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# Competitive Sharing of the Spectrum in Cognitive Radio Network: A Market Equilibrium Framework

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**Abstract**—In this paper, we consider the problem of pricing the spectrum usage in a cognitive radio network. In such a network, where licensed/primary users (who has the right to use the spectrum) and the unlicensed/secondary users coexist, a secondary spectrum market can be established where the primary network service provider charges the secondary users for the usage of the spectrum, while the secondary users also compete with each other for the access. Several important questions arise regarding the operation of such spectrum market: 1) how the spectrum should be priced; 2) how the secondary users should distributedly access the spectrum based on the spectrum price. To answer these questions, we interpret the available spectrum in the network as the aggregated interference tolerable at the receivers of the primary users, and introduce the notion of a market equilibrium in which the prices of the spectrum/interference are set correctly such that 1) the supply of the spectrum equals the demand of the spectrum, and 2) the secondary network is stable. We also developed an algorithm to distributedly compute such market equilibrium, and proved its convergence.

## I. INTRODUCTION

The traditional fixed radio spectrum allocation scheme requires that the telecommunication industry's regulator (for example, FCC) allocate licenses for the utilization of well-defined bands within the available spectrum. Consequently, unlike other commodities, no secondary market for the spectrum has ever been developed. Technological limitations as well as legal and regulatory constraints may have contributed to this fact. Today the evidence clearly points to a situation of relative under-utilization of the spectrum (see [1] [2]).

Recently, the paradigm of the cognitive radio network (CRN) has gained significant popularity, partly due to the fact that it is believed to be able to increase the efficiency of spectrum utilization [3]. In such a network, unlicensed/secondary users (SUs) are allowed to use the spectrum in a controlled fashion such that the communication of the licensed/primary users (PUs) is not affected. Various schemes have been proposed in the literature for the secondary network to efficiently and dynamically utilize the available spectrum, for example, [4], [5], [6], [7], [8], [9], [10]. The most important question is that how the SUs should dynamically and distributedly allocate their communication resources to access the available spectrum as well as to obey various regulatory constraints.

The flexibility introduced by the CRN also opens up the possibility of a secondary spectrum market. In such a market, the primary network leases the available spectrum to the

SUs and charges them for the consumption of the spectrum/bandwidth. Naturally, the price of the spectrum should be properly determined such that the available spectrum is efficiently utilized and the primary network is reasonably rewarded. In order to achieve these goals, a framework for regulating the activity of this market needs to be developed such that the spectrum access technique used by the secondary network as well as the profit structure of the primary network are fully considered. Our work serves as a first step towards developing such a framework.

The rest of the paper is organized as follows. In section II, we review the literature. In section III, we develop the CRN under consideration. In section IV, we briefly describe the secondary spectrum market and showed that the problem of spectrum allocation of the secondary network and the profit maximization of the primary network can be formulated into finding a market equilibrium. In section V, we develop a distributed algorithm to compute such equilibrium and prove its convergence. In section VI, we discuss some possible extensions and generalizations of the proposed framework. In section VII, we show some numerical results. This paper concludes in VIII.

## II. LITERATURE REVIEW

The problem of the spectrum leasing in CRN has received considerable attention in the literature. In a series of work conducted by Niyato et al [11], [12], [13], the primary network service providers (SPs) are modeled as being either competitive or cooperative for selling the spectrum, resulting in different pricing strategies as well as different profit structures. However, a successful market model for the CRN must also adequately consider the behavior of the cognitive users, especially their strategies for spectrum sharing. The previously mentioned works are not sufficient in this matter: they only consider simplified spectrum sharing techniques that are essentially centralized and are mainly orchestrated by the primary SPs, strategies which are not desirable in a distributed network like CRN. Specifically, they assume that the SUs access the available spectrum in a time-division multiple access (TDMA) fashion, in which only a single user (primary or secondary) can use the entire spectrum for each time slot. This access scheme is inherently centralized: the primary network needs to inform the secondary network which

time slots are currently available for access and the SUs also need to be coordinated by another central entity in order not to access the same time slot at the same time.

In [14], the authors considered a spectrum market where multiple SPs operating with different technologies and costs compete for potential customers. However, as in the previous cited works, this work is also insufficient in the modeling of the SU behavior. In [15], the authors considered a spectrum leasing model in which the PU also actively participates in the secondary network resource allocation game, and is well rewarded for its participation. In this scheme, the PU and the SUs can access the bandwidth simultaneously as long as some interference constraints on the PU receivers are enforced. However, only a simple network is considered (single PU and a single channel), and it is not clear how this work can be generalized to, say, multiple-PU and multiple-channel case.

The works [10], [16] considered the problem of achieving a competitive equilibrium (CE) for a wireless network in which the sum power of the users using the same channel needs to be limited. The authors model the sum power allocation on a specific channel as the demand, and the upper limit on this channel as the supply. The authors developed a set of strategies for users to set their powers and for the system manager to set the prices so that the system reaches a CE. This work is similar to our work in the modeling of the power on a channel as goods to be exchanged, and in the definition of the equilibrium solution for the system. However, this work considered a simplified user strategy space: there is no limit of the sum power for each user, nor is there any limit on the maximum amount of power each user can transmit on each channel. Moreover, the algorithm developed in this work requires that the system manager *do not* update the prices *until* all the user power profiles converge to a Nash Equilibrium (NE). In another word, the system manager needs to *coordinate* with the users in order to correctly update the prices. This coordination is hard to achieve for the following reasons: (1) the system manager might not have enough information (the individual power trajectory produced by each user) for determining if the entire system has reached a NE; (2) even the system manager does obtain such individual power trajectory, the equilibrium is still hard to determine—the diminishing differences between two successive power profiles doesn't immediate translate to the convergence of the sequence.

Our work is different from the previously cited works in several important ways. First of all, the CRN configuration under consideration is very general: we consider a complicated network in which multiple SUs and multiple PUs share the spectrum, and the spectrum is further divided into several subchannels. Secondly, the algorithm proposed is distributed in nature: (1) the secondary users access the spectrum distributedly: there is no centralized entity coordinating the secondary network, and the SUs are allowed to autonomously make spectrum decisions based on certain local measurements and the current spectrum prices; (2) the spectrum prices are computed distributedly: the spectrum manager/service provider only needs to obtain a summary statistics to compute the

instantaneous spectrum prices, and the prices can be updated without coordination with the secondary networks. Thirdly, when setting the spectrum prices, our model takes into full consideration of the spectrum access technique adopted by the secondary network: we interpret the available spectrum in the network as the allowable aggregated interference on PU receivers, subsequently transform the spectrum market into a *interference market*.

### III. SYSTEM MODEL

We consider an underlay CRN where the available bandwidth is divided into  $K$  channels. The PUs and SUs, denoted by  $\{PU_q\}_{q=1}^M$  and  $\{SU_i\}_{i=1}^N$  respectively, may concurrently establish their communications across all subchannels. Let  $\mathcal{K} \triangleq \{1, \dots, K\}$ ,  $\mathcal{M} \triangleq \{1, \dots, M\}$  and  $\mathcal{N} \triangleq \{1, \dots, N\}$ . Each SU is a transmitter-receiver pair that tries to communicate with each other. All the PUs belong to a primary network that is managed by a service provider (SP). The SP also assumes the role of charging the SUs for the interference they created to the primary network. Fig. 1 illustrates a CRN network under consideration.

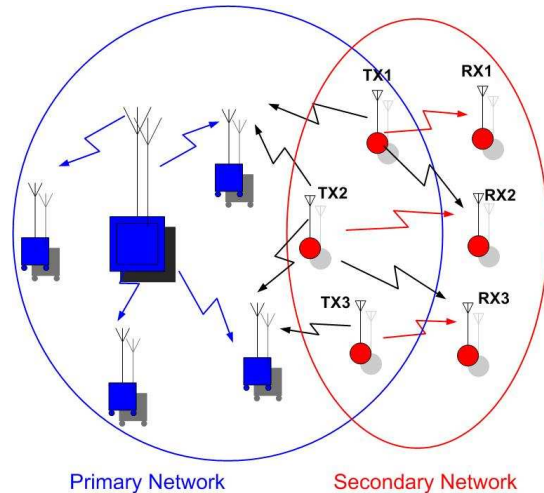


Fig. 1. The CRN configuration under consideration.

Let  $p_i(k)$  denote  $SU_i$ 's transmission power on sub-channel  $f(k)$ . We use  $\mathbf{p}_i \triangleq [p_i(1), \dots, p_i(K)]^T$  to denote the power profile of user  $SU_i$ ; use  $\mathbf{p}_{-i} \triangleq [\mathbf{p}_1^T, \dots, \mathbf{p}_{i-1}^T, \mathbf{p}_{i+1}^T, \dots, \mathbf{p}_N^T]^T$  to denote the power profile of all other SUs except  $SU_i$ ; use  $\mathbf{p} \triangleq [\mathbf{p}_1^T, \dots, \mathbf{p}_N^T]^T$  to denote the system power profile of the secondary network. We consider the scenario that there are no interference cancellations performed at the receivers of the SUs, and the interference caused by the PUs and the other SUs are considered as noises for each SU receiver. Similarly, the SUs' transmission power is considered as noise at the PU receiver.

Let  $|H_{i,j}(k)|^2$  denote the power gain between the transmitter of  $SU_i$  and the receiver of  $S_j$  on channel  $k$ ;  $n_i(k)$  and  $m_i(k)$  be the thermal noise power and total interference power at the receiver of  $SU_i$  on channel  $k$ , and we consider these two quantities fixed during the entire algorithm, partly because of

the assumption that the PUs' transmissions do not adapt to the SUs' activities. We use  $|G_{i,q}(k)|^2$  to denote the power gain between the transmitter of  $SU_i$  and the receiver of  $PU_q$  on channel  $k$ . Each  $SU_i$  is able to obtain the channel information from its transmitter to its receiver,  $\{|H_{i,i}(k)|^2\}_{k \in \mathcal{K}}$  (possibly via feedback from the receiver), and from its transmitter to all PU receivers  $\{|G_{i,q}(k)|^2\}_{k \in \mathcal{K}, q \in \mathcal{M}}$  (possibly by listening to the PU transmission, as suggested in [17]). We assume that the PUs and SUs either have fixed position or are moving slowly, such that the time scale for a typical power control interval is short enough so that the various channel coefficients  $\{|G_{i,q}(k)|^2\}$ , and  $\{|H_{i,j}(k)|^2\}$  are considered fixed during an entire power control interval.

The signal to interference plus noise ratio (SINR) measured at the receiver of  $SU_i$  on channel  $k$  can be expressed as follows:

$$SINR_i(k) = \frac{|H_{i,i}(k)|^2 p_i(k)}{n_i(k) + m_i(k) + \sum_{j \neq i} |H_{j,i}(k)|^2 p_j(k)}. \quad (1)$$

Using Shannon's capacity, the maximum transmission rate achievable for each SU is the sum rate achievable over all  $K$  channels [18]:

$$R_i(\mathbf{p}_i, \mathbf{p}_{-i}) = \sum_{i=1}^K \log(1 + SINR_i(k)). \quad (2)$$

We impose the following constraints on the power profile of the SUs:

- C-1) Total power constraint: each  $SU_i$  has a limit on the total power budget, i.e.,  $0 \leq \sum_{k=1}^K p_i(k) \leq p_i^{max}$ ,  $\forall i \in \mathcal{N}$ .
- C-2) Spectral mask constraint: this constraint is imposed to restrict the maximum power that each SU can transmit on each subchannel  $f(k)$ , i.e., we require  $0 \leq p_i(k) \leq p_{mask}(k)$ ,  $\forall k \in \mathcal{K}, \forall i \in \mathcal{N}$ .

Note that the interference caused by the SU activities measured at a primary receiver of  $PU_q$  in channel  $k$  can be expressed as follows:

$$I_q(k) = \sum_{i=1}^N p_i(k) |G_{i,q}(k)|^2. \quad (3)$$

Let's define  $\mathbf{I}_q \triangleq [I_q(1), \dots, I_q(k)]^T$ , and define  $\mathbf{I} \triangleq [\mathbf{I}_1^T, \dots, \mathbf{I}_M^T]^T$ .

#### IV. PROBLEM FORMULATION

In this section, we jointly formulate the problem of spectrum leasing for the SP and problem of spectrum allocation among the SUs as finding a market equilibrium.

The primary and secondary networks, in the process of trading the available spectrum, establish a spectrum market together. In this market, the SP (or the primary network) is the seller, the SUs are the buyers, and the available spectrum can be viewed as the commodity. Based on the current set of prices for the spectrum, the SP is interested in choosing the right amount of spectrum to sell to maximize its profit; the SUs are interested in choosing the right amount of transmission power across all the subchannels to maximize their respective

utilities. We refer to the set of spectrum made available by the SP the *supply*, and the spectrum consumed by the SUs the *demand*. In this work we assume that the SP cannot manipulate the spectrum prices, in other words, there is no market power. This assumption is reasonable in the sense that there could be many primary networks competing with each other for providing access to the secondary networks [11], [12].

However, it is difficult to measure the quantity of the spectrum that is available in the network as well as to set the price for the spectrum without considering the specific technique adopted for spectrum sharing. In an underlay CRN, where the PUs and SUs are allowed to establish communication on the same set of channels concurrently, it is natural to relate the available spectrum to the total interference tolerable on each PU receiver. In other words, the primary network will allow the secondary network to use the spectrum as long as the SUs do not generate much interference to the PUs. As such, we use the set of total interference  $\{I_q(k)\}_{k \in \mathcal{K}, q \in \mathcal{M}}$  generated by the secondary network as a measure of the usage of the spectrum by the secondary network. Then we say that the quantity  $I_q(k)$  represents the demand for the interference/spectrum on the receiver of  $PU_q$  on channel  $k$ .

##### A. Demand Side of the Market

We assume that the SP is able to communicate with SUs via some control channel (probably by a broadcast channel); it also can obtain the measurements of the instantaneous total interference experience at each PU receiver on each subchannel (probably by exchanging messages with the PUs). SP leases the spectrum to the secondary network by charging a price  $\mu_q(k) \geq 0$  on each SU in the network for contributing a unit interference to  $PU_q$  on channel  $k$ . In another word, the total charge assessed for  $SU_i$  based on its power profile  $\mathbf{p}_i$  can be expressed as:

$$C_i \triangleq \sum_{q=1}^M \sum_{k=1}^K \mu_q(k) |G_{i,q}(k)|^2 p_i(k). \quad (4)$$

We adopt the following utility function for each  $SU_i$ , which represents the SUs' preference over the choice of the power profile based on the current interference prices:

$$U_i(\mathbf{p}_i, \mathbf{p}_{-i}, \boldsymbol{\mu}) \triangleq \beta_i R_i(\mathbf{p}_i, \mathbf{p}_{-i}) - \left( \sum_{k=1}^K \sum_{q=1}^M \mu_q(k) |G_{i,q}(k)|^2 + \lambda_i \right) p_i(k) \quad (5)$$

where  $\beta_i > 0$  and  $\lambda_i > 0$  are user-specific constants that that may serve to control the power efficiency or battery usage for each user. We include these constants to represent the fact that different SUs might value the same transmission rate and spectrum charge differently. Notice that this utility function is increasing in the achievable transmission rate and decreasing in the charges. Also notice that each users' utility is a function of its own power profile, all other SUs' power profiles as well as the interference prices.

We use  $\mathcal{D}_q^k(\boldsymbol{\mu}, \mathbf{p}(k))$  to denote the demand for the interference/spectrum on channel  $k$  at  $PU_q$ 's receiver. As mentioned

previously, it is equivalent to the set of aggregated interference measured at the receivers of each  $PU_q$  on channel  $k$ :

$$\mathcal{D}_q^k(\boldsymbol{\mu}, \mathbf{p}(k)) \triangleq \sum_{i=1}^N |G_{i,q}(k)|^2 p_i(k) = I_q(k). \quad (6)$$

where  $\mathbf{p}(k) \triangleq [p_1(k), \dots, p_N(k)]$ .

Observing the set of constraints C-1) and C-2), we have that a particular secondary user  $SU_i$ 's power profile should be constrained to a feasible set, say  $\mathcal{P}_i$ :

$$\mathcal{P}_i \triangleq \left\{ \mathbf{p}_i : \sum_{k=1}^K p_i(k) \leq p_i^{max}, p_i(k) \in [0, p_{mask}(k)], \forall k \right\}.$$

Let  $\mathcal{P} \triangleq \prod_{i=1}^N \mathcal{P}_i$  be the joint feasible set for all users' power profiles.

Based on a set of fixed interference prices  $\tilde{\boldsymbol{\mu}}$  and fixed power profiles of all other users except  $SU_i$ ,  $\tilde{\mathbf{p}}_{-i}$ ,  $SU_i$ 's utility maximization problem can be expressed as:

$$\mathbf{p}_i = \arg \max_{\mathbf{p}_i \in \mathcal{P}_i} U_i(\mathbf{p}_i, \tilde{\mathbf{p}}_{-i}; \tilde{\boldsymbol{\mu}}) \quad (7)$$

### B. Supply Side of the Market

We then model the supply side of this spectrum market. Let  $I_q^{max}(k)$  denote the interference threshold that can be tolerated for  $PU_q$  on channel  $k$  such that the QoS of  $PU_q$ 's current communication is maintained at a desirable level. Note that this threshold is determined based on the type of current application that is running on  $PU_q$ , and it should be considered fixed for a particular power control interval.

We model the profit structure of the SP in the secondary spectrum market as follows. Its revenue is all the money collected from the SUs for using the spectrum. The costs is related to the compensation the SP needs to pay to the PUs for degradation of QoS. Specifically, in the case that the total interference  $I_q(k)$  is greater than the threshold  $I_q^{max}(k)$ , the SP compensates  $PU_q$  for the degradation of the QoS with the rate  $\mu_q(k)$ , which is exactly the price it charges for the SUs. The total profit for the SP then is increasing in the total revenue collected from the SUs and decreasing in the compensation it pays to the PUs, and we formulate it as in (8) on the top of the next page.

In (8),  $\mathbb{I}\{\cdot\}$  is the indicator function:

$$\mathbb{I}\{I_q(k) - I_q^{max}(k) > 0\} = \begin{cases} 1, & I_q(k) - I_q^{max}(k) > 0 \\ 0, & \text{else.} \end{cases}$$

This particular form of profit function represents the situation that the SP is severely penalized for causing the PU extra interference: when  $I_q(k) - I_q^{max}(k) > 0$ , its compensation to the  $PU_q$  is greater than the extra fees it collects from the SUs because of the use of the exponential function. Intuitively, the SPs would prefer not to allow the SUs to exceed any of the threshold  $\{I_q^{max}(k)\}$ . For the time being, we will consider this particular utility function. We shall also discuss a more general form of the utility function in section VI.

The SP's problem, assuming the prices are fixed and given<sup>1</sup>, say  $\tilde{\boldsymbol{\mu}}$ , can be expressed as finding the set of supplies  $\{\mathcal{S}_q^k(\tilde{\boldsymbol{\mu}})\}$  that maximizes its profit:

$$\mathcal{S}_q^k(\tilde{\boldsymbol{\mu}}) \triangleq \arg \max_{I_q(k)} \mathcal{P}(\mathbf{I}, \tilde{\boldsymbol{\mu}}), \forall q \in \mathcal{M}, k \in \mathcal{K}. \quad (9)$$

When using the specific form of profit for SP in (8), one of the solutions for the above optimization problem can be expressed as

$$\mathcal{S}_q^k(\boldsymbol{\mu}) = \begin{cases} I_q(k) & \text{if } I_q(k) \leq I_q^{max}(k), \mu_q(k) = 0 \\ I_q^{max}(k) & \text{otherwise} \end{cases} \quad (10)$$

Actually, when  $\mu_q(k) = 0$ , the SP is indifferent in the choice of  $\mathcal{S}_q^k(\boldsymbol{\mu})$ . We choose the above form in order to facilitate our definition of the market equilibrium in the following subsection.

### C. Market Equilibrium

We define the market equilibrium (ME) to be the set of prices  $\boldsymbol{\mu}^* \geq 0$  and the set of secondary network system power profiles  $\mathbf{p}^* \in \mathcal{P}$  that satisfy the following:

$$\mathcal{S}_q^k(\boldsymbol{\mu}^*) = \mathcal{D}_q^k(\boldsymbol{\mu}^*, \mathbf{p}^*(k)), \forall q \in \mathcal{M}, k \in \mathcal{K} \quad (11)$$

$$\mathbf{p}_i^* \in \arg \max_{\mathbf{p}_i \in \mathcal{P}_i} U_i(\mathbf{p}_i, \mathbf{p}_{-i}^*; \boldsymbol{\mu}^*), \forall i \in \mathcal{N}. \quad (12)$$

The above definition of the ME says that the interference prices should be set correctly such that the demand is equal to supply on each PU receiver on each subchannel; it also requires that based on the set of equilibrium prices, the secondary network is stable, i.e., no SU wants to change its current power allocation.

In the spectrum market point of view, the ME represents the state of the market in which the spectrum is reasonably priced such that the available spectrum is efficiently utilized: a higher price may result in less demand and inefficient utilization; a lower price may result in more demands and will subsequently decrease the SP's profit as well as PU's QoS.

To further study the property of the ME, we observe that using the specific form of the demands and supplies (6) and (10), we have the following lemma.

*Lemma 1:* The condition (11) is equivalent to the following two conditions:

$$\mu_q^*(k) \left( \sum_{i=1}^N |G_{i,q}(k)|^2 p_i^*(k) - I_q^{max}(k) \right) = 0, \forall k \in \mathcal{K} q \in \mathcal{M} \quad (13)$$

$$I_q^{max}(k) - \sum_{i=1}^N |G_{i,q}(k)|^2 p_i^*(k) \geq 0 \forall k \in \mathcal{K} q \in \mathcal{M}. \quad (14)$$

The above two conditions of the equilibrium are more intuitive than (11). First of all, all interference constraints are met (represented by (14)). Because of the fact that the control of interference at the PU receiver is extremely important in the design of the CRN, we strictly enforce this interference requirement in the equilibrium. Moreover,

<sup>1</sup>This is because of the assumption that the SP has no market power.



$$\mathcal{P}(\mathbf{I}, \boldsymbol{\mu}) = \underbrace{\sum_{q=1}^M \sum_{k=1}^K \mu_q(k) I_q(k)}_{\text{Fees collected from SUs}} - \underbrace{\sum_{q=1}^M \sum_{k=1}^K \mu_q(k) \left( \exp^{(I_q(k) - I_q^{max}(k))} \right)}_{\text{Total Compensation to the PUs}} \times \Pi \{ I_q(k) - I_q^{max}(k) > 0 \} \quad (8)$$

the prices are charged only when necessary (represented by (13)): if an interference constraint is strictly satisfied, i.e.,  $I_q^{max}(k) - \sum_{i=1}^N |G_{i,q}(k)|^2 p_i^*(k) > 0$ , then the corresponding price  $\mu_q^*(k)$  should be 0.

## V. COMPUTATION OF THE MARKET EQUILIBRIUM

In this section, we develop a distributed algorithm for the SP and SUs to compute the ME introduced in the previous section.

### A. Proposed Algorithm

In order to simplify our framework and develop distributed algorithm to compute the ME, we assign each SU  $SU_i$  with  $K$  fictitious SUs (FSUs) denoted by  $\{FSU_i(k)\}_{k \in \mathcal{K}}$ , with each FSUs, say  $FSU_i(k)$ , chooses powers on channel  $k$  for  $SU_i$ . We point out that the FSUs are only a conceptual notion that is introduced for the convenience of the analysis, similarly as in [19].

Note the FSUs  $\{FSU_i(k)\}_{k \in \mathcal{K}}$  share a common total power constraint  $\sum_{k=1}^K p_i(k) \leq p_i^{max}$ . We introduce a set of power prices  $\{\sigma_i\}_{i \in \mathcal{N}}$  to address this constraint, such that individual  $FSU_i(k)$  doesn't need to consider the total power constraint, but will incur a charge when this constraint is violated. Specifically, the feasible set for each FSU  $FSU_i(k)$  is reduced to the following:

$$\bar{\mathcal{P}}_i(k) \triangleq \left\{ p_i(k) : 0 \leq p_i(k) \leq p_{mask}(k) \right\} \quad (15)$$

and its net surplus can be expressed as:

$$\begin{aligned} \bar{U}_i^k(p_i(k), \mathbf{p}_{-i}(k), \boldsymbol{\mu}(k), \sigma_i) & \quad (16) \\ & \triangleq \beta_i \log \left( 1 + \frac{|H_{i,i}(k)|^2 p_i(k)}{n_i(k) + m_i(k) + \sum_{j \neq i} |H_{j,i}(k)|^2 p_j(k)} \right) \\ & - \left( \sum_{q=1}^M \mu_q(k) |G_{i,q}(k)|^2 + \sigma_i + \lambda_i \right) p_i(k). \end{aligned}$$

with  $\boldsymbol{\mu}(k) \triangleq [\mu_1(k), \dots, \mu_M(k)]^T$ , and  $\mathbf{p}_{-i}(k) \triangleq [p_1(k), \dots, p_{i-1}(k), p_{i+1}(k), \dots, p_N(k)]^T$ .

Now we are ready to describe the proposed algorithm. Note that for each FSU  $FSU_i(k)$  the solution to the maximization of its utility  $\bar{U}_i^k(p_i(k), \mathbf{p}_{-i}(k), \boldsymbol{\mu}(k), \sigma_i)$ , say  $\Phi_i^k(\mathbf{p}_{-i}(k), \boldsymbol{\mu}(k), \sigma_i)$ , assuming that  $\mathbf{p}_{-i}$ ,  $\boldsymbol{\mu}$  and  $\sigma_i$  are known and fixed, can be written in closed form as follows:

$$\Phi_i^k(\mathbf{p}_{-i}(k), \boldsymbol{\mu}(k), \sigma_i) \triangleq \left[ \frac{\beta_i}{\sum_{q=1}^M \mu_q(k) |G_{i,q}(k)|^2 + \sigma_i + \lambda_i} - \frac{n_i(k) + m_i(k) + \sum_{j \neq i} |H_{j,i}(k)|^2 p_j(k)}{|H_{i,i}(k)|^2} \right]_0^{p_{mask}(k)}. \quad (17)$$

where  $[x]_0^{p_{mask}(k)} = \max\{0, \min\{x, p_{mask}(k)\}\}$ .

Notice that in order to compute this solution, each SU only need to *i*) measure locally the power of the total interference plus noise (IPN), represented by the term  $n_i(k) + m_i(k) + \sum_{j \neq i} |H_{j,i}(k)|^2 p_j(k)$  in (17), and *ii*) to have the knowledge of the set of power and interference prices  $\boldsymbol{\mu}$  and  $\sigma_i$ .

The proposed algorithm can be succinctly described as follows:

- (*Initialization*) At  $t = 0$ , for all  $i \in \mathcal{N}$ ,  $k \in \mathcal{K}$ ,  $q \in \mathcal{M}$ . Initialize  $p_i^0(k)$  to a feasible value in  $\bar{\mathcal{P}}_i(k)$ ; Initialize  $\mu_q^0(k) \geq 0$ ,  $\sigma_i^0 \geq 0$ ;
- (*Dynamic Updating of Power Profiles*) At iteration  $t + 1$ , the SUs' power profiles are updated *simultaneously* using the measurements of IPN and prices obtained at time  $t$

$$p_i^{t+1}(k) = (1 - \alpha_t) p_i^t(k) + \alpha_t \Phi_i^k(\mathbf{p}_{-i}^t(k), \boldsymbol{\mu}(k)^t, \sigma_i^t). \quad (18)$$

- (*Dynamic Updating of Prices*) Let  $\rho \in (0, 1)$  and  $\nu \in (0, 1)$  be small stepsizes. At iteration  $t + 1$ , prices are updated (after the completion of power updates in this iteration) as follows

– SP updates the interference prices:

$$\mu_q^{t+1}(k) = \left[ \mu_q^t(k) + \rho (I_q^{t+1}(k) - I_q^{max}(k)) \right]^+, \forall k, q. \quad (19)$$

– Each  $SU_i$  updates the power price:

$$\sigma_i^{t+1} = \left[ \sigma_i^t + \nu \left( \sum_{k=1}^K p_i^{t+1}(k) - p_i^{max} \right) \right]^+, \forall i. \quad (20)$$

In the above algorithm,  $\{0 < \alpha_t \leq 1\}_{t=1}^\infty$  is a sequence of positive numbers that have the following properties:

$$\lim_{T \rightarrow \infty} \alpha_T = 0; \quad \lim_{T \rightarrow \infty} \sum_{t=0}^T \alpha_t = \infty; \quad \lim_{T \rightarrow \infty} \sum_{t=0}^T \alpha_t^2 = b < \infty.$$

A typical choice of  $\alpha_t$  is  $\alpha_t = \frac{1}{t+1}$ .

The intuition for the adjustment of the interference price  $\mu^t$  is as follows: when the demand is more than the supply, the price must be increased to suppress the demand, in the hope that in the next iteration the difference between the demand and the supply will be decreased; conversely, when the demand is less than the supply, the price should be decreased. For now, we let the prices to be updated *in every iteration* after the SUs updated their power profiles. We will relax this requirement in the following subsection. Also notice that the adjustment of a particular  $\sigma_i^t$  requires the knowledge of  $\{p_i^{t+1}(k)\}_{k \in \mathcal{K}}$ , which is completely available to this particular  $SU_i$ . Consequently, the power price adjustment can be carried out locally by individual SUs.

In words, the proposed algorithm works as follows. At time instance 0, the SUs choose their transmission profile  $\{\mathbf{p}_i^0\}_{i \in \mathcal{N}}$  randomly; the SP then calculates and announces the prices  $\boldsymbol{\mu}^1$  based on the current measurements of total interference  $\{I_q^0(k)\}$  at PUs receivers. At time instance  $t$ , the SUs update their transmission strategy based on  $\boldsymbol{\mu}^t$  and  $\{\mathbf{p}_i^t\}_{i \in \mathcal{N}}$ ; the SP calculates the prices as in (19) after all SUs have chosen their strategy. The algorithm goes on until an ME has been reached.

We should emphasize here that the above algorithm is sufficiently distributed in two ways. (1) The spectrum access part of the algorithm (represented by (18)) is distributed: the SUs can update their individual power profiles without detailed knowledge of the other SUs' current power profiles and the current spectrum usage of the entire system. They only need to have the knowledge of the power of IPN on all channels (which can be measured at the receiver of each SU) and the set of current interference prices (which can be obtained via the broadcast channel). (2) The computation of the spectrum price is distributed: the SP, upon measuring the total received interference power on channel  $k$  of  $PU_q$ , can update  $\mu_q(k)$  without knowing the individual power profile of each SU; moreover, different from the algorithms proposed in [20] and [10], the SP can update the interference prices *in each iteration* of the algorithm, thus, no explicit coordination is needed between the SP and the secondary networks.

### B. Convergence Property of the Proposed Algorithm

In this subsection, we provide sufficient conditions for the proposed algorithm to converge to an ME.

Define  $K$  nonnegative matrices  $\mathbf{H}(k) \in \mathbb{R}_+^{N \times N}$  with their elements defined as follows:

$$[\mathbf{H}]_{qr}(k) \triangleq \begin{cases} \frac{|H_{r,q}(k)|^2}{|H_{q,q}(k)|^2} & \text{if } r \neq q \\ 0 & \text{otherwise} \end{cases} \quad (21)$$

*Theorem 1:* Suppose that  $\alpha_t$  is restricted to the following form:  $\lim_{t \rightarrow \infty} \frac{1}{\alpha_t t} = 1$ . If for all  $k \in \mathcal{K}$ , we have that:

$$\|\mathbf{H}(k)\| < 1 \quad (22)$$

where  $\|\cdot\|$  is an induced matrix norm with its corresponding vector norm monotone, i.e., we have the following implication for two arbitrary vectors  $\mathbf{v}$ ,  $\mathbf{u}$  ([21], Definition 5.5.9):

$$\|[\mathbf{v}]_i\| \leq \|[\mathbf{u}]_i\| \quad \forall i \Rightarrow \|\mathbf{v}\| \leq \|\mathbf{u}\|. \quad (23)$$

Then we have that for all  $i \in \mathcal{N}$ ,  $k \in \mathcal{K}$  and  $q \in \mathcal{M}$ :

$$\lim_{t \rightarrow \infty} p_i^{t+1}(k) = p_i^*(k) \quad (24)$$

$$\lim_{t \rightarrow \infty} \mu_q^{t+1}(k) = \mu_q^*(k); \quad \lim_{t \rightarrow \infty} \sigma_i^{t+1} = \sigma_i^* \quad (25)$$

where  $(\mathbf{p}^*; \boldsymbol{\mu}^*)$  is a market equilibrium. Furthermore, in the limit the total power constraints C-1) will be met. Additionally, if we allow several rounds of power updates between two consecutive price updates, the above results are still true.

*Proof:* The proof is given in [22]. We first show that the prices converge. Based on this result, we show that when (22) is true for all  $k$ , the power profiles of SUs also converge. ■

The condition that  $\|\mathbf{H}(k)\| < 1$  can be viewed as a weak interference condition. Specifically, taking the norm to be the maximum norm, which is defined as follows [23]:

$$\|\mathbf{A}\|_\infty^{\mathbf{w}} \triangleq \max_q \frac{1}{w_q} \sum_{r=1}^Q [\mathbf{A}]_{qr} w_r, \quad \mathbf{A} \in \mathbf{R}^{Q \times Q}. \quad (26)$$

where  $\mathbf{w} = [w_1, \dots, w_N]$  is any positive vector. We have that  $\|\mathbf{H}(k)\|_{\infty, \text{mat}}^{\mathbf{w}} < 1$  is equivalent to

$$\|\mathbf{H}(k)\|_\infty^{\mathbf{w}} = \max_j \frac{1}{w_j} \sum_{i \neq j} \frac{|H_{i,j}(k)|^2}{|H_{j,j}(k)|^2} w_i < 1. \quad (27)$$

which essentially says that if each user  $i$  in the system transmit  $w_i$  unit of power on channel  $k$ , then at a particular user  $j$ 's receiver, the useful signal strength  $|H_{j,j}(k)|^2 w_j$  will be larger than the sum of interference  $\sum_{i \neq j} |H_{i,j}(k)|^2 w_i$ . We refer the readers to [24] for more detailed physical interpretation of the condition (22).

## VI. EXTENSIONS AND SOME DISCUSSION

We also consider a more flexible and general form of SP's utility function (hence the supply function). In the case that the total interference  $I_q(k)$  is greater than the threshold  $I_q^{\text{max}}(k)$ , the SP compensate  $PU_q$  for the degradation of the QoS with a predefined rate  $\{r_q(k)\}_{k \in \mathcal{K}}$ . The total profit for the SP is again increasing in the total charge collected from the SUs and decreasing in the compensation it pays to the PUs, and we formulate it as in (28) on the top of the next page.

In (28), each function  $f_q(x) : \mathbf{R}_+ \rightarrow \mathbf{R}_+$  is an increasing convex function with respect to  $x$ . This choice of the profit function is different with (8) in an important way: depending on the choice of  $r_q(k)$  and the choice of the function  $f(\cdot)$ , the total interference at certain PU receivers might exceed the threshold  $I_q^{\text{max}}(k)$  in the market equilibrium. In another word, the SP may obtain higher profit by leasing extra spectrum to the SUs while letting certain PUs suffer from low QoS. We will demonstrate this property by choosing a specific functional form for  $f(x)$ . We need to mention here that when adopting this more general profit function, the SP needs to be extremely careful in determining the free parameters  $\{r_q(k)\}_{k \in \mathcal{K}, q \in \mathcal{M}}$  and the functions  $\{f_q\}_{q \in \mathcal{M}}$ , in order to avoid the decreasing of PUs satisfactory level. The SP should certainly make these choices adaptively based on the type of applications/services that is currently ran on the PU terminal; the PUs should also be aware of these choices because they will affect their communication quality and monthly bill.

In the following, we consider a specific form of (28) by choosing  $f_q(x)$  to be  $x^2$  for all  $q \in \mathcal{M}$ . In this case, one of the solution for the SP's optimization problem can be expressed as

$$S_q^k(\boldsymbol{\mu}) = \begin{cases} I_q(k) & \text{if } I_q(k) \leq I_q^{\text{max}}(k), \mu_q(k) = 0 \\ I_q^{\text{max}}(k) & \text{if } I_q(k) \leq I_q^{\text{max}}(k), \mu_q(k) \neq 0 \\ I_q^{\text{max}}(k) & \text{if } I_q(k) \geq I_q^{\text{max}}(k), \mu_q(k) = 0 \\ I_q^{\text{max}}(k) + \frac{\mu_q(k)}{2r_q(k)} & \text{otherwise} \end{cases} \quad (29)$$

$$\mathcal{P}(\mathbf{I}, \boldsymbol{\mu}) = \sum_{q=1}^M \sum_{k=1}^K \mu_q(k) I_q(k) - \sum_{q=1}^M \sum_{k=1}^K r_q(k) f_q(I_q(k) - I_q^{max}(k)) \times \Pi \{I_q(k) - I_q^{max}(k) > 0\} \quad (28)$$

Actually, when  $\mu_q(k) = 0$ ,  $\mathcal{S}_q^k(\boldsymbol{\mu})$  can take any value less than or equal to  $I_q^{max}(k)$ . We choose the above form in order to facilitate the definition of the market equilibrium.

According to the above supply function, the interference prices should be updated as in (30) on the top of the next page.

In this case, the first defining condition of the ME (11) is no longer equivalent to (13)–(14): clearly, it is possible that we have  $I_q^*(k) = I_q^{max}(k) + \frac{\mu_q^*(k)}{2r_q(k)} > I_q^{max}(k)$ . Intuitively, the smaller the value of  $r_q(k)$ , the more the aggregated interference will exceed the threshold. We will further study the property of the ME in the simulation section.

## VII. NUMERICAL RESULTS

In this section, we give simulation results to demonstrate various properties of the proposed algorithm. We considered the CRN that consisted of 20 SUs, 2 PUs, and 64 Channels. The positions of the SUs and PUs were random, and the SU to SU channel gains  $\{|H_{i,j}(k)|^2\}_{i,j \in \mathcal{N}, k \in \mathcal{K}}$  were chosen such that condition (22) was satisfied; the SU to PU channel gains were selected randomly. We set the maximum total power  $\{p_i^{max}\}_{i \in \mathcal{N}}$  to be the same for all SUs, and the channel mask  $p_{mask}(k)$  to be the same across all the channels. We also set  $p_i^{max} < \sum_{k=1}^K p_{mask}(k)$  to avoid triviality [25].

### A. Convergence to the ME

We first show that when using the profit function (8), the ME can be reached. In the top part of Fig. 2, we observe that the power profiles of individual SU converge. This means that no SU has the intention to deviate from its current power profile, which implies that the condition (12) is satisfied. Note that we only show the power output  $p_i(k)$  of selected SUs on selected channels to avoid making the figure overcrowded. In the bottom part of Fig. 2 we also show that the total power constraints for each SU can be met after several iterations of the computation.

In the top part of Fig. 3, we observe that the set of interference prices  $\boldsymbol{\mu}^t$  that produced by the algorithm converges. We can also observe in the bottom part of this figure that all the interference thresholds  $\{I_q^{max}(k)\}$  are enforced, as predicted by the defining condition of the ME (14). Furthermore, the condition (13) is also met, as can be seen by the fact that whenever an interference constraint is strictly satisfied (e.g., the blue line in the bottom part of Fig.3), the corresponding interference price converges to 0 (e.g., the blue line in the top part of Fig.3).

We argue that the ability to enforce the interference constraints (see (14)) is unique to our algorithm. We compare the proposed algorithm with a popular Iterative Water Filling (IWF) algorithm [26], [24] that is used for dynamic spectrum allocation *without* considering the presence of the primary network. In Fig. 4, we compare the aggregated interference

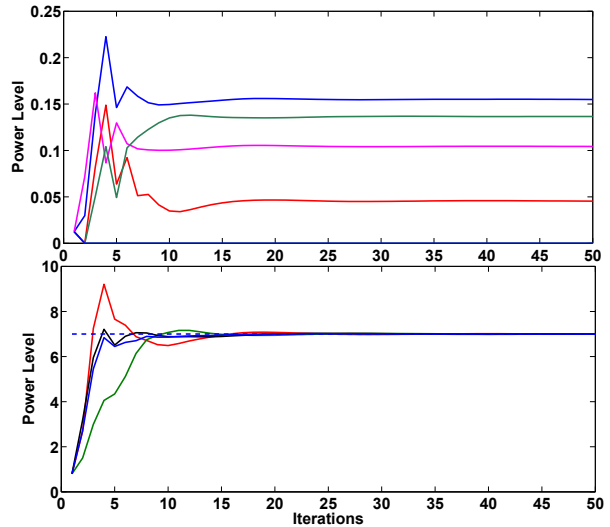


Fig. 2. Top: Selected Power profiles  $p_i^t(k)$ . Bottom: Total power output of different SUs. The dotted line represents individual power limit.

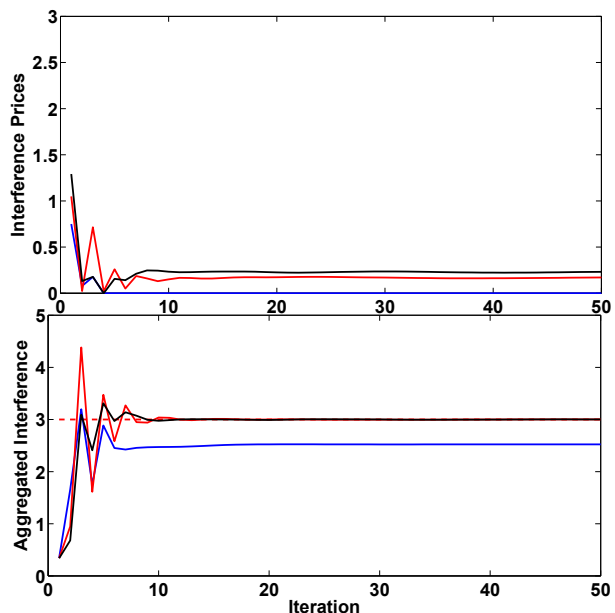


Fig. 3. Top: Selected Interference Prices  $\mu_q^t(k)$ . Bottom: Corresponding aggregated interference  $I_q^t(k)$ . The dotted line represents the threshold  $I_q^{max}(k)$ .

resulted from adopting these two different algorithms on all subchannels/carriers measured by a particular primary user  $P_1$ . It is clear that the proposed algorithm is able to enforce the interference constraints on all subchannels/carriers, while the IWF algorithm fails to do so.



$$\mu_q^{t+1}(k) = \left[ \mu_q^t(k) + \rho (\mathcal{D}_q(k) - \mathcal{S}_q(k)) \right]^+ = \begin{cases} 0 & \text{if } I_q(k) \leq I_q^{max}(k), \mu_q(k) = 0 \\ \left[ \mu_q^t(k) + \rho (I_q^t(k) - I_q^{max}(k)) \right]^+ & \text{if } I_q(k) \leq I_q^{max}(k), \mu_q(k) \neq 0 \\ \left[ \mu_q^t(k) + \rho (I_q^t(k) - I_q^{max}(k)) \right]^+ & \text{if } I_q(k) \geq I_q^{max}(k), \mu_q(k) = 0 \\ \left[ \mu_q^t(k) + \rho \left( I_q^t(k) - I_q^{max}(k) - \frac{\mu_q^t(k)}{2r_q(k)} \right) \right]^+ & \text{otherwise.} \end{cases} \quad (30)$$

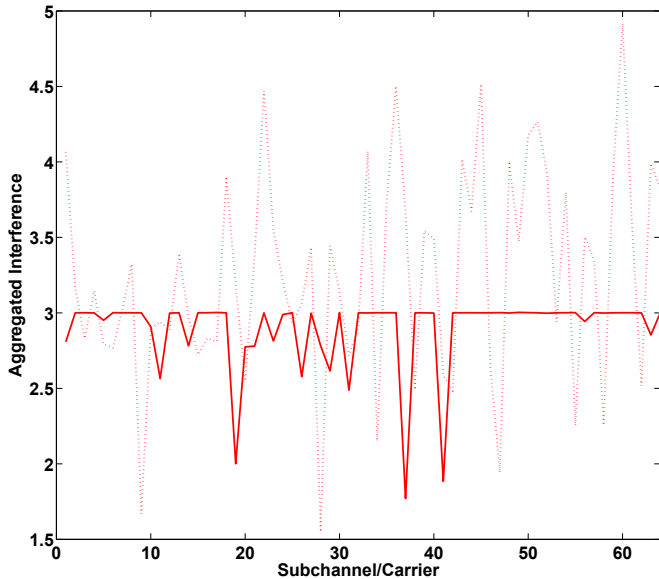


Fig. 4. Comparison of aggregated interference across all subchannels. The interference threshold is set to be 3. The dotted line and the solid line are the interference produced by IWF algorithm and the proposed algorithm, respectively.

### B. The Choice of the Profit Function

We then show that by adopting different forms of the profit functions, the SP can have the ability to affect the performance of the entire network. Specifically, we consider the form of profit function (28) by choosing  $f_q(x)$  to be  $x^2$  for all  $q \in \mathcal{M}$ . The resulting price update procedure has been derived in (30).

We observe that in the equilibrium, some of the aggregated interference  $\{I_q^*(k)\}$  indeed exceeds the thresholds  $\{I_q^{max}(k)\}$ . Moreover, the less the SP compensates the PU (with smaller  $r_q(k)$ ), the more that the aggregated interference  $\{I_q^*(k)\}$  tends to exceed the thresholds. Fig. 5 shows exactly such an instance. The top part of the figure shows the selected  $\{I_q^*(k)\}$  when setting  $r_q(k) = 0.5$  for all  $q \in \mathcal{M}$  and  $k \in \mathcal{K}$ , while the bottom part of the figure shows a similar case but with  $r_q(k) = 0.1$ .

We further investigate the properties of the ME by looking at relationship between different values of the interference constraints with the network performance, as well as the effects of the choice of  $\{r_q(k)\}$  on the network performance. The performance metrics we considered are the sum communication rates achievable by the secondary network and the total profit obtained by the SP.

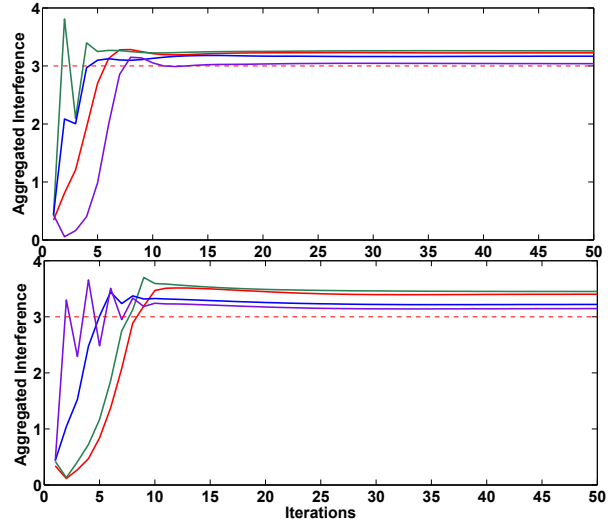


Fig. 5. Comparison of Aggregated Interference with Different Choices of  $r_q(k)$ . Left:  $r_q(k) = 0.5$ . Right:  $r_q(k) = 0.1$ .

Fig. 6 shows the relationship between the sum rates of the secondary network and the values of the interference thresholds. Each point in this figure is obtained by running the algorithm on 50 independently generated network configurations. For ease of demonstration, we show the results where the thresholds  $\{I_q^{max}(k)\}$  are set to be equal for all  $k$  and  $q$ . It is clear that the sum rate is increasing with the value of the thresholds. We also observe that different choices of  $r_q(k)$  results in very different sum rate performance. This is intuitive since in our profit model, the less the SP needs to compensate the primary users, the more it is willing to sell the spectrum to the SUs. When the compensation is low enough, the SP might willing to let the secondary network use the entire spectrum without even considering the existence of the PUs. Consequently, the interference thresholds do not really affect the spectrum decision of the secondary network when the compensation is too small. Obviously, this is not a desirable situation, hence extreme caution needs to be exercised when designing custom profit functions.

Fig. 7 shows the relationship between SP's profits and the values of the interference thresholds. It is clear that the profit is decreasing with the value of the thresholds. This is also intuitive because when the thresholds are large enough, the primary and secondary network can be considered as separable: the activity of the secondary network cannot affect

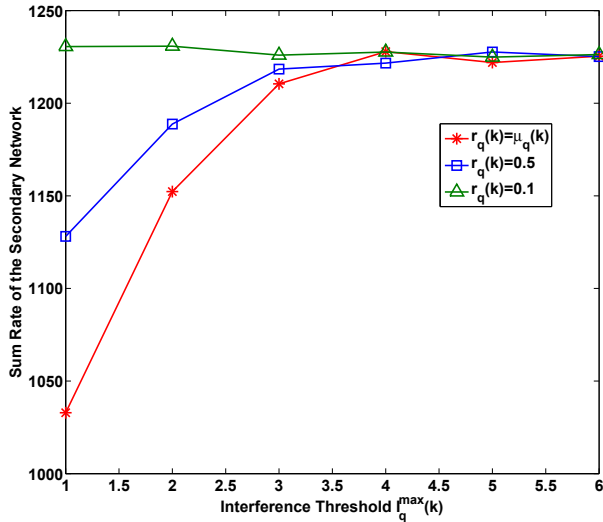


Fig. 6. Relationship between the sum rate and the interference thresholds as well as the compensation rates.

the primary network in anyway, as a result, the SUs can freely access the entire spectrum without incurring any additional charges.

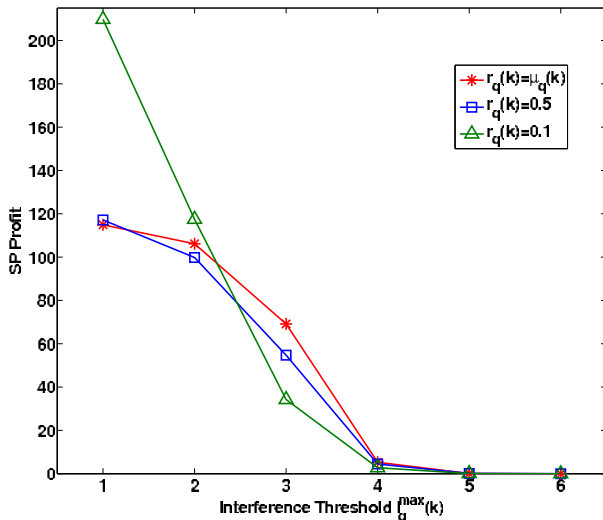


Fig. 7. Relationship between the SP profit and the interference thresholds as well as the compensation rates.

One might falsely conclude from the previous simulation results that it is beneficial for both SUs and the SPs if the system is to operate on the point where the compensation as well as the thresholds are small (e.g.  $r_q(k) = 0.1$  and  $I_q^{max}(k) = 1$ ), because it seems that the SP can obtain the highest profit and the secondary network can achieve the highest sum rate. However, we should note that low interference thresholds on the primary receivers imply that the PUs are running applications that require high QoS (such as video streaming). Recall that we have observed in Fig. 5 that when the value of  $r_q(k)$  is small, the interference constraints of the primary receivers will be severely violated, consequently causing significant degradation of the QoS for the PUs. The

SPs are simply not allowed to compensate this little to the PUs in this situation. The point is that it is of extreme importance to properly choose the profit function for the SP, so that the both the interest of the PUs and the SUs can be properly balanced and the spectrum can be efficiently shared.

## VIII. CONCLUSION

In this work, we study the structure of a potential spectrum market in which the primary network is the seller and the secondary network is the buyer of the spectrum, and the seller cannot manipulate the spectrum prices (i.e., no market power). The objective of the primary network service provider is to choose the amount of the spectrum to sell in order to maximize its profit; the objectives of the secondary users are to choose their power profiles to maximize their own utility functions. We formulated the notion of a market equilibrium, which is essentially a stable state of the network where both the service provider and the secondary users do not want to change their current strategies and the spectrum is efficiently shared. We also developed an algorithm such that the ME can be reached distributedly.

In reality, due to the large amount of capital needed for the purchasing of the spectrum and for the establishing of the operation on this spectrum, usually there are only a few competing NSPs (thus small number of the the seller of the market), which implies that there exists a certain degree of market power. Consequently, we plan to extend this work to the market form including oligopoly and monopoly.

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