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Local Decisions and Optimal Distributed Detection in Mobile Wireless Sensor Networks

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Abstract—A set of mobile wireless sensors observe the environment as they move about and make decisions based on their observations. They send/relay their decisions to a sensor, called the Cluster-Head (CH), that has requested all decisions made about observations from a given region during a specified time-interval. There are two sources of error facing the multi-hop cluster of sensors that results from this scenario: observations are corrupted by noise and transmissions suffer communication errors. Once the sensors' decisions have reached the CH, the optimal maximum a posteriori (MAP) detector is known to be a weighted order statistic of these noisy decisions.

We characterize the performance and energy usage of this decision fusion algorithm by: determining when local fusion reduces the CH's decision error rate and characterizing the tradeoff between the energy saved by compression of local decisions and the performance of the decision algorithm. Large deviation techniques, simulations and direct calculation are used to determine the performance of these strategies and to demonstrate that hybrids of them perform best.

I. INTRODUCTION

In an ad-hoc mobile wireless sensor network, mobile devices sample their environment, process these samples to make local decisions, and transmit/relay these decisions as needed. We consider a group of sensors that have each collected a sample in a specified spatial region during a specified time-interval. They process their samples as they continue to move. The first sensor to get a positive result floods the network with a request for decisions from all other sensors that took samples within the same spatiotemporal region. The other sensors report back to the requesting sensor, which is called the Cluster Head (CH) [1]. The CH fuses these local decisions to obtain a more reliable decision [2].

Our goal is to develop strategies that improve the performance or minimize the energy usage of decision fusion algorithms [3], [4]. This is a challenge because: (i) the decision each mobile sensor makes may be incorrect because of measurement errors; and, (ii) these noisy decisions must be transmitted/relayed to the CH over one or more noisy wireless hops because the sensors may have moved a significant distance during the time required to process their samples.

In the 2D version of this problem, the communication architecture for this sensor fusion problem may look like the 2-hop cluster shown in Fig. 1. The sensor in the center of the cluster, the CH, is the one that requested the decisions from other motes. The sensors shown in the first ring are

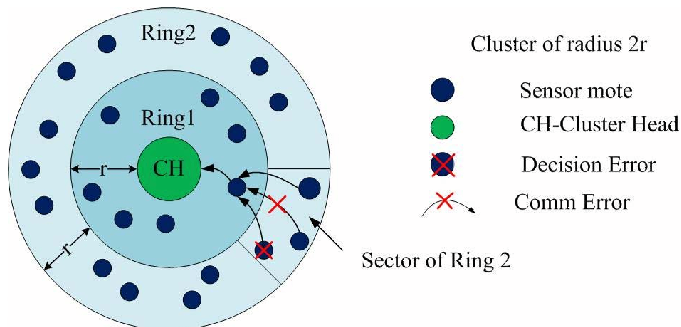


Fig. 1. A 2-hop, 2-D cluster. Each mote's decision may be incorrect because of measurement noise; transmitted packets may suffer bit errors because of noisy communication channels.

one wireless hop from the CH; the ones in the second ring are two hops away. Three sensors in a sector of ring 2 are shown forwarding their decisions to one sensor in ring 1. Each mote's observation is affected by measurement noise and all communications throughout the network will be affected by channel noise, fading, and transceiver errors. Because the cluster is multi-hop, decisions forwarded from the outer rings will suffer repeated exposure to the sources of communication error. Understanding the combined effects of these communication and measurement errors, and the development of energy-efficient strategies to mitigate these effects, are goals of this paper.

This paper is organized as follows: Section 2 describes the distributed detection problem in clustered sensor networks and summarizes results for the MAP detector derived in [5]. In Sections 3 through 5, large deviation techniques are used to bound the decision error probability of the MAP detector and show precisely when local fusion and coding provide better performance than simple forwarding – without increasing energy usage. In Section 6, simulations are used to compare the performance of a pure-relay strategy, a local-decision/repetition strategies, a hybrid strategy, and other strategies that use compression to trade detection performance for energy savings.

II. SYSTEM ASSUMPTIONS AND PRIOR RESULTS

A MAP approach to the distributed detection problem in a multi-hop sensor cluster was developed in [5]. It considered

a cluster with K rings and N_k motes in the k th ring. Each mote makes a decision between two hypotheses, $s_0 = 0$ and $s_1 = 1$: “1” denotes that an event has occurred and “0” that it has not. The decisions by different motes are assumed to be independent and identically distributed (i.i.d.) bernoulli variables, each with error probability $p_m < 1/2$. The noise processes in different wireless channels are assumed to be independent. The bit error rate in each hop in the network is modeled w.l.o.g. as a binary symmetric channel (BSC) with cross-over probability $p_c < 1/2$.

The decisions made by motes in the outer rings are relayed by the motes in inner rings to the CH. Let the error probability of the detection results received by the CH from the k th ring be $p_{e,k}$. Then, for example, $p_{e,1} = p_c(1 - p_m) + p_m(1 - p_c)$. Denote the detection result received by the CH from the i th mote in the k th ring by $r_{k,i}$ and arrange the detection results in the same ring in a vector $\bar{r}_k = (r_{k,1}, r_{k,2}, \dots, r_{k,N_k}), k = 1, 2, \dots, K$. Let E denote the event that a decision error happens at the CH.

It is known [5] that for a mote that is k hops away from the CH, the detection result received by the CH after being relayed over these hops has error probability: $p_{e,k} = \frac{1}{2} - \frac{1}{2}(1 - 2p_m)(1 - 2p_c)^k, k \geq 1$.

Suppose w.l.o.g. that the prior probabilities are $p(s = s_0) = p(s = s_1) = 1/2$. Define $\chi_k = \ln((1 - p_{e,k})/p_{e,k})$, and assume these χ_k 's can be scaled up such that $\chi_1 : \chi_2 : \dots : \chi_K = W_1 : W_2 : \dots : W_K$, where the W_k 's are positive integers with $\gcd(W_1, W_2, \dots, W_K) = 1$. The MAP-based decision bit is then given by [5]: $\hat{r} = \text{Median}(W_1 \diamond \bar{r}_1, W_2 \diamond \bar{r}_2, \dots, W_K \diamond \bar{r}_K)$, where $W \diamond x$ means x is duplicated W times. The decision error probability at the cluster head is then:

$$P(E) = \sum_{\sum_{k=1}^K (2c_k - N_k) > 0} \prod_{k=1}^K \binom{N_k}{c_k} (p_{e,k})^{c_k} (1 - p_{e,k})^{N_k - c_k} \quad (1)$$

where c_k is the number of occurrences of $1 - s$ in the vector $\bar{r}_k, k = 1, 2, \dots, K$.

Since the weighted median is the MAP detector, we assume – even as we introduce local fusion algorithms – that the CH calculates this weighted median after receiving inputs from all motes in the cluster. To understand what happens when local fusion is allowed, we first determine the asymptotic behavior of the weighted median for both the single-hop case and the more complex multi-hop case.

III. BEHAVIOR OF WEIGHTED MEDIAN MAP DETECTORS

The asymptotic behavior of the median filter in one-hop clusters was studied using large deviation techniques in [6]. We now find the error exponent for and bound the decision error probability for the more complex multi-hop case. This is the first step in determining when local fusion – which is not useful in single-hop clusters – is needed to combat repeated exposure of decisions to communication error in multi-hop networks.

With the weighted median filter, the decision error proba-

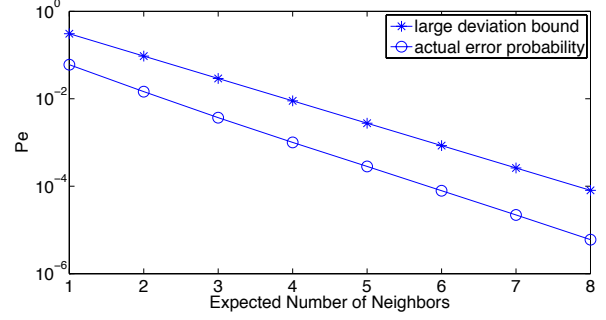


Fig. 2. Comparison for a 3-ring cluster of the log of the error probability from both simulations and the large deviations bound. The bound is *parallel* to the actual error probability; thus, maximizing the error exponent maximizes the *rate of decay* of the actual error probability. We can thus compare the performance of complex strategies via their error exponents to determine which ones are best. Simulations confirm these results also apply for small/moderate numbers of motes.

bility at the CH is found to be:

$$P(E) = P\left(\sum_{k=1}^K \sum_{i=1}^{N_k} W_k(r_{k,i} - 1/2) \geq 0\right) \quad (2)$$

Since $P(Z \geq 0) \leq E[e^{Zt}]$, let $r'_{k,i} = r_{k,i} - 1/2$, and $E[r'_{k,i}] = p_{e,k} - 1/2 < 0$. We have:

$$P(E) \leq E[e^{t \sum_{k=1}^K \sum_{i=1}^{N_k} W_k r'_{k,i}}] = \prod_{k=1}^K \prod_{i=1}^{N_k} E[e^{t W_k r'_{k,i}}] \quad (3)$$

For each term in the product, $E[e^{t W_k r'_{k,i}}] = p_{e,k} e^{\frac{1}{2} t W_k} + (1 - p_{e,k}) e^{-\frac{1}{2} t W_k}$. Setting $\frac{\partial E[e^{t W_k r'_{k,i}}]}{\partial t} = 0$, we find:

$$\frac{1}{2} W_k p_{e,k} e^{\frac{1}{2} t W_k} = \frac{1}{2} W_k (1 - p_{e,k}) e^{-\frac{1}{2} t W_k} \quad (4)$$

which results in $t = 1$. This last equality follows because the results for the MAP detector in (4) state that $W_k = \ln((1 - p_{e,k})/p_{e,k})$. The optimal t 's are thus same for every ring. When $t = 1$, $E[e^{t W_k r'_{k,i}}] = 2 \sqrt{p_{e,k}(1 - p_{e,k})}$. Thus:

$$P(E) \leq \prod_{k=1}^K \prod_{i=1}^{N_k} E[e^{W_k r'_{k,i}}] = \prod_{k=1}^K \left[2 \sqrt{p_{e,k}(1 - p_{e,k})}\right]^{N_k} \quad (5)$$

Fig. 2 compares this large deviation bound on the error probability with the error probability from simulations for a 3-ring cluster. It shows the desired results that: (a) the difference between them approaches zero as the number of motes increases (it is log plot); and (b) the large deviation *error exponent* is very accurate – the bound is *parallel* with the simulation result over the entire range of spatial densities. The bound is off by a multiplicative factor, which is typical for large deviation techniques. The conclusion is that it is reasonable to maximize the magnitude of the error exponent because that will maximize the *rate of decay* of the actual error probability. Simulations are used later to confirm this.

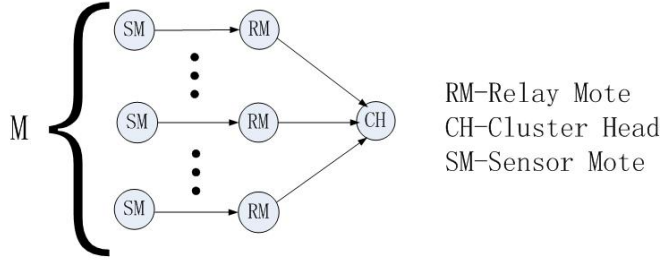


Fig. 3. The *pure-relay* strategy in a two-hop network.

IV. LOCAL-DECISION/REPETITION STRATEGY

We now analyze a decision and relay strategy that reduces the decision error probability at the CH. We consider the case that one mote in ring i receives a set of individual detection results from M motes in ring $i + 1$. Assume w.l.o.g. that each individual detection result has an equal probability to be “0” or “1”.

The relay mote can either: (i) Relay every received bit toward the CH – this will be called the *pure-relay* decision strategy and is the most commonly assumed strategy; or (ii) Make a local decision by computing the locally optimal weighted median of the M decision bits it receives from the motes in a sector of the next furthest ring. The relay mote then transmits this decision M times to the CH. This will be called the *local-decision/repetition* strategy.

The number of bits transmitted by the relay mote is the same in both the pure-relay and the local-decision/repetition strategies, so both use the same communication energy. The local-decision/repetition strategy uses slightly more processing energy because the mote must compute the median of the bits it has received.

Figs. 3 and 4 show diagrams of the pure-relay and local-decision/repetition strategies. Assume w.l.o.g. that the CH makes a decision based on relayed detection results using the median filter. Then, since both strategies use the same energy, we want to know which one has a lower error probability at the CH. We will show that the local-decision/repetition strategy is *significantly* better for the motes in the outer rings because it counteracts the effect of a series of communication errors as the decisions are hopped to the CH.

Denote the error probability of each individual detection result received by the relay mote by p_d . Denote the cross-over probability of the BSC between the relay mote and the CH by p_c , with $0 < p_d, p_c < 1/2$. Define $h(M, p)$ to be the decision error probability when applying the median filter to M independent and identically distributed (i.i.d.) bernoulli variables, each with failure probability p . Also define $l(a, b) = a(1 - b) + b(1 - a)$. In the global decision strategy, the error probability of individual detection results received by the CH is then $l(p_d, p_c)$. Also note that in the local-decision/repetition

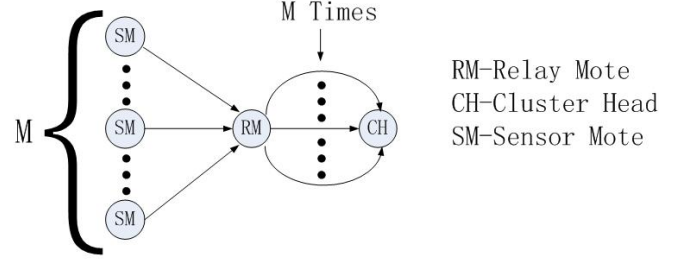


Fig. 4. The *local-decision/repetition* strategy in a two-hop network.

strategy, the multiple copies of the local-decision result sent by the relay mote to the CH are not mutually independent but are conditionally independent given the local decision result. Let E_G and E_L denote the events of the decision error that happens at the CH using the pure-relay and local-decision/repetition strategies, respectively. Then: $P(E_G) = h(M, l(p_d, p_c))$ and $P(E_L) = l(h(M, p_d), h(M, p_c))$.

When M is an even number, we always have $h(M, p) = h(M - 1, p)$, so it is thus not economical to use an even number of bits because one is wasted. We now prove the hypothesis in the odd number case. We compare the two strategies when the source code of the repetition code is “1”. Denote the number of “1”s received by the relay sensor by S . The decision error probabilities are, $P(E_G) = \sum_{D=0}^M P(E_G|S = D)P(S = D)$ and $P(E_L) = \sum_{D=0}^M P(E_L|S = D)P(S = D)$, where $P(S = D) = \binom{M}{D} p_d^D (1 - p_d)^{M-D}$.

In the pure-relay strategy, D “1”s and $M - D$ “0”s are transmitted from the relay mote to the CH. In the local-decision/repetition strategy, the median filter is adopted at the relay mote. Since M is assumed to be odd, when $0 \leq D \leq \lfloor M/2 \rfloor$, the local decision result is “0”, and M “0”s are relayed to the CH. When $\lceil M/2 \rceil \leq D \leq M$, the local decision result is “1”, and M “1”s are relayed. Denote the individual detection results received by the relay mote by s_i 's, $1 \leq i \leq M$. We partition them into two sets. The received “0”s are classified to T_0 , while the received “1”s are classified to T_1 , i.e., $T_0 = \{i|s_i = 0, 1 \leq i \leq M, i \in N\}$ and $T_1 = \{i|s_i = 1, 1 \leq i \leq M, i \in N\}$. The cardinality of T_1 is D ; for T_0 it is $M - D$.

In the pure-relay strategy, after the transmissions from the relay mote to the CH, among the elements in T_1 , suppose A ones remain “1” and $D - A$ ones flip to “0”. Also, among the elements in T_0 , suppose B zeros flip to “1” and $M - D - B$ zeros remain “0”. Certainly, all the (A, B) pairs are under the constraints that $0 \leq A \leq D$ and $0 \leq B \leq M - D$. If a pair (A, B) satisfies the condition $A + B \leq \lfloor M/2 \rfloor$, a decision error will occur at the CH. We call such pairs decision error characteristic pairs and denote the set containing all these (A, B) pairs by F_0 . Given M and p_c , the probability of

achieving (A, B) from $(D, M - D)$ after transmission is:

$$f_1(M, D, A, B, p_c) = \binom{D}{A} (1 - p_c)^A p_c^{D-A} \binom{M-D}{B} p_c^B (1 - p_c)^{M-D-B} \quad (6)$$

In the local-decision/repetition strategy, partition the detection results received by the relay mote in the same way as the pure-relay strategy. Assume $\lceil M/2 \rceil \leq D \leq M$, so all the s_i 's that belong to T_0 are converted to "1" to transmit. The probability of achieving (A, B) from $(D, M - D)$ after the transmission is

$$f_2(M, D, A, B, p_c) = \binom{D}{A} (1 - p_c)^A p_c^{D-A} \binom{M-D}{B} (1 - p_c)^B p_c^{M-D-B} \quad (7)$$

Note that the following two equalities always hold,

$$f_2(M, D, D - A, B, p_c) = f_1(M, D, A, B, 1 - p_c) \quad (8)$$

$$f_2(M, D, A, M - D - B, p_c) = f_1(M, D, A, B, p_c) \quad (9)$$

Denote the set containing all the (A, B) pairs satisfying both $A + B \leq \lfloor M/2 \rfloor$ and $A - B \leq \lfloor D - M/2 \rfloor$ by F_1 . For all such pairs, we can further get $A + (M - D - B) \leq \lfloor M/2 \rfloor$, which implies that the pair $(A, M - D - B)$ is also a decision error characteristic pair. Denote the set containing all such $(A, M - D - B)$ pairs by F_2 . Denote the set containing all the (A, B) pairs satisfying both $A + B \leq \lfloor M/2 \rfloor$ and $A - B \geq \lceil D - M/2 \rceil$ by F_3 . For all such pairs, we can further get $(D - A) + B \leq \lfloor M/2 \rfloor$, which implies that the pair $(D - A, B)$ is also a decision error characteristic pair. Denote the set containing all such $(D - A, B)$ pairs by F_4 . There is a one-to-one mapping between F_1 and F_2 , and another one between F_3 and F_4 . So $|F_1| = |F_2|$, and $|F_3| = |F_4|$. Apparently, $F_1 \cap F_3 = \emptyset$, and $F_1 \cup F_3 = F_0$. Thus, F_1 and F_3 are mutually exclusive and collectively exhaustive subsets in F_0 . Assume $F_2 \cap F_4 \neq \emptyset$, which means that at least one decision error characteristic pair falls into the intersection of the sets F_2 and F_4 . Suppose this pair is denoted as $(A_1, M - D - B_1)$ and $(D - A_2, B_2)$ in F_2 and F_4 , respectively. Then A_1, A_2, B_1, B_2 must satisfy both $D - A_2 = A_1$ and $B_2 = M - D - B_1$, which induces $A_1 + A_2 + B_1 + B_2 = M$. Since $A_1 + B_1 \leq \lfloor M/2 \rfloor$ and $A_2 + B_2 \leq \lfloor M/2 \rfloor$, $A_1 + A_2 + B_1 + B_2 \leq M - 1$. But this contradicts our assumption. Thus we have proved that $F_2 \cap F_4 = \emptyset$. $F_2, F_4 \in F_0$, and we simply have $|F_2 \cup F_4| = |F_2| + |F_4| = |F_1| + |F_3| = |F_1 \cup F_3| = |F_0|$. Therefore, we can declare that F_2 and F_4 are also mutually exclusive and collectively exhaustive subsets in F_0 .

Define $g(M, D, p_c) = P(E_G | S = D) - P(E_L | S = D)$. For $\lceil M/2 \rceil \leq D \leq M$,

$$g(M, D, p_c) = \sum_{F_1} f_1(M, D, A, B, p_c) + \sum_{F_3} f_1(M, D, A, B, p_c) - \sum_{F_2} f_2(M, D, A, M - D - B, p_c) - \sum_{F_4} f_2(M, D, D - A, B, p_c) \quad (10)$$

Since $D - A + B \leq \lfloor M/2 \rfloor$ for all (A, B) pairs in F_3 , and $0 < p_c < 1/2$, each term in the above equation is positive, so that the entire sum is positive. Note that for $\lceil M/2 \rceil \leq D \leq M$ and all (A, B) pairs in F_0 ,

$$f_1(M, M - D, B, A, p_c) = f_1(M, D, A, B, 1 - p_c) \quad (11)$$

$$f_2(M, M - D, B, A, p_c) = f_2(M, D, A, B, 1 - p_c) \quad (12)$$

Therefore, $g(M, M - D, p_c) = \sum_{F_0} f_1(M, M - D, B, A, p_c) - \sum_{F_0} f_2(M, M - D, B, A, p_c) = g(M, D, 1 - p_c) = -g(M, D, p_c)$. For each $\lceil M/2 \rceil \leq D \leq M$, we have $0 \leq M - D \leq \lfloor M/2 \rfloor$. Now we examine the difference between $P(E_G)$ and $P(E_L)$.

$$\Delta P = \sum_{D=\lceil M/2 \rceil}^M (P(S = D) - P(S = M - D))g(M, D, p_c) \quad (13)$$

When $\lceil M/2 \rceil \leq D \leq M$ and $0 < p_c < 1/2$, $P(S = D) - P(S = M - D) > 0$. We also know $g(M, D, p_c) > 0$ from (10). So we have proved that $\Delta P > 0$ for M odd, and thus for M even as well.

Thus, the local-decision/repetition strategy performs much better than the pure-relay strategy. We now see a corollary of this conclusion in a multi-hop system. Suppose there are K hops before reaching the CH. Define G_0 as the strategy that relays every bit to the cluster head and then makes a global decision. Define $G_k, 1 \leq k \leq K - 1$ as strategies that relay every bit to the mote k hops away from the CH and then, at each hop after that uses the local-decision/repetition strategy to forward M bits to the next hop. Comparing their decision error probabilities, $P_e(G_k)$, we can easily see that

$$P_e(G_0) > P_e(G_1) > \dots > P_e(G_{K-1}) > P_e(G_{K-1}) \quad (14)$$

This result leads to two conclusions. First, with the same energy consumption, one way to improve performance is to use the local-decision/relay strategy from the very beginning; i.e., starting at the first hop along the relay path. Second, the performance degradation is cumulative from right to left in (15). In other words, the benefit of the local-decision/repetition strategy is greater for motes further from the CH.

Fig. 5 shows the results of simulations to compare the decision error probability at the CH between the pure-relay and local-decision/repetition strategies.

V. HYBRID DECISION STRATEGY

We now design relay strategies for the entire multi-hop sensor network. In each ring k , partition the motes into different sectors. Assume each sector has M motes that each use the same relay mote in ring $k-1$. For each sector, the decision error probability is reduced by adopting the local-decision/repetition strategy. However, the number of independent detection results received by the CH is also reduced M times. So in our proposed relay strategy, we should trade the accuracy for the number of the individual detection results under the framework of the large deviation bound. Here, we assume that all N_k 's are divisible by M . Since M is usually small and N_k is large, the number of sectors, N_k/M , is still large. For the pure-relay

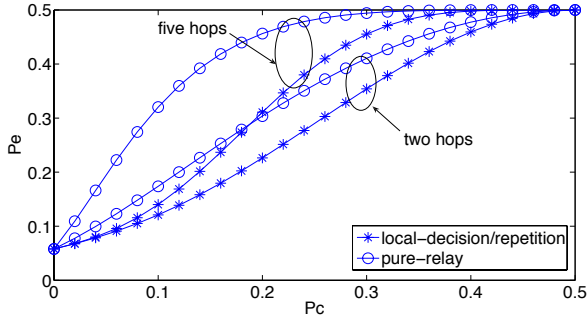


Fig. 5. Calculations of the probability of error for the local-decision/repetition strategy for small numbers of motes show how much it improves the detection performance for a range of communication error probabilities. The improvement increases with both the number of wireless hops and the number of motes, M , whose decisions are fused locally.

and local-decision/repetition strategies, we have

$$\begin{aligned} \lim_{N_k \rightarrow \infty} -\frac{1}{N_k} P(E_G) &= C(p_{e,k}) \\ C(p_{e,k}) &= -\ln(2) - \frac{1}{2} \ln(p_{e,k}(1-p_{e,k})) \\ p_{e,k} &= \frac{1}{2} - \frac{1}{2}(1-2p_m)(1-2p_c)^k \end{aligned} \quad (15)$$

and

$$\begin{aligned} \lim_{N_k \rightarrow \infty} -\frac{1}{N_k} P(E_L) &= C'(M, p'_{e,k}) \\ C'(M, p'_{e,k}) &= \frac{1}{M} [-\ln(2) - \frac{1}{2} \ln(p'_{e,k}(1-p'_{e,k}))] \\ p'_{e,k} &= \frac{1}{2} - \frac{1}{2}(1-2h(M, p_{e,1}))(1-2h(M, p_c))^{k-1} \end{aligned} \quad (16)$$

respectively, where $p'_{e,k}$ can be obtained by solving the first-order difference equation $p'_{e,k} = p'_{e,k-1}(1-h(M, p_c)) + (1-p'_{e,k-1})h(M, p_c)$, $k \geq 2$ with the initial condition $p'_{e,1} = h(M, p_{e,1})$. In each ring, the motes can compare these two large deviation rates, and adopt the relay strategy corresponding to the larger rate. We consider two extreme cases. First, when $k=1$, there is no relay step during the transmission between the motes and the CH. In this case, the pure-relay strategy is already known to minimize the decision error probability. Hence, motes in the first ring should always adopt the pure-relay strategy. To complete the analysis, we give a concise proof that $C(p_{e,1}) > C'(M, p'_{e,1})$. For any positive odd number M , based on the Cauchy-Schwarz inequality,

$$\begin{aligned} h(M, p_{e,1})(1-h(M, p_{e,1})) &\geq \left(\sum_{i=\lceil M/2 \rceil}^M \sqrt{\binom{M}{i} \binom{M}{M-i}} p_{e,1}^M (1-p_{e,1})^M \right)^2 \\ &= (2^M/2)^2 p_{e,1}^M (1-p_{e,1})^M \end{aligned} \quad (17)$$

Note that the equality can not hold unless the two vectors are proportional to each other; i.e., $p_{e,1} = 0, 1/2$ or 1 . We can further get $C'(M, p'_{e,1}) < C(p_{e,1})$. For any positive even number M , since $h(M, p_{e,1}) = h(M-1, p_{e,1})$, $C'(M, p'_{e,1}) <$

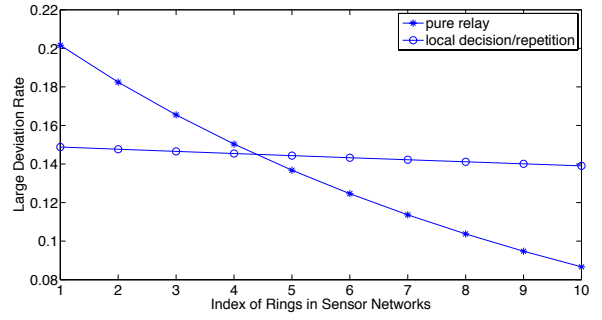


Fig. 6. Large deviation rates as a function of the number of rings for pure-relay and local-decision/repetition strategies. The crossing of the rates implies that the error probabilities for a finite number of motes cross as well. Motes in the outer rings should thus use the local-decision/repetition strategy; those in the inner rings should use the pure-relay strategy.

$C'(M-1, p'_{e,1})$, and the inequality still holds.

Now, assuming K is sufficiently large, we consider the case for $k=K$. Since $\ln(1+x) \cong x$ when x is sufficiently small, we have $C(p_{e,K}) \cong \frac{1}{2}(1-2p_m)^2(1-2p_c)^{2K}$ and $C'(M, p'_{e,K}) \cong \frac{1}{2M}(1-2h(M, p_{e,1}))^2(1-2h(M, p_c))^{2(K-1)}$. Comparing $C(p_{e,K})$ and $C'(M, p'_{e,K})$, we can get $\ln(C(p_{e,K})) - \ln(C'(M, p'_{e,K})) \cong 2K[\ln(1-2p_c) - \ln(1-2h(M, p_c))] + 2\ln(1-2p_m) + \ln(M) - 2\ln(1-2h(M, p_{e,1})) + 2\ln(1-2h(M, p_c))$.

Note that $p_c > h(M, p_c)$, so $\ln(1-2p_c) - \ln(1-2h(M, p_c)) < 0$. Since all the other terms in the difference are independent of K , when K is sufficiently large, $C(p_{e,K}) < C'(M, p'_{e,K})$. Hence, motes many hops away from the cluster head should always adopt the local-decision/relay strategy.

Thus, in one-hop sensor networks, the pure-relay strategy outperforms the local-decision/repetition strategy. In multi-hop sensor networks, though, the local-decision/repetition strategy is more resilient to communication error than the pure-relay strategy. This advantage is especially apparent when the number of hops is large. Fig.6 shows the comparison of the large deviation rate between the global and local decision strategies in different cases. The number of hops at which the bounds cross increases as the one-hop communication error probability decreases.

To minimize the upper bound on the overall decision error probability, the large deviation rate can be used to determine which rings should fuse local decisions. The motes will switch from the global decision strategy to the local decision strategy when k is large enough to make $C'(M, p'_{e,k})$ greater than $C(p_{e,k})$. We call this the *hybrid-decision* strategy. Denote the event that the decision error happens at the CH using the hybrid decision strategy by E_H . Given M , the upper bound of the decision error probability at the cluster head is

$$\ln(P(E_H)) \leq -\sum_{k=1}^K N_k \max\{C(p_{e,k}), C'(M, p'_{e,k})\} \quad (18)$$

These three decision and relay strategies are now compared via simulation. When the motes are uniformly deployed in a plane, the average number of motes in ring k is $N_k = d(2k-1)$,

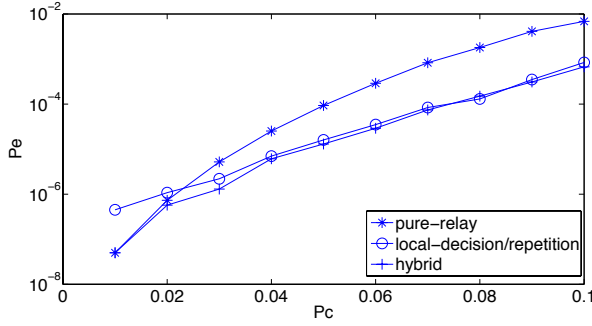


Fig. 7. Comparisons via simulation of the pure-relay, local-decision/repetition, and hybrid strategies. The hybrid decision strategy lowers the probability of decision error by using local decisions when needed.

where $d \in N$ is the distribution density. Fig. 7 compares the three relay strategies when $P_m = 0.2, K = 5, d = 3, M = 3$. The hybrid strategy is the best for all p_c 's. In the high p_c interval, the hybrid decision strategy is close to the local-decision/repetition strategy; in the low p_c interval, the hybrid decision strategy is close to the pure-relay strategy.

VI. ADDITIONAL STRATEGIES

In this section, six strategies for distributed detection in mobile sensor networks are compared via simulation. The first three were analyzed above; the others are variations of them that either save energy or improve performance.

- 1) The *pure-relay* strategy: The decision reached by each sensor mote is relayed to the CH by other motes over as many hops as necessary. No fusion of decisions takes place anywhere along the route
- 2) The *local-decision/repetition* strategy: For each i , a local decision (median) is made by a relay mote in ring i based on the M independent decisions transmitted to it by the M motes in a sector of ring $i + 1$. This local decision is transmitted M times by the relay mote in ring i to a relay mote in ring $i - 1$. The relay mote in ring $i - 1$ computes the median of these M bits and then transmits its decision M times to a relay mote in ring $i - 2$. This *fuse/repeat* process continues until M bits reach the CH. It is also performed for every set of M motes in every ring except the first ring.
- 3) The *hybrid* strategy: Sensor motes in different rings adopt the *pure-relay* (1) or *local-decision/repetition* (2) strategy by choosing the one with the biggest large-deviation exponent. When all motes have the same measurement error probability and every transmission has the same communication error probability, the innermost rings might adopt the *pure-relay* strategy while all others adopt the *local-decision/repetition* strategy.
- 4) The *hierarchical decision* strategy: The local decisions are made in a hierarchical manner. Each mote in a set of M motes in ring i transmit their decisions to a relay mote in ring $i - 1$. The relay mote fuses these received bits by computing the median and then relays this bit M

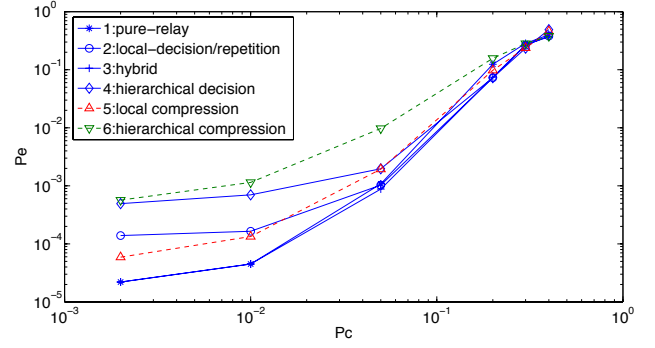


Fig. 8. Comparisons via simulation of decision error probabilities achieved by 6 different strategies in a 3 ring network. The ones in solid use the same amount of communication energy. The compression strategies (dashed) use less energy because they make fewer transmissions.

times to a relay mote in ring $i - 2$. The relay mote in ring $i - 2$ fuses the M bits repeated to it by each of M relay motes in ring $i - 1$ and then transmits its decision M^2 times to the relay mote in ring $i - 3$.

- 5) The *local compression* strategy: This is the *local-decision/repetition* strategy in (2) but without repetition. Thus, M sensor motes in ring i send their independent decisions to a relay mote in ring $i-1$. The relay mote fuses (median) these M bits and transmits the single bit that results to a relay mote in ring $i-1$. This bit is forwarded by each relay mote after that until it reaches the CH – no further fusion takes place.
- 6) The *hierarchical compression* strategy: It is the same as the *hierarchical decision* strategy, but local fusions at each hop are transmitted only once to the next hop. This significantly reduces the number of bits transmitted and thus the energy used.

We first compare the number of transmissions, and thus the energy used, by these strategies for a sensor network that has a tree topology with the CH as the root. Suppose there are Q motes in the inner-most ring around the CH and that the k 'th ring has $N_k = L^{k-1}Q$, $1 \leq k \leq K$ motes, where L is an integer. There are thus $(L^K - 1)Q/(L - 1)$ motes in the network. All motes except those in the outermost ring are responsible for relaying the decision results transmitted from the L motes in the corresponding sector in the next closest ring. In this case, strategies (1) through (4) each require a total of $(KL^{K+1} - (K + 1)L^K + 1)Q/(L - 1)^2$ transmissions. Strategy (5) requires $(L^K - 1)Q/(L - 1) + ((K - 1)L^K - KL^{K-1} + 1)Q/(L - 1)^2$ transmissions and strategy (6) requires $(L^{K+1} - KL + K - 1)Q/(L - 1)^2$.

Fig. 8 shows the results of simulations to determine the detection error performance of the 6 strategies when $P_m = 0.2, K = 3, Q = 3, L = 3, M = 3$. When the communication noise is severe, strategies 2 and 4 perform very well and the detection error probabilities of 1, 5, and 6 are similar. When the communication noise is negligible, strategy 1 is always the best, 5 and 6 are the worst, and 2 and 4 are in the middle.

VII. CONCLUSION

This paper developed tools to determine when local fusion of decisions improves the performance of distributed detection algorithms operating in the multi-hop clusters that result from distributed detection tasks in mobile sensor nets. Our current efforts include development of optimization tools that enable the best tradeoffs between detection-performance, delay, energy-usage, etc. to be found for any sensor network application.

REFERENCES

- [1] S. Bandyopadhyay, Q. Tian and E.J. Coyle, "Spatio-temporal sampling rates and energy efficiency in wireless sensor networks," *IEEE/ACM Trans. Netw.*, vol. 13, no. 6, pp. 1339–1352, Dec. 2005.
- [2] S. Bandyopadhyay and E.J. Coyle, "Minimizing communication costs in hierarchically clustered networks of wireless sensors," *Comput. Netw.*, vol. 44, no. 1, pp. 1–16, Jan. 2004.
- [3] R. Viswanathan and P. K. Varshney, "Distributed detection with multiple sensors: Part I–Fundamentals," *Proc. IEEE*, vol. 85, no. 1, pp. 54–63, Jan. 1997.
- [4] R. S. Blum, S. A. Kassam, and H. V. Poor, "Distributed detection with multiple sensors: Part II–Advanced topics," *Proc. IEEE*, vol. 85, no. 1, pp. 64–79, Jan. 1997.
- [5] Q. Tian and E.J. Coyle, "Optimal distributed detection in clustered wireless sensor networks," *IEEE Trans. Signal Processing*, vol. 55, no. 7, pp. 3892–3904, Jul. 2007.
- [6] J.-F. Chamberland and V.V. Veeravalli, "Asymptotic results for decentralized detection in power constrained wireless sensor networks," *IEEE J. Sel. Areas of Commun.*, vol. 22, no. 6, pp. 1007–1015, Aug. 2004.