_{118} about 30%.

The (1,2_m^s)-CMA-ES outperforms the function-wise best algorithm of BBOB-2009 on f_{103} by about 12%, on f_{109} and f_{112} by about 30%, and on f_{130} by about 65% for several low target values in 20D.

2.3 Comparing (1,2)- and (1,2_m^s)-CMA-ES

The third comparison yields similar results for the (1,2_m^s)-CMA-ES than for the (1,2_m)-CMA-ES above when compared to the (1,2)-CMA-ES, except that the improvement over the (1,2)-CMA-ES is even larger—not surprisingly af-

ter the results of Sec. 2.2. In 20D (5D), the (1,2)-CMA-ES is outperformed by the (1,2_m^s)-CMA-ES on 8 (12) functions (with statistical significance). The expected running time of the (1,2_m^s)-CMA-ES is always smaller than the one of the (1,2)-CMA-ES (with a few exceptions in 2D and 3D, cp. 3). Note that due to space limitations, we cannot show the plots and tables of this comparison.

3. CONCLUSIONS

The idea behind derandomization by means of mirroring introduced in [1] is to use only one random sample from a multivariate normal distribution to create two (negatively correlated or *mirrored*) offspring. Thereby, the first offspring is generated by adding a random sample to the parent solution and the second offspring then equals the solution which is symmetric to the first offspring with respect to the parent (by adding the negative sample to the parent). Here, this concept of mirroring has been integrated within two variants of a simple (1,2)-CMA-ES (of which the (1,2_m^s)-CMA-ES uses sequential selection [1] in addition and the (1,2_m)-CMA-ES does not). The three algorithms are then compared on the noisy BBOB-2010 testbed.

The experiments clearly show a ranking among the three algorithms: both the (1,2_m)-CMA-ES and the (1,2_m^s)-CMA-ES are never worse than the (1,2)-CMA-ES and have lower expected running times than the baseline by at least 50% on 5 functions. Moreover, the two algorithms employing mirroring solve three additional functions in 20D which the (1,2)-CMA-ES cannot solve (or only with small probability). The comparison between the (1,2_m)-CMA-ES and the (1,2_m^s)-CMA-ES is in favor of the (1,2_m^s)-CMA-ES outperforming the (1,2_m)-CMA-ES on five functions by at least 17% whereas no statistically significant worsening can be observed. Both the (1,2_m)-CMA-ES and the (1,2_m^s)-CMA-ES also outperform the function-wise best algorithm of the BBOB-2009 benchmarking on three (respectively four) functions comprising Cauchy noise by up to 65%.

Acknowledgments

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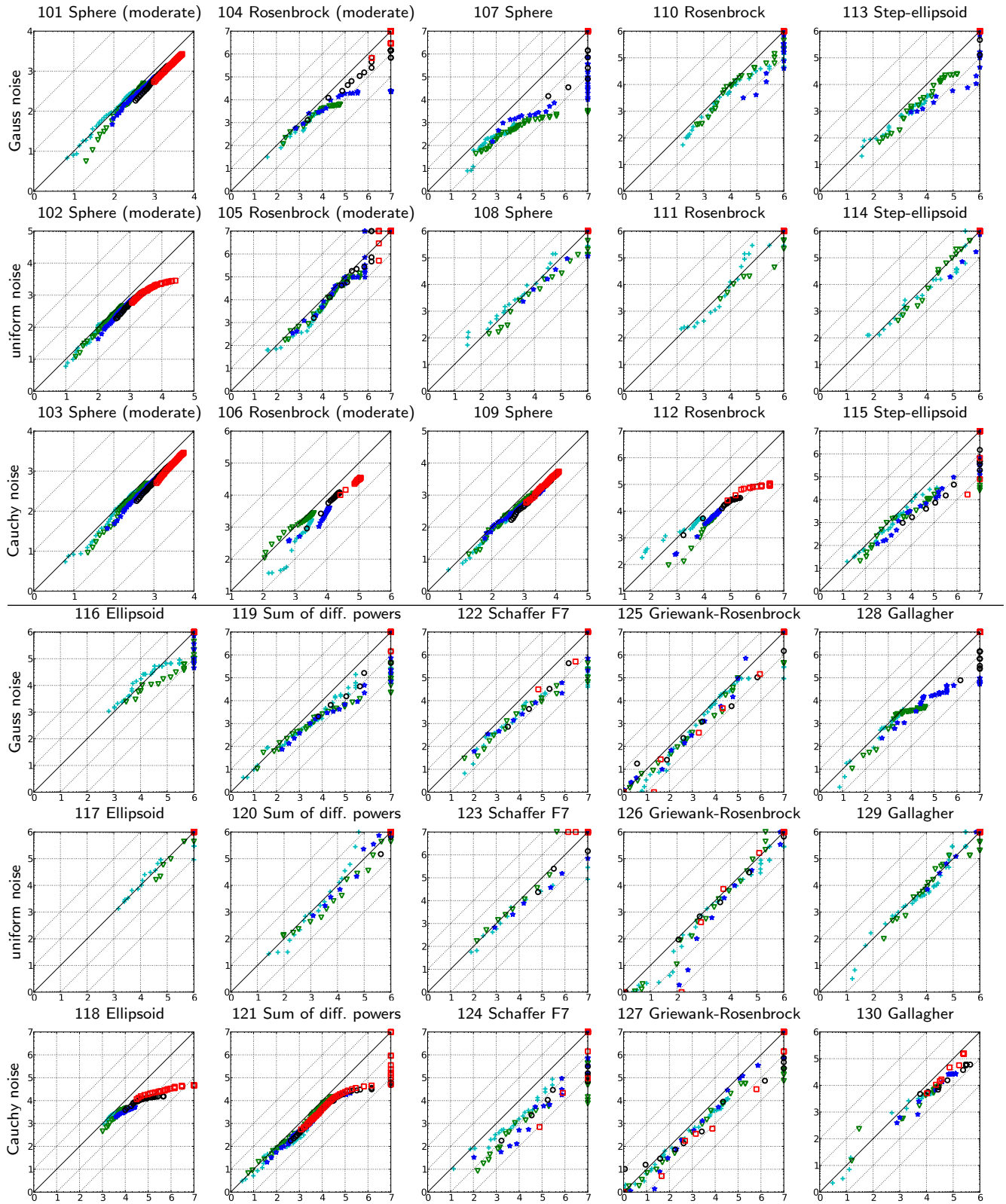


Figure 1: Expected running time (ERT in log10 of number of function evaluations) of $(1,2_m)$ -CMA-ES versus $(1,2)$ -CMA-ES for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension for functions $f_{101}-f_{130}$. Markers on the upper or right edge indicate that the target value was never reached by $(1,2_m)$ -CMA-ES or $(1,2)$ -CMA-ES respectively. Markers represent dimension: 2:+, 3: ∇ , 5:*, 10:o, 20: \square .

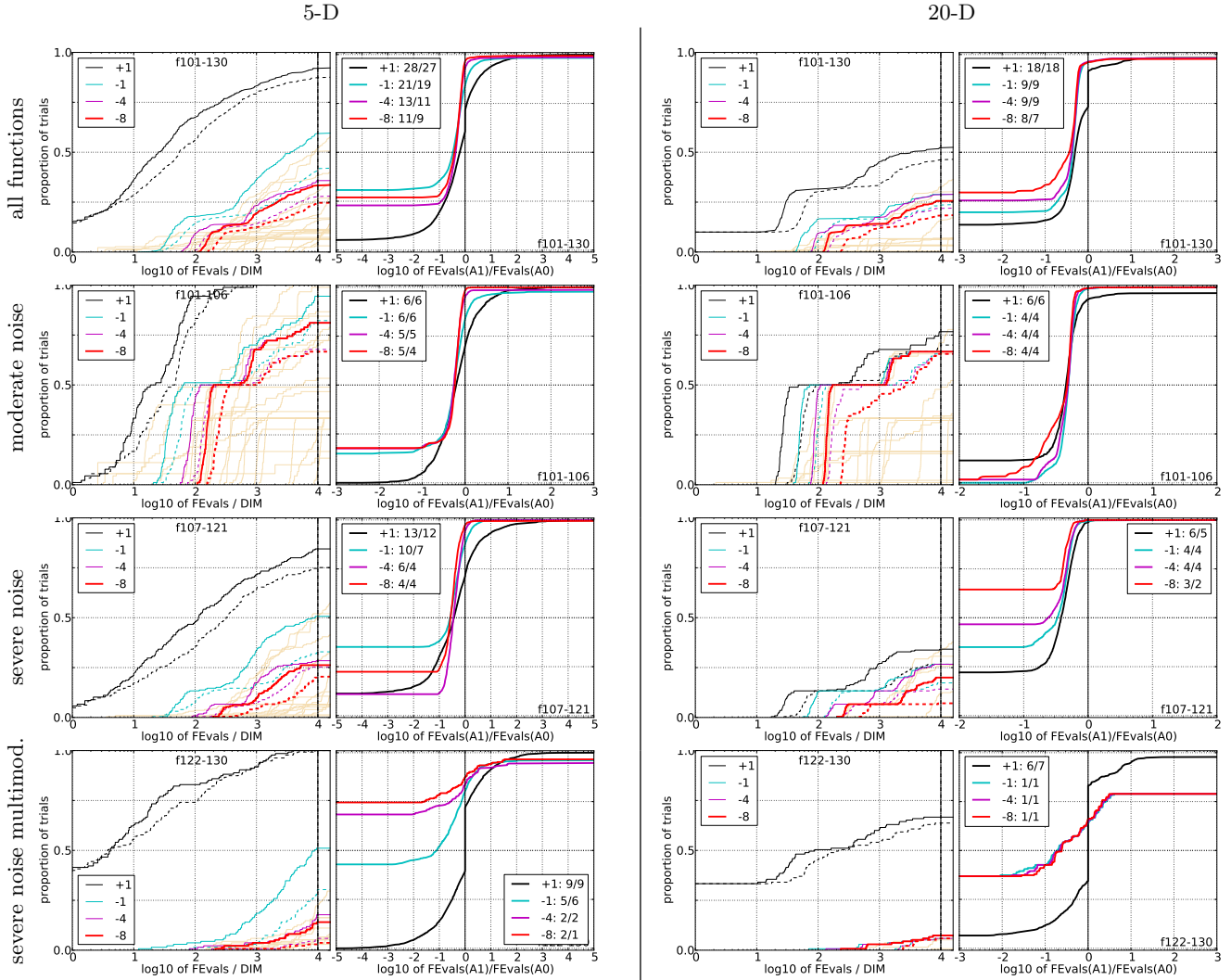


Figure 2: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of necessary function evaluations divided by dimension D (FEvals/ D) to reached a target value $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for (1,2_m)-CMA-ES (solid) and (1,2)-CMA-ES (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of all algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of (1,2_m)-CMA-ES divided by (1,2)-CMA-ES, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1 . The legends indicate the number of functions that were solved in at least one trial ((1,2_m)-CMA-ES first).

5-D

Δf	1e+1	1e+0	1e-1	1e-3	1e-5	1e-7	#succ
f_{101}	11	37	44	62	69	75	15/15
(1,2)-CMA-ES	8.3	4.4	6.3	8.5	11	13	15/15
(1,2 _m)-CMA-ES	4.3	3.3	4.9	5.8	7.4*	8.9*2	15/15
f_{102}	11	35	50	72	86	99	15/15
(1,2)-CMA-ES	9.2	5.8	6.2	7.7	9.7	11	15/15
(1,2 _m)-CMA-ES	3.9	3.3	3.6*	4.9*2	6*3	6.8*3	15/15
f_{103}	11	28	30	31	35	120	15/15
(1,2)-CMA-ES	6.2	6.7	9	18	25	10	15/15
(1,2 _m)-CMA-ES	3.4	3.8*	5.5*2	10*3	15*3	6.1*3	15/15
f_{104}	170	770	1300	1800	2000	2300	15/15
(1,2)-CMA-ES	3.9	13	54	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	3.3	4.3	13	13*3	12*3	11*3	13/15
f_{105}	170	1400	5200	1.0e4	1.1e4	1.1e4	15/15
(1,2)-CMA-ES	3.1	7.8	19	70	∞	∞	0/15
(1,2 _m)-CMA-ES	2.1	6	9.8	20	∞	∞	0/15
f_{106}	86	530	1100	2700	2900	3100	15/15
(1,2)-CMA-ES	7.5	13	8.6	4.1	3.9	3.8	15/15
(1,2 _m)-CMA-ES	4.3	3.1	2.5*2	1.3*3	1.3*3	1.3*3	15/15
f_{107}	40	230	450	940	1400	1900	15/15
(1,2)-CMA-ES	17	43	330	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	3.8	7.7*	6.7*	76*3	∞	∞	0/15
f_{108}	87	5100	1.4e4	3.1e4	5.9e4	8.1e4	15/15
(1,2)-CMA-ES	43	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	27	22	∞	∞	∞	∞	0/15
f_{109}	11	57	220	570	870	950	15/15
(1,2)-CMA-ES	5.5	2.8	1.8	1.9	2.5	3.4	15/15
(1,2 _m)-CMA-ES	4.1	2.5	1.2	1*	1*2	1.3*2	15/15
f_{110}	950	3.4e4	1.2e5	5.9e5	6.0e5	6.1e5	15/15
(1,2)-CMA-ES	32	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	3.4*	2.3*2	2.8*2	∞	∞	∞	0/15
f_{111}	6900	6.1e5	8.8e6	2.3e7	3.1e7	3.1e7	3/15
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
f_{112}	110	1700	3400	4500	5100	5600	15/15
(1,2)-CMA-ES	8	9.3	7	7.8	7.4	7.1	12/15
(1,2 _m)-CMA-ES	2.2	2.7	2.1*	2.1*2	2*2	1.9*2	15/15
f_{113}	130	1900	8100	2.4e4	2.4e4	2.4e4	15/15
(1,2)-CMA-ES	20	51	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	7.1	3.1*3	13*3	∞	∞	∞	0/15
f_{114}	770	1.5e4	5.6e4	8.3e4	8.3e4	8.5e4	15/15
(1,2)-CMA-ES	100	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	25	∞	∞	∞	∞	∞	0/15
f_{115}	64	490	1800	2600	2600	3000	15/15
(1,2)-CMA-ES	5.3	5.7	66	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	1.9	2.4	6.6*2	140*2	140*	240*	0/15
f_{116}	5700	1.4e4	2.2e4	2.7e4	3.0e4	3.2e4	15/15
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	8*3	16*3	∞	∞	∞	∞	0/15
f_{117}	2.7e4	7.6e4	1.1e5	1.4e5	1.7e5	1.9e5	15/15
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
f_{118}	430	1200	1600	2000	2400	2900	15/15
(1,2)-CMA-ES	9.2	6.5	6.4	7.8	8.2	8.8	15/15
(1,2 _m)-CMA-ES	4.6	2.4*2	2.2*3	2.2*3	2*3	1.8*3	15/15
f_{119}	12	660	1100	1.0e4	3.5e4	5.0e4	15/15
(1,2)-CMA-ES	15	7.8	86	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	6.4	2.5	6.1*2	∞	∞	∞	0/15
f_{120}	16	2900	1.9e4	7.2e4	3.3e5	5.5e5	15/15
(1,2)-CMA-ES	77	33	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	46	79	40	∞	∞	∞	0/15
f_{121}	8.6	110	270	1600	3900	6200	15/15
(1,2)-CMA-ES	4.5	2.9	1.9	2.9	4.5	9	4/15
(1,2 _m)-CMA-ES	2.4	1.2	0.94	1.1*2	1.9*	2.8*2	14/15
f_{122}	10	1700	9200	3.0e4	5.4e4	1.1e5	15/15
(1,2)-CMA-ES	11	30	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	6	4.7*2	∞	∞	∞	∞	0/15
f_{123}	11	1.6e4	8.2e4	3.4e5	6.7e5	2.2e6	15/15
(1,2)-CMA-ES	75	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	61	44	∞	∞	∞	∞	0/15
f_{124}	9.7	200	1000	2.0e4	4.5e4	9.5e4	15/15
(1,2)-CMA-ES	11	130	700	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	3.4	2.7*3	30*3	∞	∞	∞	0/15
f_{125}	1	1	1	2.4e5	2.4e5	2.5e5	15/15
(1,2)-CMA-ES	1	120	5.9e4	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	1.3	69	1.5e4	∞	∞	∞	0/15
f_{126}	1	1	1	∞	∞	∞	0
(1,2)-CMA-ES	1.1	2.0e3	7.4e5	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	1	620	∞	∞	∞	∞	0/15
f_{127}	1	1	1	3.4e5	3.9e5	4.0e5	15/15
(1,2)-CMA-ES	1	85	1.4e4	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	1	31	5.2e3	∞	∞	∞	0/15
f_{128}	110	4200	7800	1.2e4	1.7e4	2.1e4	15/15
(1,2)-CMA-ES	4.8	5.9	9.6	29	∞	∞	0/15
(1,2 _m)-CMA-ES	2.1	1.7	2.1	1.8*2	3.1*2	2.9*2	6/15
f_{129}	64	1.1e4	5.9e4	2.8e5	5.1e5	5.8e5	15/15
(1,2)-CMA-ES	170	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	110	∞	∞	∞	∞	∞	0/15
f_{130}	55	810	3000	3.3e4	3.4e4	3.5e4	10/15
(1,2)-CMA-ES	14	48	23	2.9	2.8	3.6	5/15
(1,2 _m)-CMA-ES	7.2	17	8.7	0.82	0.8	0.79	13/15

20-D

Δf	1e+1	1e+0	1e-1	1e-3	1e-5	1e-7	#succ
f_{101}	59	360	510	700	740	780	15/15
(1,2)-CMA-ES	17	4	3.7	4	5	5.8	15/15
(1,2 _m)-CMA-ES	9.1*3	2.2*3	2*3	2.1*3	2.7*3	3.1*3	15/15
f_{102}	230	400	580	920	1200	1400	15/15
(1,2)-CMA-ES	5	4.1	3.7	4.1	5.3	9.8	15/15
(1,2 _m)-CMA-ES	2.5*3	2*3	1.8*3	1.7*3	1.7*3	1.8*3	15/15
f_{103}	65	420	630	1300	1900	2500	14/15
(1,2)-CMA-ES	18	3.7	3.1	2.2	2.1	2	15/15
(1,2 _m)-CMA-ES	7.7*3	1.7*3	1.6*3	1.1*3	1.1*3	1.1*3	15/15
f_{104}	2.4e4	8.6e4	1.7e5	1.8e5	1.9e5	2.0e5	15/15
(1,2)-CMA-ES	63	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	28	34	∞	∞	∞	∞	0/15
f_{105}	1.9e5	6.1e5	6.3e5	6.5e5	6.6e5	6.7e5	15/15
(1,2)-CMA-ES	16	4.9	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	2.7	∞	∞	∞	∞	∞	0/15
f_{106}	1.1e4	2.2e4	2.4e4	2.5e4	2.6e4	2.7e4	15/15
(1,2)-CMA-ES	2.3	3.9	3.9	4.1	4.1	4.1	14/15
(1,2 _m)-CMA-ES	0.88*	1.2*3	1.3*3	1.3*3	1.3*3	1.3*3	15/15
f_{107}	8600	1.4e4	1.6e4	2.7e4	5.2e4	6.5e4	15/15
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
f_{108}	5.8e4	9.7e4	2.0e5	4.5e5	6.3e5	9.0e5	15/15
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
f_{109}	330	630	1100	2300	3600	5000	15/15
(1,2)-CMA-ES	3.6	3.5	2.7	2.4	2.3	2.2	15/15
(1,2 _m)-CMA-ES	1.7*3	1.5*3	1.4*3	1.1*3	1*3	0.96*3	15/15
f_{110}	∞	∞	∞	∞	∞	∞	0
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
f_{111}	∞	∞	∞	∞	∞	∞	0
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
f_{112}	2.6e4	6.4e4	7.0e4	7.4e4	7.6e4	7.8e4	15/15
(1,2)-CMA-ES	3.1	8.4	20	40	39	38	1/15
(1,2 _m)-CMA-ES	0.98*3	1.2*2	1.2*3	1.2*3	1.3*3	1.3*3	15/15
f_{113}	5.0e4	3.6e5	5.6e5	5.9e5	5.9e5	5.9e5	15/15
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
f_{114}	2.1e5	1.1e6	1.4e6	1.6e6	1.6e6	1.6e6	15/15
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
f_{115}	2400	3.0e4	9.2e4	1.3e5	1.3e5	1.3e5	15/15
(1,2)-CMA-ES	1.2e3	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	7*3	∞	∞	∞	∞	∞	0/15
f_{116}	5.0e5	6.9e5	8.9e5	1.0e6	1.1e6	1.1e6	15/15
(1,2)-CMA-ES	∞	$\$					

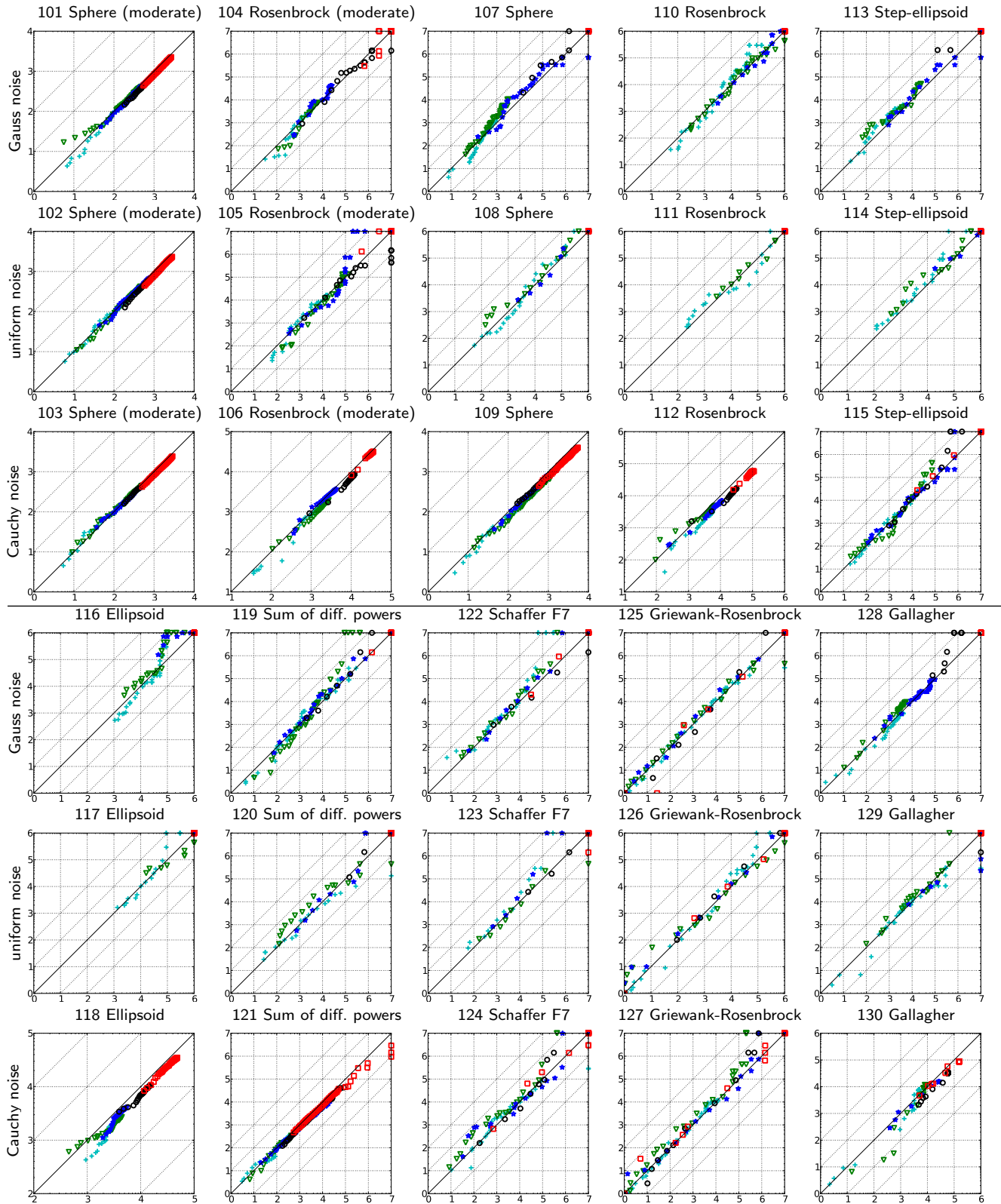


Figure 3: Expected running time (ERT in log10 of number of function evaluations) of $(1,2_m^s)$ -CMA-ES versus $(1,2_m)$ -CMA-ES for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension for functions f_{101} – f_{130} . Markers on the upper or right edge indicate that the target value was never reached by $(1,2_m^s)$ -CMA-ES or $(1,2_m)$ -CMA-ES respectively. Markers represent dimension: 2: +, 3: ∇, 5: *, 10: ○, 20: □.

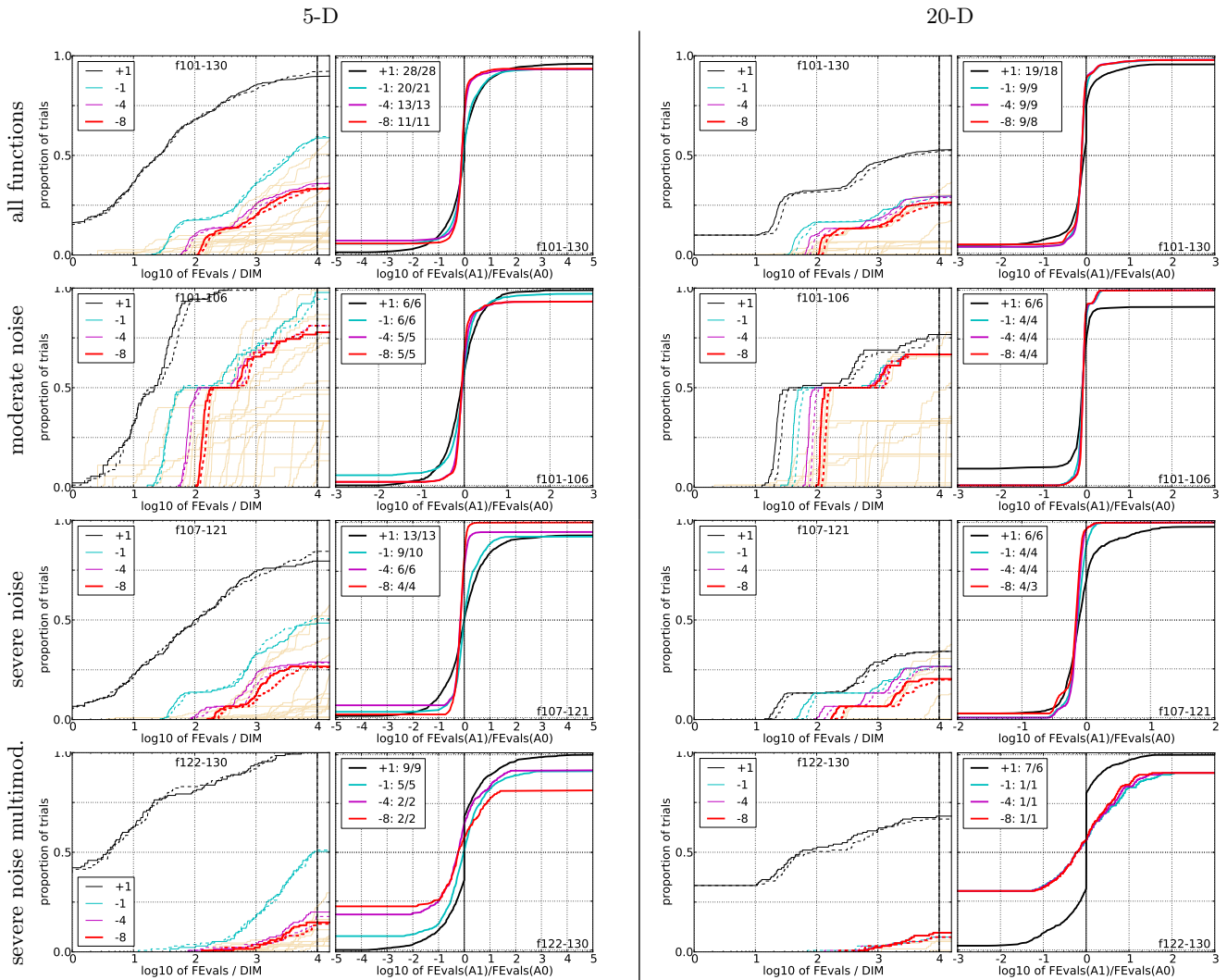


Figure 4: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right) as in Fig.2 but now for $(1,2_m^s)$ -CMA-ES (solid) and $(1,2_m)$ -CMA-ES (dashed) and ratios of $(1,2_m^s)$ -CMA-ES divided by $(1,2_m)$ -CMA-ES respectively.

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