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Mirrored Variants of the (1,2)-CMA-ES Compared on the Noiseless BBOB-2010 Testbed

[Black-Box Optimization Benchmarking Workshop]

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ABSTRACT

Derandomization by means of mirroring has been recently introduced to enhance the performances of $(1, \lambda)$ -Evolution-Strategies (ESs) with the aim of designing fast robust local search stochastic algorithms. This paper compares on the BBOB-2010 noiseless benchmark testbed two variants of the $(1,2)$ -CMA-ES where the mirroring method is implemented. Independent restarts are conducted till a total budget of $10^4 D$ function evaluations per trial is reached, where D is the dimension of the search space. The results show that the improved variants increase the success probability on 5 (respectively 7) out of 24 test functions in 20D and at the same time are significantly faster on 9 (10) functions in 20D by a factor of about 2–3 (2–4) for a target value of 10^{-7} while in no case, the baseline $(1,2)$ -CMA-ES is significantly faster on any tested target function value in 5D and 20D.

Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

General Terms

Algorithms

1. INTRODUCTION

Evolution Strategies (ESs) are robust stochastic search algorithms for numerical optimization where the function to be minimized, f , maps the continuous search space \mathbb{R}^D into \mathbb{R} . Recently, a new derandomization technique replacing the independent sampling of new solutions by mirrored sampling has been introduced to enhance the performances of ESs [1]. While mirrored samples were introduced with

the aim of designing fast robust local search algorithms, investigation of convergence speed was mainly carried out on the sphere function [1]. In this paper, we want to assess quantitatively the improvements that can be brought with the mirroring method on a wider range of problems. To do so, we compare on the BBOB-2010 noiseless testbed the $(1,2)$ -CMA-ES with two variants implementing the mirrored samples: first the $(1,2_m)$ -CMA-ES where every second mutation step is derandomized, and second the $(1,2_m^s)$ -CMA-ES that in addition to the mirroring idea implements sequential selection [1]. Both variants are described in Sec. 2.

2. THE ALGORITHMS TESTED

The three algorithms tested are variants of the well-known CMA-ES [8] where at each iteration n , λ new solutions are generated by sampling *independently* λ random vectors $(\mathcal{N}_i(\mathbf{0}, \mathbf{C}_n))_{1 \leq i \leq \lambda}$ following a multivariate normal distribution with mean vector $\mathbf{0}$ and covariance matrix \mathbf{C}_n . The vectors are added to the current solution \mathbf{X}_n to create the λ new solutions or offspring $\mathbf{X}_n^i = \mathbf{X}_n + \sigma_n \mathcal{N}_i(\mathbf{0}, \mathbf{C}_n)$, where σ_n is the strictly positive step-size. In the standard $(1,2)$ -CMA-ES, the number of offspring λ equals 2 and \mathbf{X}_{n+1} is set to the best solution among \mathbf{X}_n^1 and \mathbf{X}_n^2 , i.e., $\mathbf{X}_{n+1} = \operatorname{argmin}\{f(\mathbf{X}_n^1), f(\mathbf{X}_n^2)\}$.

In the mirrored variant, denoted $(1,2_m)$ -CMA-ES, the second offspring is symmetric to the first offspring with respect to \mathbf{X}_n , namely $\mathbf{X}_n^2 = \mathbf{X}_n - \sigma_n \mathcal{N}_1(\mathbf{0}, \mathbf{C}_n)$, where $\sigma_n \mathcal{N}_1(\mathbf{0}, \mathbf{C}_n)$ is the random vector *added* to \mathbf{X}_n to create \mathbf{X}_n^1 . We see that the first and second added vector are negatively correlated (with correlation coefficient one). The update of \mathbf{X}_{n+1} is then identical to the $(1,2)$ -CMA-ES, namely $\mathbf{X}_{n+1} = \operatorname{argmin}\{f(\mathbf{X}_n^1), f(\mathbf{X}_n^2)\}$.

In the $(1,2_m^s)$ -CMA-ES, sequential selection is implemented. The offspring solutions are generated with mirrored sampling. Evaluations are carried out in a sequential manner. After evaluating \mathbf{X}_n^1 , it is compared to \mathbf{X}_n and if $f(\mathbf{X}_n^1) \leq f(\mathbf{X}_n)$, the sequence of evaluations is stopped and $\mathbf{X}_{n+1} = \mathbf{X}_n^1$. In case both offspring are worse than \mathbf{X}_n , $\mathbf{X}_{n+1} = \operatorname{argmin}\{f(\mathbf{X}_n^1), f(\mathbf{X}_n^2)\}$ according to the comma selection. The number of offspring evaluated is a random variable ranging from 1 to 2—reducing the number of offspring adaptively as long as improvements are easy to achieve [1].

Covariance matrix and step-size are updated using the selected steps [8, 1].

Independent restarts: Similar to [2], we independently restarted the $(1,2)$ -CMA-ES, $(1,2_m)$ -CMA-ES and $(1,2_m^s)$ -

CMA-ES as long as function evaluations were left, where $10^4 \cdot D$ was the maximal number of function evaluations.

2.1 Parameter setting

We used the default parameter and termination settings (cf. [1, 4, 7]) found in the source code on the WWW¹ with two exceptions. We rectified the learning rate of the rank-one update of the covariance matrix for small values of λ , setting $c_1 = \min(2, \lambda/3)/((D+1.3)^2 + \mu_{\text{eff}})$. The original value was not designed to work for $\lambda < 5$. We modified the damping parameter for the step-size to $d_\sigma = 0.3 + 2\mu_{\text{eff}}/\lambda + c_\sigma$. The setting was found by performing experiments on the sphere function, f_1 : d_σ was set as large as possible while still showing close to optimal performance, but, at least as large such that decreasing it by a factor of two did not lead to unacceptable performance. For $\mu_{\text{eff}}/\lambda = 0.35$ and $\mu_{\text{eff}} \leq D + 2$ the former setting of d_σ is recovered. For a smaller ratio of μ_{eff}/λ or for $\mu_{\text{eff}} > D + 2$, the new setting allows larger (i.e. faster) changes of σ . Here, $\mu_{\text{eff}} = 1$. For $\lambda \geq 3$, the new setting might be harmful in a noisy or too rugged landscape. Finally, the step-size multiplier was clamped from above at $\exp(1)$, while we do not believe this had any effect in the presented experiments. Each initial solution \mathbf{X}_0 was uniformly sampled in $[-4, 4]^D$ and the step-size σ_0 was initialized to 2. The source code used for the experiments is available at².

As the same parameter setting has been used in all experiments for all test functions, the crafting effort CrE of all three algorithms is 0.

3. CPU TIMING EXPERIMENTS

For the timing experiment, all three algorithms were run on f_8 with a maximum of $10^4 D$ function evaluations and restarted until at least 30 seconds have passed (according to Figure 2 in [5]). The experiments have been conducted with an 8 core Intel Xeon E5520 machine with 2.27 GHz under Ubuntu 9.1 linux and Matlab R2008a. The time per function evaluation was 5.9; 5.9; 6.2; 6.1; 6.8; 9.1 times 10^{-4} seconds for (1,2)-CMA-ES, 6.6; 6.0; 5.9; 6.0; 6.8; 8.3 times 10^{-4} seconds for (1,2_m)-CMA-ES, and 8.4; 8.6; 8.6; 9.2; 9.2; 11 times 10^{-4} seconds for (1,2_m^s)-CMA-ES in dimensions 2; 3; 5; 10; 20; 40 respectively. Note that MATLAB distributes the computations over all 8 cores only for 20D and 40D.

4. RESULTS

4.1 Comparing (1,2)- and (1,2_m)-CMA-ES

Results from experiments comparing (1,2)-CMA-ES and (1,2_m)-CMA-ES according to [5] on the benchmark functions given in [3, 6] are presented in Figures 1 and 2 and in Table 1. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value, $f_t = f_{\text{opt}} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t , summed over all trials and divided by the number of trials that actually reached f_t [5, 9]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t using, for each trial, either the number of needed function evaluations to

reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

The experiments show a big improvement of the (1,2_m)-CMA-ES over the (1,2)-CMA-ES, in particular on the sphere (f_1 , in 20D 44% faster), ellipsoid (f_2 and f_{10} , both about 2 times faster), Rosenbrock (f_8 and f_9 , 2-3 times faster), as well as on f_{11} (42% better), f_{12} (about 50% better), and f_{14} (factor of 2.4 better). Figure 2 suggests that the speedup is larger in 20D than in 5D where the largest improvement can be seen for the moderate and separable functions.

The attractive sector function f_6 , that is not solved by the (1,2)-CMA-ES, is reliably solved (for 13 of the 15 instances) by the mirrored version up to the target value 10^{-8} (Table 1). The success probability for the (1,2_m)-CMA-ES is also slightly higher on the Gallagher functions f_{21} and f_{22} but the differences in expected running time are not statistically significant. Overall, the (1,2_m)-CMA-ES outperforms the (1,2)-CMA-ES statistically significantly on 9 (respectively 10) out of the 24 functions in 20D (5D) and is never worse. However, there are still 12 problems that the (1,2_m)-CMA-ES cannot solve in dimension 20 within $10^4 D$ function evaluations, compared to 13 problems for the (1,2)-CMA-ES; in 5D, the (1,2)-CMA-ES has no successful run for 11 test problems and the (1,2_m)-CMA-ES for 9.

4.2 Comparing (1,2_m)- and (1,2_m^s)-CMA-ES

The results from experiments comparing the (1,2)-CMA-ES and the (1,2_m^s)-CMA-ES according to [5] on the benchmark functions given in [3, 6] are presented in Figures 3 and 4 and in Table 2. The statistical tests and the definition of the ERT is the same than above.

The results indicate that the (1,2_m^s)-CMA-ES is even faster than the (1,2_m)-CMA-ES for 11 functions in 20D of which only 6 functions show a significant outperformance ($p \leq 0.05$). The largest speedups are on the sphere function, on f_2 (on both functions approx. 10% faster), on f_6 (about a factor of 2 faster, but not significant), and on f_8 (factor of 1.6 faster). On the other hand, the (1,2_m)-CMA-ES is never significantly faster (only on f_{13} and f_{21} it is slightly faster than the (1,2_m^s)-CMA-ES which is not statistically significant). Results on 5D are similar, although the difference between the algorithms is larger in higher dimensions.

4.3 Comparing (1,2)- and (1,2_m^s)-CMA-ES

Regarding this comparison, we cannot show the results in the same form as above due to space limitations but can state that, in principle, the results are comparable to the first comparison above—showing an even larger improvement for the (1,2_m^s)-CMA-ES here.

5. CONCLUSIONS

The idea behind derandomization by means of mirroring introduced in [1] is to use only one random sample from a multivariate normal distribution to create two (negatively correlated or *mirrored*) offspring. Thereby, the first offspring is generated by adding a random sample to the parent solution and the second offspring then equals the solution which is symmetric to the first offspring with respect to the parent (by adding the negative sample to the parent). Here, this concept of mirroring has been integrated within two variants of a simple (1,2)-CMA-ES (of which one uses se-

¹cmaes.m, version 3.41.beta, from http://www.lri.fr/~hansen/cmaes_inmatlab.html

²<http://coco.gforge.inria.fr/doku.php?id=bbob-2010-results>

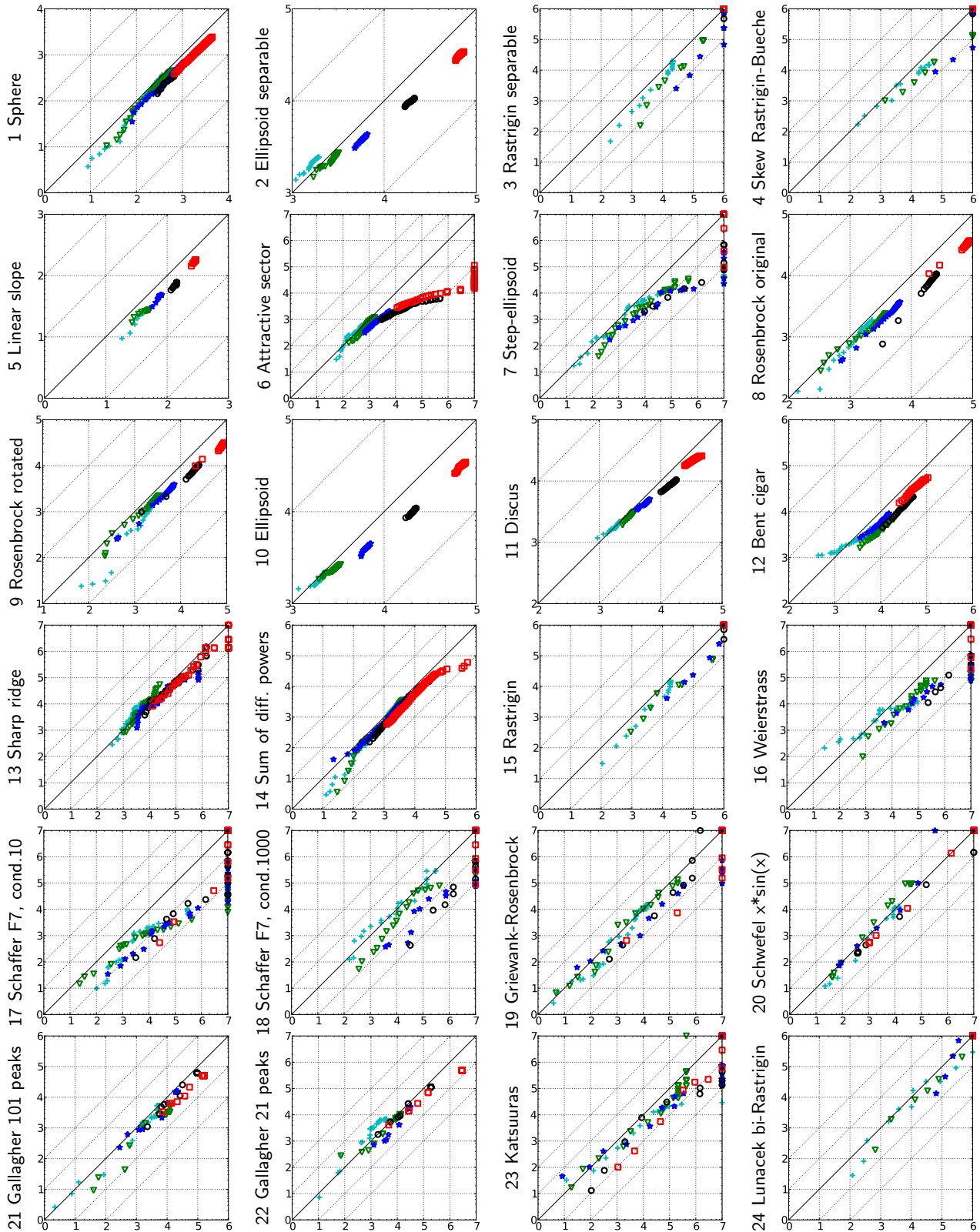


Figure 1: Expected running time (ERT in log10 of number of function evaluations) of $(1,2_m)$ -CMA-ES versus $(1,2)$ -CMA-ES for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension for functions f_1 – f_{24} . Markers on the upper or right edge indicate that the target value was never reached by $(1,2_m)$ -CMA-ES or $(1,2)$ -CMA-ES respectively. Markers represent dimension: 2:+, 3:∇, 5:*, 10:○, 20:□.

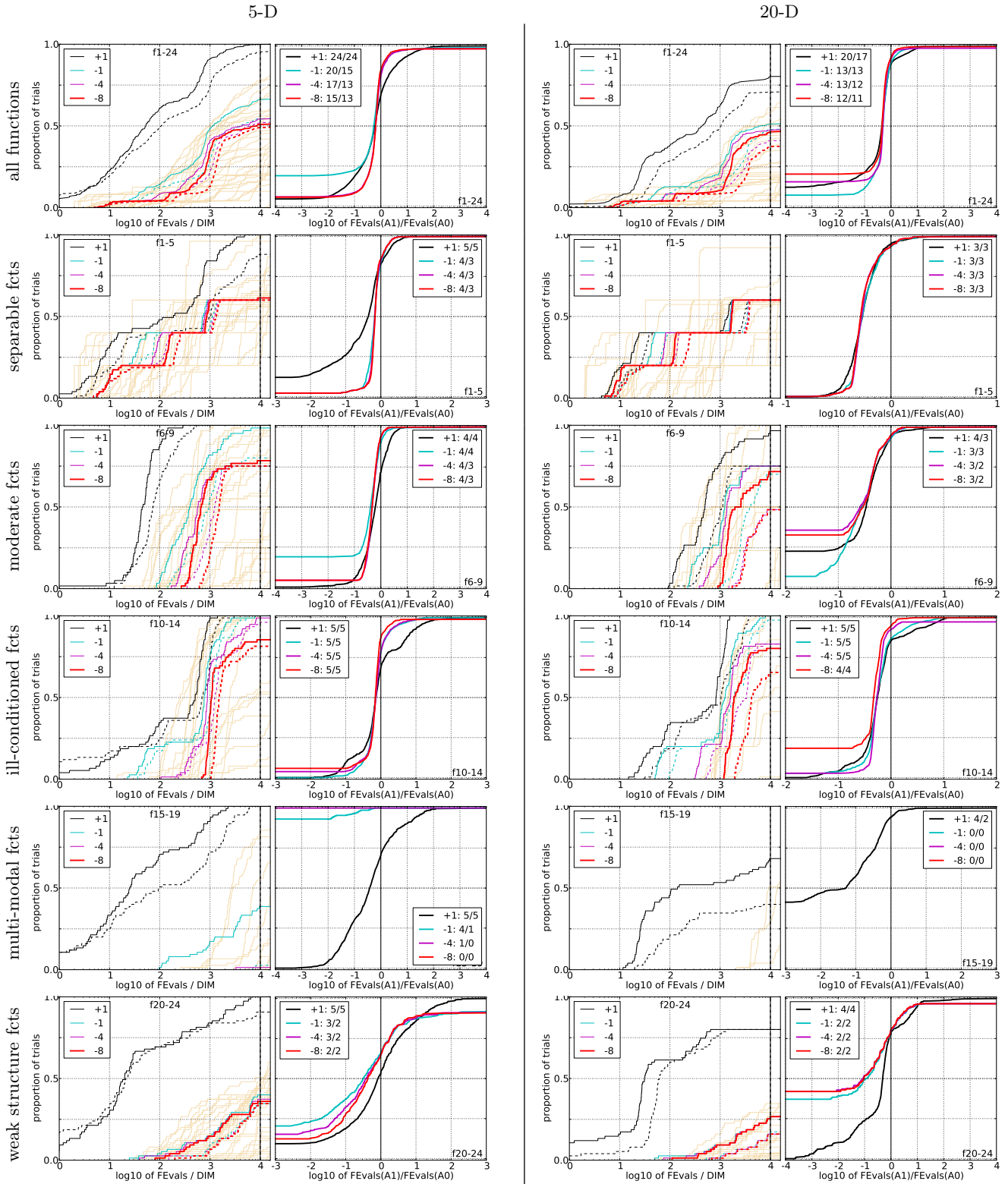


Figure 2: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/ D) to reach a target value $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for $(1,2_m)$ -CMA-ES (solid) and $(1,2)$ -CMA-ES (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of $(1,2_m)$ -CMA-ES divided by $(1,2)$ -CMA-ES, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1 . The legends indicate the number of functions that were solved in at least one trial ($(1,2_m)$ -CMA-ES first).

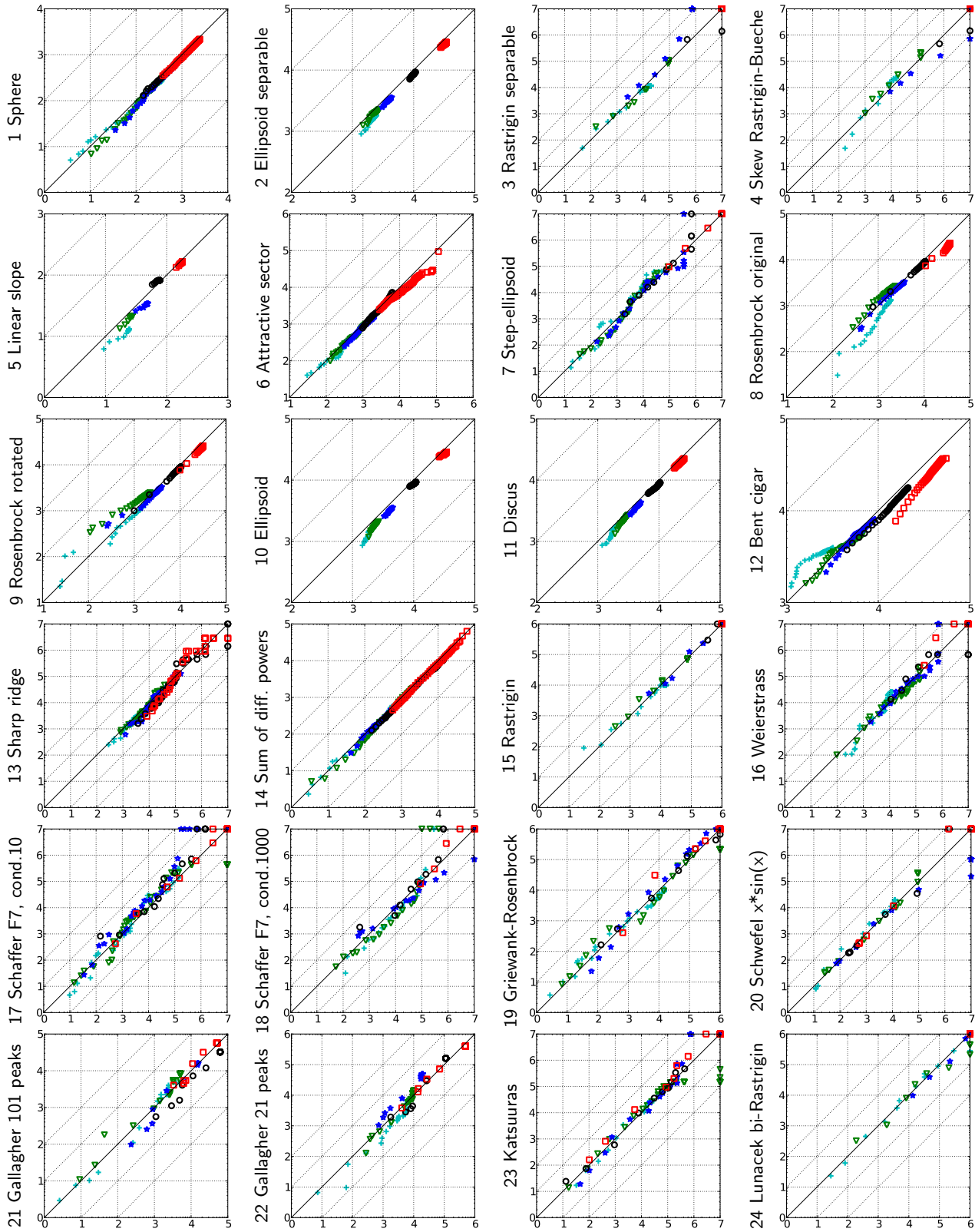


Figure 3: Expected running time (ERT in log10 of number of function evaluations) of $(1,2^s_m)$ -CMA-ES versus $(1,2_m)$ -CMA-ES for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension for functions f_1 - f_{24} . Markers on the upper or right edge indicate that the target value was never reached by $(1,2^s_m)$ -CMA-ES or $(1,2_m)$ -CMA-ES respectively. Markers represent dimension: 2:+, 3:∇, 5:*, 10:○, 20:□.

5-D

Δf	1e+1	1e+0	1e-1	1e-3	1e-5	1e-7	#succ
f_1	11	12	12	12	12	12	15/15
(1,2)-CMA-ES	7.4	15	27	47	63	80	15/15
(1,2 _m)-CMA-ES	3.2	10	15*2	28*3	40*3	53*3	15/15
f_2	83	87	88	90	92	94	15/15
(1,2)-CMA-ES	57	61	64	66	68	69	15/15
(1,2 _m)-CMA-ES	37*	40*2	42*3	44*3	45*3	46*3	15/15
f_3	720	1600	1600	1600	1700	1700	15/15
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	3.5*3	150*2	450*2	450*2	450*2	450*2	1/15
f_4	810	1600	1700	1800	1900	1900	15/15
(1,2)-CMA-ES	75	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	11*	∞	∞	∞	∞	∞	0/15
f_5	10	10	10	10	10	10	15/15
(1,2)-CMA-ES	5.7	7.5	8.1	8.1	8.1	8.1	15/15
(1,2 _m)-CMA-ES	3.1	4.8	4.8	4.9	4.9	4.9	15/15
f_6	110	210	280	580	1000	1300	15/15
(1,2)-CMA-ES	5.9	5.1	5	4.3	3.9	4	15/15
(1,2 _m)-CMA-ES	2.7	2.4	2.5*2	1.8*3	1.4*3	1.4*3	15/15
f_7	24	320	1200	1600	1600	1600	15/15
(1,2)-CMA-ES	19	28	190	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	7.2	5.3*2	11*2	220*2	220*2	220*2	2/15
f_8	73	270	340	390	410	420	15/15
(1,2)-CMA-ES	9.7	10	14	15	15	15	15/15
(1,2 _m)-CMA-ES	5.6	5.7	7.2*	8.1*2	8.4*2	8.7*2	15/15
f_9	35	130	210	300	340	370	15/15
(1,2)-CMA-ES	12	27	24	21	20	19	15/15
(1,2 _m)-CMA-ES	7.4	14	12*	11*	11*	10*	15/15
f_{10}	350	500	570	630	830	880	15/15
(1,2)-CMA-ES	16	12	11	10	8.2	8	15/15
(1,2 _m)-CMA-ES	9.4*3	7.6*3	6.8*3	6.5*3	5.1*3	5.1*3	15/15
f_{11}	140	200	760	1200	1500	1700	15/15
(1,2)-CMA-ES	28	25	7.3	5.1	4.3	3.9	15/15
(1,2 _m)-CMA-ES	24	20	5.6*	3.9*2	3.3*2	3*2	15/15
f_{12}	110	270	370	460	1300	1500	15/15
(1,2)-CMA-ES	33	26	24	24	10	9.8	15/15
(1,2 _m)-CMA-ES	25	16	14	14	6	5.9	15/15
f_{13}	130	190	250	1300	1800	2300	15/15
(1,2)-CMA-ES	25	27	37	22	28	98	1/15
(1,2 _m)-CMA-ES	9.4	30	34	14	27	37	4/15
f_{14}	9.8	41	58	140	250	480	15/15
(1,2)-CMA-ES	2.3	5	5.9	8.3	13	13	15/15
(1,2 _m)-CMA-ES	4.3	3.5	3.9	5.8	11	9.3*2	15/15
f_{15}	510	9300	1.9e4	2.0e4	2.1e4	2.1e4	14/15
(1,2)-CMA-ES	27	76	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	8.1	26	∞	∞	∞	∞	0/15
f_{16}	120	610	2700	1.0e4	1.2e4	1.2e4	15/15
(1,2)-CMA-ES	42	260	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	16	29*	45*2	69*2	62*2	∞	0/15
f_{17}	5.2	210	900	3700	6400	7900	15/15
(1,2)-CMA-ES	52	54	400	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	6.7	5.3*	6.2*3	64*3	∞	∞	0/15
f_{18}	100	380	4000	9300	1.1e4	1.2e4	15/15
(1,2)-CMA-ES	36	200	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	3.7	28*	23*2	∞	∞	∞	0/15
f_{19}	1	1	240	1.2e5	1.2e5	1.2e5	15/15
(1,2)-CMA-ES	31	1.6e4	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	62	4.6e3	940	∞	∞	∞	0/15
f_{20}	16	850	3.8e4	5.4e4	5.5e4	5.5e4	14/15
(1,2)-CMA-ES	4.7	19	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	4.5	11	∞	∞	∞	∞	0/15
f_{21}	41	1200	1700	1700	1700	1800	14/15
(1,2)-CMA-ES	6.9	8.6	13	13	13	12	14/15
(1,2 _m)-CMA-ES	5.7	5.8	9	8.9	8.9	8.8	13/15
f_{22}	71	390	940	1000	1000	1100	14/15
(1,2)-CMA-ES	18	31	31	30	29	29	12/15
(1,2 _m)-CMA-ES	10	11	20	19	18	18	14/15
f_{23}	3	520	1.4e4	3.2e4	3.3e4	3.4e4	15/15
(1,2)-CMA-ES	2.8	100	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	15	37	16	23	22	21	0/15
f_{24}	1600	2.2e5	6.4e6	9.6e6	1.3e7	1.3e7	3/15
(1,2)-CMA-ES	40	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	8.3	∞	∞	∞	∞	∞	0/15

20-D

Δf	1e+1	1e+0	1e-1	1e-3	1e-5	1e-7	#succ
f_1	43	43	43	43	43	43	15/15
(1,2)-CMA-ES	15	25	34	53	72	93	15/15
(1,2 _m)-CMA-ES	8.9*3	15*3	20*3	30*3	41*3	52*3	15/15
f_2	380	390	390	390	390	390	15/15
(1,2)-CMA-ES	150	170	170	180	180	180	15/15
(1,2 _m)-CMA-ES	71*3	80*3	82*3	84*3	85*3	86*3	15/15
f_3	5100	7600	7600	7600	7600	7700	15/15
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
f_4	4700	7600	7700	7700	7800	1.4e5	9/15
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
f_5	41	41	41	41	41	41	15/15
(1,2)-CMA-ES	6.1	7	7.2	7.2	7.2	7.2	15/15
(1,2 _m)-CMA-ES	3.5*	4.4	4.4	4.4	4.4	4.4	15/15
f_6	1300	2300	3400	5200	6700	8400	15/15
(1,2)-CMA-ES	8.9	12	26	570	∞	∞	0/15
(1,2 _m)-CMA-ES	2.2*3	1.8*3	1.8*3	2.5*3	3.5*3	6.7*3	13/15
f_7	1400	4300	9500	1.7e4	1.7e4	1.7e4	15/15
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	71*3	∞	∞	∞	∞	∞	0/15
f_8	2000	3900	4000	4200	4400	4500	15/15
(1,2)-CMA-ES	9.5	19	20	20	20	20	14/15
(1,2 _m)-CMA-ES	5.3*	7.8	8.2*	8.3*	8.2*	8.1*	15/15
f_9	1700	3100	3300	3500	3600	3700	15/15
(1,2)-CMA-ES	12	24	25	25	24	24	15/15
(1,2 _m)-CMA-ES	5.8*	8.1*2	8.5*2	8.6*2	8.5*3	8.4*3	15/15
f_{10}	7400	8700	1.1e4	1.5e4	1.7e4	1.7e4	15/15
(1,2)-CMA-ES	7.9	7.4	6.4	4.9	4.4	4.3	15/15
(1,2 _m)-CMA-ES	3.5*3	3.5*3	2.9*3	2.2*3	2*3	2*3	15/15
f_{11}	1000	2200	6300	9800	1.2e4	1.5e4	15/15
(1,2)-CMA-ES	24	22	4.8	3.6	3.2	2.9	15/15
(1,2 _m)-CMA-ES	18*3	8.6*3	3.3*3	2.3*3	2*3	1.7*3	15/15
f_{12}	1000	1900	2700	4100	1.2e4	1.4e4	15/15
(1,2)-CMA-ES	24	21	19	16	6.4	6.7	14/15
(1,2 _m)-CMA-ES	15	13	12	9.3	3.6	3.6*	15/15
f_{13}	650	2000	2800	1.9e4	2.4e4	3.0e4	15/15
(1,2)-CMA-ES	20	25	41	47	∞	∞	0/15
(1,2 _m)-CMA-ES	13	12	24	33	55	93	0/15
f_{14}	75	240	300	930	1600	1.6e4	15/15
(1,2)-CMA-ES	16	8.2	8.2	7.8	14	4.8	5/15
(1,2 _m)-CMA-ES	7.7*2	3.8*3	4.1*3	4.2*3	8.1*3	2*3	15/15
f_{15}	3.0e4	1.5e5	3.1e5	3.2e5	4.5e5	4.6e5	15/15
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
f_{16}	1400	2.7e4	7.7e4	1.9e5	2.0e5	2.2e5	15/15
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	150*3	∞	∞	∞	∞	∞	0/15
f_{17}	63	1000	4000	3.1e4	5.6e4	8.0e4	15/15
(1,2)-CMA-ES	400	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	8.6*	2.8e3*3	∞	∞	∞	∞	0/15
f_{18}	620	4000	2.0e4	6.8e4	1.3e5	1.5e5	15/15
(1,2)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	140*3	∞	∞	∞	∞	∞	0/15
f_{19}	1	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
(1,2)-CMA-ES	2.3e3	∞	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	660*2	∞	∞	∞	∞	∞	0/15
f_{20}	82	4.6e4	3.1e6	5.5e6	5.6e6	5.6e6	14/15
(1,2)-CMA-ES	13	30	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	6.5*3	30	∞	∞	∞	∞	0/15
f_{21}	560	6500	1.4e4	1.5e4	1.6e4	1.8e4	15/15
(1,2)-CMA-ES	13	8.3	10	11	10	9.1	11/15
(1,2 _m)-CMA-ES	5.9	3.3	3.6	3.4	3.3	2.9	15/15
f_{22}	470	5600	2.3e4	2.5e4	2.7e4	1.3e5	12/15
(1,2)-CMA-ES	10	27	120	120	110	22	1/15
(1,2 _m)-CMA-ES	8.8	13	21	20	18	3.7	5/15
f_{23}	3.2	1600	6.7e4	4.9e5	8.1e5	8.4e5	15/15
(1,2)-CMA-ES	340	1.8e3	∞	∞	∞	∞	0/15
(1,2 _m)-CMA-ES	32	140*	∞	∞	∞	∞	0/15

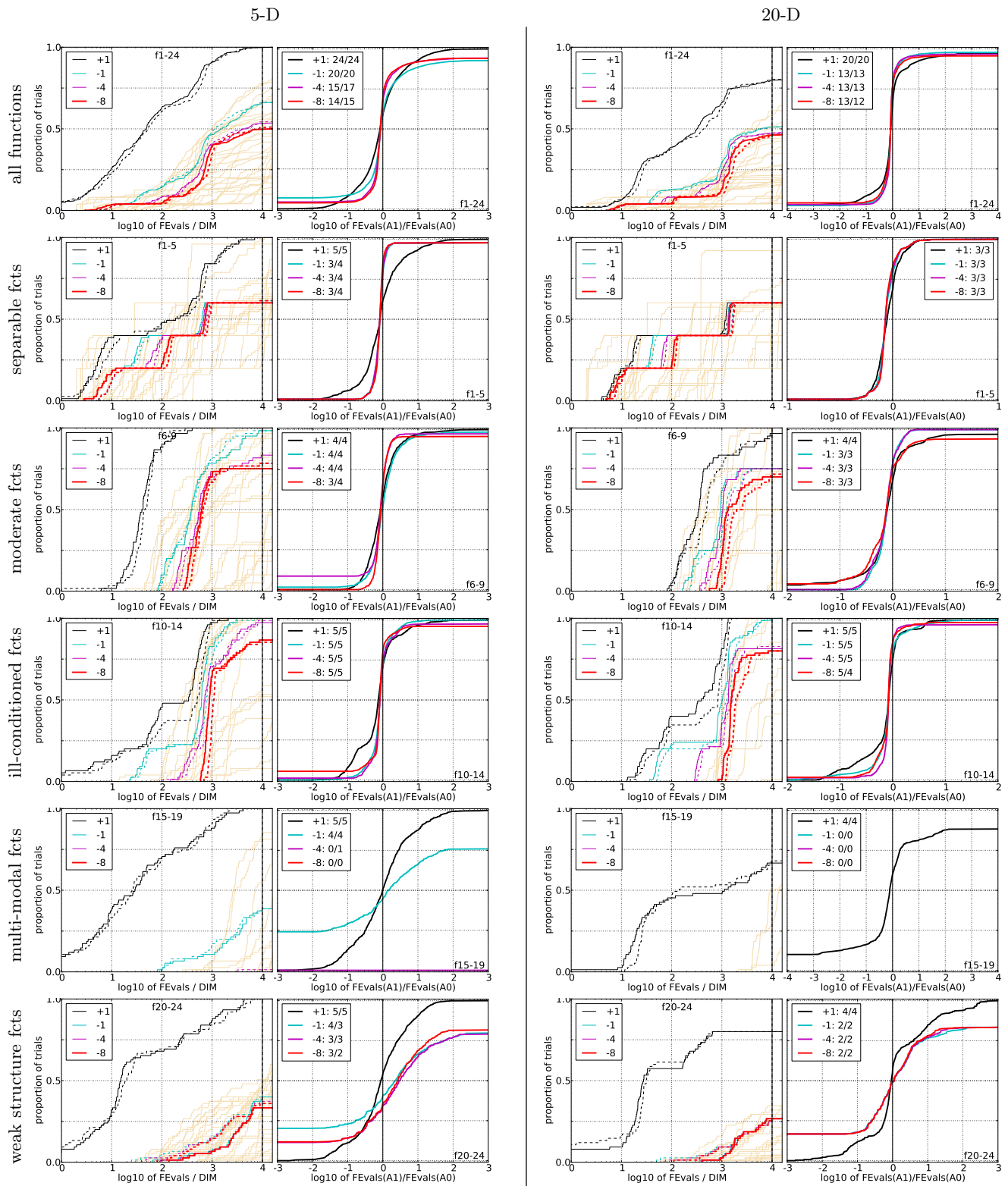


Figure 4: Empirical cumulative distributions of run lengths for $(1,2_m^s)$ -CMA-ES (solid) and $(1,2_m)$ -CMA-ES (dashed) and speed-up ratios of the former divided by the latter in 5-D (left) and 20-D (right) as in Fig. 2.

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5-D

Δf	1e+1	1e+0	1e-1	1e-3	1e-5	1e-7	#succ
f_1	11	12	12	12	12	12	15/15
(1,2 _m)-CMA-ES	3.2	10	15	28	40	53	15/15
(1,2 _m ^s)-CMA-ES	2.1	7.2	13	24	35	45	15/15
f_2	83	87	88	90	92	94	15/15
(1,2 _m)-CMA-ES	37	40	42	44	45	46	15/15
(1,2 _m ^s)-CMA-ES	30	34*	35*2	36*2	37*3	37*3	15/15
f_3	720	1600	1600	1600	1700	1700	15/15
(1,2 _m)-CMA-ES	3.5	150	450	450	450	450	1/15
(1,2 _m ^s)-CMA-ES	6.2	440	∞	∞	∞	∞	0/15
f_4	810	1600	1700	1800	1900	1900	15/15
(1,2 _m)-CMA-ES	11	∞	∞	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	8.8	450	∞	∞	∞	∞	0/15
f_5	10	10	10	10	10	10	15/15
(1,2 _m)-CMA-ES	3.1	4.8	4.8	4.9	4.9	4.9	15/15
(1,2 _m ^s)-CMA-ES	2.6	3.3	3.5	3.5	3.5	3.5	15/15
f_6	110	210	280	580	1000	1300	15/15
(1,2 _m)-CMA-ES	2.7	2.4	2.5	1.8	1.4	1.4	15/15
(1,2 _m ^s)-CMA-ES	2.2	2	2.1	1.6	1.1*	1.1*	15/15
f_7	24	320	1200	1600	1600	1600	15/15
(1,2 _m)-CMA-ES	7.2	5.3	11	220	220	220	2/15
(1,2 _m ^s)-CMA-ES	5.8	5	17	81	81	100	0/15
f_8	73	270	340	390	410	420	15/15
(1,2 _m)-CMA-ES	5.6	5.7	7.2	8.1	8.4	8.7	15/15
(1,2 _m ^s)-CMA-ES	4.2	5.8	6.8	7.5	7.6	7.8	15/15
f_9	35	130	210	300	340	370	15/15
(1,2 _m)-CMA-ES	7.4	14	12	11	11	10	15/15
(1,2 _m ^s)-CMA-ES	14	12	11	9.6	9.2	8.8	15/15
f_{10}	350	500	570	630	830	880	15/15
(1,2 _m)-CMA-ES	9.4	7.6	6.8	6.5	5.1	5.1	15/15
(1,2 _m ^s)-CMA-ES	7.3	5.8*2	5.4*2	5.2*2	4.1*3	4*3	15/15
f_{11}	140	200	760	1200	1500	1700	15/15
(1,2 _m)-CMA-ES	24	20	5.6	3.9	3.3	3	15/15
(1,2 _m ^s)-CMA-ES	19	16	4.8	3.3	2.8*	2.6*	15/15
f_{12}	110	270	370	460	1300	1500	15/15
(1,2 _m)-CMA-ES	25	16	14	14	6	5.9	15/15
(1,2 _m ^s)-CMA-ES	20	15	14	13	5.4	5.2	15/15
f_{13}	130	190	250	1300	1800	2300	15/15
(1,2 _m)-CMA-ES	9.4	30	34	14	27	37	4/15
(1,2 _m ^s)-CMA-ES	4.6	17	21	14	18	36	5/15
f_{14}	9.8	41	58	140	250	480	15/15
(1,2 _m)-CMA-ES	4.3	3.5	3.9	5.8	11	9.3	15/15
(1,2 _m ^s)-CMA-ES	3.1	3.2	3.3	4.8	9.2	8.1	15/15
f_{15}	510	9300	1.9e4	2.0e4	2.1e4	2.1e4	14/15
(1,2 _m)-CMA-ES	8.1	26	∞	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	11	25	∞	∞	∞	∞	0/15
f_{16}	120	610	2700	1.0e4	1.2e4	1.2e4	15/15
(1,2 _m)-CMA-ES	16	29	45	69	62	∞	0/15
(1,2 _m ^s)-CMA-ES	15	43	38	∞	∞	∞	0/15
f_{17}	5.2	210	900	3700	6400	7900	15/15
(1,2 _m)-CMA-ES	6.7	5.3	6.2	64	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	5.3	5	13	∞	∞	∞	0/15
f_{18}	100	380	4000	9300	1.1e4	1.2e4	15/15
(1,2 _m)-CMA-ES	3.7	28	23	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	8.2	27	21	∞	∞	∞	0/15
f_{19}	1	1	240	1.2e5	1.2e5	1.2e5	15/15
(1,2 _m)-CMA-ES	62	4.6e3	940	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	22	8.5e3	1.4e3	∞	∞	∞	0/15
f_{20}	16	850	3.8e4	5.4e4	5.5e4	5.5e4	14/15
(1,2 _m)-CMA-ES	4.5	11	∞	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	4.7	9.3	18	13	13	13	1/15
f_{21}	41	1200	1700	1700	1700	1800	14/15
(1,2 _m)-CMA-ES	5.7	5.8	9	8.9	8.9	8.8	13/15
(1,2 _m ^s)-CMA-ES	2.4	5	9.5	9.4	9.3	9.2	15/15
f_{22}	71	390	940	1000	1000	1100	14/15
(1,2 _m)-CMA-ES	10	11	20	19	18	18	14/15
(1,2 _m ^s)-CMA-ES	15	21	51	48	47	46	9/15
f_{23}	3	520	1.4e4	3.2e4	3.3e4	3.4e4	15/15
(1,2 _m)-CMA-ES	15	37	16	23	22	21	0/15
(1,2 _m ^s)-CMA-ES	6.4	23	9.5	∞	∞	∞	0/15
f_{24}	1600	2.2e5	6.4e6	9.6e6	1.3e7	1.3e7	3/15
(1,2 _m)-CMA-ES	8.3	∞	∞	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	6	∞	∞	∞	∞	∞	0/15

20-D

Δf	1e+1	1e+0	1e-1	1e-3	1e-5	1e-7	#succ
f_1	43	43	43	43	43	43	15/15
(1,2 _m)-CMA-ES	8.9	15	20	30	41	52	15/15
(1,2 _m ^s)-CMA-ES	8.1	13*	18	27	36*	47*	15/15
f_2	380	390	390	390	390	390	15/15
(1,2 _m)-CMA-ES	71	80	82	84	85	86	15/15
(1,2 _m ^s)-CMA-ES	61	67*2	69*2	70*3	71*3	72*3	15/15
f_3	5100	7600	7600	7600	7600	7700	15/15
(1,2 _m)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
f_4	4700	7600	7700	7700	7800	1.4e5	9/15
(1,2 _m)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
f_5	41	41	41	41	41	41	15/15
(1,2 _m)-CMA-ES	3.5	4.4	4.4	4.4	4.4	4.4	15/15
(1,2 _m ^s)-CMA-ES	3.3	3.9	4.1	4.1	4.1	4.1	15/15
f_6	1300	2300	3400	5200	6700	8400	15/15
(1,2 _m)-CMA-ES	2.2	1.8	1.8	2.5	3.5	6.7	13/15
(1,2 _m ^s)-CMA-ES	1.9	1.6	1.5	1.7	2.1	3.1	12/15
f_7	1400	4300	9500	1.7e4	1.7e4	1.7e4	15/15
(1,2 _m)-CMA-ES	71	∞	∞	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	71	∞	∞	∞	∞	∞	0/15
f_8	2000	3900	4000	4200	4400	4500	15/15
(1,2 _m)-CMA-ES	5.3	7.8	8.2	8.3	8.2	8.1	15/15
(1,2 _m ^s)-CMA-ES	3.6	4.5*	4.8*2	5*2	5*2	5*2	15/15
f_9	1700	3100	3300	3500	3600	3700	15/15
(1,2 _m)-CMA-ES	5.8	8.1	8.5	8.6	8.5	8.4	15/15
(1,2 _m ^s)-CMA-ES	4.5	6.5	6.8	6.9	6.9	6.8	15/15
f_{10}	7400	8700	1.1e4	1.5e4	1.7e4	1.7e4	15/15
(1,2 _m)-CMA-ES	3.5	3.5	2.9	2.2	2	2	15/15
(1,2 _m ^s)-CMA-ES	3.2	3*2	2.5*2	1.8*3	1.6*3	1.6*3	15/15
f_{11}	1000	2200	6300	9800	1.2e4	1.5e4	15/15
(1,2 _m)-CMA-ES	18	8.6	3.3	2.3	2	1.7	15/15
(1,2 _m ^s)-CMA-ES	16	7.7	2.9	2	1.7	1.5*	15/15
f_{12}	1000	1900	2700	4100	1.2e4	1.4e4	15/15
(1,2 _m)-CMA-ES	15	13	12	9.3	3.6	3.6	15/15
(1,2 _m ^s)-CMA-ES	7.4	8	7.6	6.4	2.6	2.6	15/15
f_{13}	650	2000	2800	1.9e4	2.4e4	3.0e4	15/15
(1,2 _m)-CMA-ES	13	12	24	33	55	93	0/15
(1,2 _m ^s)-CMA-ES	4.6	6.9	23	49	120	94	1/15
f_{14}	75	240	300	930	1600	1.6e4	15/15
(1,2 _m)-CMA-ES	7.7	3.8	4.1	4.2	8.1	2	15/15
(1,2 _m ^s)-CMA-ES	6	3.1	3.3*	3.6*	6.6*2	1.6*	14/15
f_{15}	3.0e4	1.5e5	3.1e5	3.2e5	4.5e5	4.6e5	15/15
(1,2 _m)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	∞	∞	∞	∞	∞	∞	0/15
f_{16}	1400	2.7e4	7.7e4	1.9e5	2.0e5	2.2e5	15/15
(1,2 _m)-CMA-ES	150	∞	∞	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	190	∞	∞	∞	∞	∞	0/15
f_{17}	63	1000	4000	3.1e4	5.6e4	8.0e4	15/15
(1,2 _m)-CMA-ES	8.6	2.8e3	∞	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	6.7	2.9e3	∞	∞	∞	∞	0/15
f_{18}	620	4000	2.0e4	6.8e4	1.3e5	1.5e5	15/15
(1,2 _m)-CMA-ES	140	∞	∞	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	130	∞	∞	∞	∞	∞	0/15
f_{19}	1	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
(1,2 _m)-CMA-ES	660	∞	∞	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	410	∞	∞	∞	∞	∞	0/15
f_{20}	82	4.6e4	3.1e6	5.5e6	5.6e6	5.6e6	14/15
(1,2 _m)-CMA-ES	6.5	30	∞	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	5.3	∞	∞	∞	∞	∞	0/15
f_{21}	560	6500	1.4e4	1.5e4	1.6e4	1.8e4	15/15
(1,2 _m)-CMA-ES	5.9	3.3	3.6	3.4	3.3	2.9	15/15
(1,2 _m ^s)-CMA-ES	7.2	4.9	4	3.9	3.7	3.3	14/15
f_{22}	470	5600	2.3e4	2.5e4	2.7e4	1.3e5	12/15
(1,2 _m)-CMA-ES	8.8	13	21	20	18	3.7	5/15
(1,2 _m ^s)-CMA-ES	8.2	13	17	16	15	3	6/15
f_{23}	3.2	1600	6.7e4	4.9e5	8.1e5	8.4e5	15/15
(1,2 _m)-CMA-ES	32	140	∞	∞	∞	∞	0/15
(1,2 _m ^s)-CMA-ES	50	390	∞				