



**HAL**  
open science

## Taxation for Green Communication

Eitan Altman, Konstantin Avrachenkov, Andrey Garnaev

► **To cite this version:**

Eitan Altman, Konstantin Avrachenkov, Andrey Garnaev. Taxation for Green Communication. WiOpt'10: Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks, May 2010, Avignon, France. pp.209-213. inria-00496479

**HAL Id: inria-00496479**

**<https://inria.hal.science/inria-00496479>**

Submitted on 30 Jun 2010

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Taxation for Green Communication

Eitan Altman

Konstantin Avrachenkov

Andrey Garnaev

INRIA Sophia Antipolis, France  
Email: altman@sophia.inria.fr

INRIA Sophia Antipolis, France  
Email: k.avrachenkov@sophia.inria.fr

St. Petersburg State University, Russia  
Email: garnaev@yahoo.com

**Abstract**—Nowadays energy saving and reduction of electromagnetic pollution become important issues. One approach to these problems is the introduction of taxes on the energy dissipation. In this paper we investigate a taxation game between a user or a provider or a group of users and the taxation authority. This is a Stackelberg game where the taxation authority acts as a leader and users or service providers act as followers. Clearly too big taxes will strongly discourage the user of wireless technology and hinder the progress and too small taxes will lead to a wasteful use of the energy resources and may also lead to reckless use of the radio resources. We study several important particular cases with complete and partial information. We focus on the problem of finding taxation strategy in closed form and investigating how incomplete information of authorities about users impacts the equilibrium strategy.

## I. INTRODUCTION

Nowadays energy saving and reduction of electromagnetic pollution become important issues. One approach to these problems is the introduction of taxes on the energy dissipation. In the present paper we consider the problem of taxation on energy as a Stackelberg game. The taxation authority acts as a leader and users or service providers act as followers. Clearly too big taxes will strongly discourage the user of wireless technology and hinder the progress and too small taxes will lead to a wasteful use of the energy resources and may also lead to reckless use of the radio resources. In this work we consider two important cases: In the first case we consider one user or a service provider (it depends if we consider uplink or downlink transmission) and one taxation authority. In the second case we consider several users (uplink transmission) and again one single taxation authority. The first case then has several important sub-cases. Specifically, we consider the sub-case of complete information and sub-cases of partial information on the amount of resources and on the type of users. In most cases we provide explicit expressions for the optimal values of taxation. On one hand, this optimal value allows users to transmit with the best possible rate and on the other hand, this optimal taxation value allows the taxation authority to accumulate the maximal taxation and to ensure the reasonable level of the resource spending. Finally we notice that in [2] a problem how energy taxation can impact on the quality of signal transmission under jamming was investigated, but the taxation there was fixed a priori. The following papers [1], [3] have analyzed the Stackelberg game approach for

power control in wireless network. However, the problem formulations of [1], [3] are different from ours, namely, we focus on the problem of finding taxation politics in closed form and investigating how incomplete information of authorities about users impacts equilibrium strategies. Also, note that in [4] influence of different tax systems on competitive homogeneous production was investigated.

## II. A TAXATION AUTHORITY AND A USER OR PROVIDER

In this section we consider the following Taxation Game. A User (which can also be a service provider; it depends if we consider uplink or downlink) intends to use a resource (energy, radio bandwidth) for signal transmission. The user or the provider has to pay to the taxation authority some taxes for using common resources like energy or radio bandwidth. Of course the amount of taxation impacts the user's behaviour. If taxation level is zero then user tends to exploit the resources as much as possible and this can lead to strong electromagnetic pollution which can represent health hazard. On the other hand, big taxation can completely discourage the user from network utilization. A natural question arises for the taxation authority: which taxation level the authority has to assign to get the maximal profit and at the same time encourage sensible user behaviour. This problem can be modelled by a Stackelberg game with the taxation authority as a leader and a user or provider as a follower. At the first step the taxation level is fixed. It is natural to consider as a strategy for user the power of transmitted signal, and as the user's payoff the Shannon capacity minus taxes, i.e.

$$v(T) = \ln \left( 1 + \frac{hT}{N_0} \right) - CT, \quad (1)$$

where  $h$  is the fading channel gains,  $N_0$  is the background noise,  $C$  is the taxation rate per power unit and  $T \in [0, \bar{T}]$  is the user's strategy, namely, it is the power of transmitted signal and  $\bar{T}$  is the maximal power which user can apply.

### A. Complete information case

First let us consider the case of complete information. Namely, we assume that the maximal power  $\bar{T}$ , fading channel

gains  $h$  and background noise are known to both players. To solve this problem first note that since

$$T = \frac{1}{C} - \frac{N_0}{h}$$

is the root of the equation

$$\frac{dv}{dT} = \frac{h}{N_0 + hT} - C = 0$$

we obtain that the optimal user strategy is

$$T(C) = \min \left\{ \left[ \frac{1}{C} - \frac{N_0}{h} \right]_+, \bar{T} \right\}$$

which is equivalent to

$$T(C) = \begin{cases} \left[ \frac{1}{C} - \frac{N_0}{h} \right]_+, & \frac{1}{\bar{T} + N_0/h} \leq C \\ \bar{T}, & \frac{1}{\bar{T} + N_0/h} > C. \end{cases} \quad (2)$$

Now we can find the optimal taxation rate  $C$  from the taxation authority point of view. The payoff to the taxation authority is  $v_P(C) = CT(C)$ . Then, by (2), we have

$$v_P(C) = \begin{cases} \bar{T}C, & C < \frac{1}{\bar{T} + N_0/h}, \\ \max\{1 - \frac{C}{h}, 0\}, & C \geq \frac{1}{\bar{T} + N_0/h}. \end{cases} \quad (3)$$

It is clear that  $v_P(C)$  is increasing for  $C \in \left(0, \frac{1}{\bar{T} + N_0/h}\right)$  and it is decreasing for  $C \in \left(\frac{1}{\bar{T} + N_0/h}, h\right)$ . Thus,  $v_P(C)$  achieves its maximum at  $C = \frac{1}{\bar{T} + N_0/h}$  which implies the following result on optimal taxation rate.

*Theorem 1:* The optimal taxation rate is

$$C = \frac{1}{\bar{T} + N_0/h}$$

which brings to the taxation authority the total profit

$$v_P = \frac{h\bar{T}}{h\bar{T} + N_0}.$$

This taxation rate allows user to transmit with the best possible rate, so his optimal strategy is  $T = \bar{T}$ . Of course, in reality the taxation office might choose the taxation rate slightly bigger than the optimal value to decrease the use of energy. In such a case the optimal taxation values serves as a reference point. The optimal taxation for the taxation authority is the one which allows the user to transmit all he wants but for the maximal price he agrees to pay. Thus, the authority's strategy tells him to be maximally greedy without overdoing it so that do not discourage the user.

## B. Unknown Fading Channel Gains

In this subsection we consider a modification of the game from the previous subsection for the case when only distribution of fading channel gains known to the user but not its exact value. Thus, the payoff to the user is given as follows:

$$v(T) = \int_0^\infty \ln(1 + hT) \sigma(h) dh - CT, \quad (4)$$

where  $\sigma(h)$  is the distribution of the fading channel gain.

To find the optimal user strategy we have to look at the root of the equation

$$\frac{dv}{dT} = \int_0^\infty \frac{h}{1 + hT} \sigma(h) dh - C = 0. \quad (5)$$

Since  $f(T) := \int_0^\infty \frac{h}{1 + hT} \sigma(h) dh$  is decreasing in  $T$ , the equation (5) has the unique root  $T = \tau(C)$  for any  $C \in (f(\bar{T}), f(0))$ . Thus the optimal user strategy is

$$T = T(C) := \begin{cases} \bar{T} & C < f(\bar{T}), \\ \tau(C), & C \in [f(\bar{T}), f(0)], \\ 0 & C > f(0). \end{cases}$$

Now we can turn our attention to finding the optimal taxation level  $C$ . Note that by (4) the function  $\tau(C)$  is decreasing for  $C \in (f(0), \infty)$ . Since, by (4), we have

$$\int_0^\infty \frac{h}{C + \tau(C)Ch} \sigma(h) dh = 1, \quad C \in [f(\bar{T}), f(0)]$$

the function  $\tau(C)C$  is also decreasing for  $C \in [f(\bar{T}), f(0)]$ . Meanwhile for  $C \in (0, f(\bar{T}))$ ,  $\tau(C) = \bar{T}$ , and  $\tau(C)C = \bar{T}C$  is increasing. Then, since the optimal taxation rate  $C$  is the one which gets the maximum gain for the taxation authority  $v_P(C) = CT(C)$  we have obtained the following result.

*Theorem 2:* The optimal taxation  $C$  is given as follows

$$C = C_* := \int_0^\infty \frac{h}{1 + h\bar{T}} \sigma(h) dh.$$

## C. Discrete Model of Incomplete Information on Total User Power

In this subsection we consider a scenario when the taxation authority does not know the total power which user intends to use for signal transmission but just knows its distribution. We study discrete distribution case as well as continuous one.

First we investigate the case of discrete distribution, namely, the authority knows that with probability  $q^r$  the total user power is  $\bar{T}^r$  where  $r \in [1, M]$ . Of course, the user knows what power is available to him. To deal with this situation we introduce  $M$  types of user, i.e. user has type  $r$  if the total power available for transmission is  $\bar{T}^r$ ,  $r \in [1, M]$ . The notation  $T^r$

implies the user of type  $r$ . Thus, the payoff to the taxation authority is

$$v_P(C) = C \sum_{r=1}^M q^r T^r$$

meanwhile the payoff to the user of type  $r$  is given as follows:

$$v^r(T^r) = \ln(1 + hT^r/N_0) - CT^r.$$

Thus, we have

$$T^r(C) = \begin{cases} \frac{1}{C} - \frac{N_0}{h}, & \frac{1}{T^r + N_0/h} \leq C, \\ \bar{T}^r, & \frac{1}{T^r + N_0/h} > C. \end{cases} \quad (6)$$

Without loss of generality we can assume that

$$\bar{T}^1 \leq \bar{T}^2 \leq \dots \leq \bar{T}^M.$$

Then

$$v_P(C) = C \sum_{r=1}^M q^r T^r(C) = \begin{cases} C \sum_{r=1}^M q^r \bar{T}^r, & C < \frac{1}{\bar{T}^M + N_0/h}, \\ C \sum_{r=1}^{k-1} q^r \bar{T}^r + \left(1 - \sum_{r=1}^{k-1} q^r\right) \left(1 - C \frac{N_0}{h}\right), & \frac{1}{\bar{T}^k + N_0/h} < C < \frac{1}{\bar{T}^{k-1} + N_0/h}, \\ 1 - C \frac{N_0}{h}, & \frac{1}{\bar{T}^1 + N_0/h} < C. \end{cases} \quad (7)$$

Introduce the following auxiliary sequence:

$$\varphi_k = \sum_{r=1}^{k-1} q^r \bar{T}^k - \frac{N_0}{h} \sum_{r=k}^M q^r \text{ for } k \in [1, M+1].$$

Then  $\varphi_k$  is increasing from  $-1/h$  for  $k = 1$  to  $\sum_{r=1}^M q^r \bar{T}^k$  for  $k = M+1$ . Thus, we can summarize the obtained result in the following theorem supplying the optimal taxation rate.

*Theorem 3:* The optimal taxation rate is given as follows:

$$C = \frac{1}{\bar{T}^{k_*} + N_0/h},$$

where  $k_*$  is such that

$$\varphi_{k_*+1} \leq 0 < \varphi_{k_*}.$$

In particular in two mass case, i.e.  $M = 2$ , we have the following result.

*Theorem 4:* The optimal taxation rate for two mass distribution case is given by

$$C = \begin{cases} \frac{1}{\bar{T}^2 + N_0/h}, & q^1 \bar{T}^1 > q^2 \frac{N_0}{h}, \\ \frac{1}{\bar{T}^1 + N_0/h}, & q^1 \bar{T}^1 < q^2 \frac{N_0}{h}. \end{cases}$$

#### D. Continuous Model of Incomplete Information on Total User Power

Of course we can also consider a continuous version of the game, namely, the taxation authority knows that with probability  $q(t)$  the total user's power is  $t$  where  $t \in [0, \infty)$ .

To deal with this situation we introduce a continuous spectrum of types of the user, i.e. user has type  $t$  if his total power to transmit is  $t$ . Denote by  $T(t)$  the strategy implied by user of type  $t$ . Then the payoff to the taxation authority is

$$v_P(C) = C \int_0^\infty T(t)q(t) dt$$

meanwhile the payoff to a user of type  $t$  is given by

$$v^t(T(t)) = \ln(1 + hT(t)) - CT(t).$$

Thus,

$$T(t) = T(t, C) := \begin{cases} \frac{1}{C} - \frac{1}{h}, & \frac{1}{t + 1/h} \leq C \\ t, & \frac{1}{t + 1/h} > C. \end{cases} \quad (8)$$

Then the payoff to the taxation authority is

$$v_P(C) = \left(1 - \frac{C}{h}\right) \int_{1/C-1/h}^\infty q(t) dt + C \int_{1/C-1/h}^\infty tq(t) dt \quad (9)$$

and its derivative can be found as follows:

$$\frac{dv_P(C)}{dC} = \varphi(C) := \frac{1}{h} \left( \int_0^{1/C-1/h} tq(t) dt - \int_{1/C-1/h}^\infty q(t) dt \right). \quad (10)$$

Obviously,  $\varphi(C)$  is decreasing function from  $(\int_0^\infty tq(t) dt)/h$  for  $C = 0$  to  $-1/h$  for  $C = \infty$ . So, the equation  $dv_P(C)/dC = 0$  has the unique root. This implies the following result supplying the optimal taxation level.

*Theorem 5:* The optimal taxation rate is given by

$$C = C_*,$$

where  $C_*$  is the unique positive root of the equation:

$$\int_0^{1/C-1/h} tq(t) dt - \int_{1/C-1/h}^\infty q(t) dt = 0.$$

In particular if  $q(t)$  is uniform on an interval  $[0, \bar{T}]$  then the optimal taxation rate is given by

$$C_* = \max \left\{ \frac{h-1+h\sqrt{1+2\bar{T}}}{2\bar{T}h^2+2h-1}h, \frac{h-1-h\sqrt{1+2\bar{T}}}{2\bar{T}h^2+2h-1}h \right\}.$$

### III. A TAXATION AUTHORITY AND TWO USERS

In this section we consider a generalization of the Taxation Game for the case when two users present in the network and user  $k$  would like to transmit signal of total power  $\bar{T}_k$  through the interference channel. The payoff to a user is the Shannon capacity minus taxes, i.e.

$$\begin{aligned} v_1 &= \ln \left( 1 + \frac{h_1 T_1}{N_0 + h_{21} T_2} \right) - C T_1, \\ v_2 &= \ln \left( 1 + \frac{h_2 T_2}{N_0 + h_{12} T_1} \right) - C T_2, \end{aligned} \quad (11)$$

where  $h_1, h_2, h_{21}, h_{12}$  are fading channel gains ( $h_1 \geq h_{21}, h_2 \geq h_{12}$ ) and  $N_0$  is the background noise which are all known to each player as well as the total powers  $\bar{T}_1$  and  $\bar{T}_2$ .

The payoff to the taxation authority is given as follows:

$$v_P = C(T_1 + T_2). \quad (12)$$

It is clear that the taxation level impacts on user's behaviour essentially. Namely, too big taxation stops just any transmission and too small taxation leads to wasteful behaviour. Therefore, there is a question which taxation rate  $C$  the taxation authority has to assign to maximize its profit while users employ the network selfishly trying maximize own profit under the assigned taxation rate. So, we have here again a Stackelberg game with the taxation authority as a leader and two users as followers. Thus, at the first step the taxation rate is fixed and the users find their equilibrium strategies. At the second step the taxation authority knowing the choice made by the users assigns final taxation level to maximize its gain.

First let the taxation rate be fixed and let find the equilibrium strategies for selfish users. We can find derivatives of the payoffs as follows:

$$\begin{aligned} \frac{\partial v_1}{\partial T_1} &= \frac{h_1}{N_0 + h_{21} T_2 + h_1 T_1} - C, \\ \frac{\partial v_2}{\partial T_2} &= \frac{h_2}{N_0 + h_{12} T_1 + h_2 T_2} - C. \end{aligned} \quad (13)$$

Thus, the best response strategies are given by relations:

$$\begin{aligned} T_1 &= \min \left\{ \left[ \frac{1}{C} - \frac{N_0 + h_{21} T_2}{h_1} \right]_+, \bar{T}_1 \right\}, \\ T_2 &= \min \left\{ \left[ \frac{1}{C} - \frac{N_0 + h_{12} T_1}{h_2} \right]_+, \bar{T}_2 \right\}. \end{aligned} \quad (14)$$

It is useful to rewrite the relations (14) in the following extended form:

$$T_1 = \begin{cases} 0, & \frac{1}{C} \leq \frac{N_0 + h_{21} T_2}{h_1}, \\ \frac{1}{C} - \frac{N_0 + h_{21} T_2}{h_1}, & \frac{N_0 + h_{21} T_2}{h_1} < \frac{1}{C} \\ & < \frac{N_0 + h_{21} T_2 + h_1 \bar{T}_1}{h_1}, \\ \bar{T}_1, & \frac{N_0 + h_{21} T_2 + h_1 \bar{T}_1}{h_1} \leq \frac{1}{C}, \end{cases} \quad (15)$$

$$T_2 = \begin{cases} 0, & \frac{1}{C} \leq \frac{N_0 + h_{12} T_1}{h_2}, \\ \frac{1}{C} - \frac{N_0 + h_{12} T_1}{h_2}, & \frac{N_0 + h_{12} T_1}{h_2} < \frac{1}{C} \\ & < \frac{N_0 + h_{12} T_1 + h_2 \bar{T}_2}{h_2}, \\ \bar{T}_2, & \frac{N_0 + h_{12} T_1 + h_2 \bar{T}_2}{h_2} \leq \frac{1}{C}. \end{cases} \quad (16)$$

Applying (15) and (16) allows us to obtain which form as functions on taxation rate  $C$  the user equilibrium strategies have to be.

*Theorem 6:* For a given taxation rate  $C$  the users equilibrium strategies  $(T_1, T_2) = (T_1(C), T_2(C))$  have to have the following form:

(00) if  $1/C \in I_{00}$  where

$$I_{00} = \left( 0, \min \left\{ \frac{N_0}{h_1}, \frac{N_0}{h_2} \right\} \right)$$

then

$$T_1(C) := 0 \text{ and } T_2(C) := 0,$$

(i0) if  $1/C \in I_{i0}$  where

$$I_{i0} = \left[ 0, \min \left\{ \frac{N_0 + h_1 \bar{T}_1}{h_1}, \frac{N_0}{h_1} \frac{h_1 - h_{12}}{h_2 - h_{12}} \right\} \right)$$

then

$$T_1(C) := \frac{1}{C} - \frac{N_0}{h_1} \text{ and } T_2(C) := 0,$$

(10) if  $1/C \in I_{10}$  where

$$I_{10} = \left[ \frac{N_0 + h_1 \bar{T}_1}{h_1}, \frac{N_0 + h_{12} \bar{T}_1}{h_2} \right)$$

then

$$T_1(C) := \bar{T}_1 \text{ and } T_2(C) := 0,$$

(0i) if  $1/C \in I_{0i}$  where

$$I_{0i} = \left( \frac{N_0}{h_2}, \min \left\{ \frac{N_0 + h_2 \bar{T}_2}{h_2}, \frac{(h_2 - h_{21}) N_0}{(h_1 - h_{21}) h_2} \right\} \right)$$

then

$$T_1(C) := 0 \text{ and } T_2(C) := \frac{1}{C} - \frac{N_0}{h_2},$$

(ii) if  $1/C \in I_{ii}$  where

$$\begin{aligned} I_{ii} &= \left( \max \left\{ \frac{(h_2 - h_{21}) N_0}{(h_1 - h_{21}) h_2}, \frac{(h_1 - h_{12}) N_0}{(h_2 - h_{12}) h_1} \right\}, \right. \\ &\quad \min \left\{ \frac{(h_2 - h_{21}) N_0 + (h_1 h_2 - h_{12} h_{21}) \bar{T}_1}{h_2 (h_1 - h_{21})}, \right. \\ &\quad \left. \left. \frac{(h_1 - h_{12}) N_0 + (h_1 h_2 - h_{12} h_{21}) \bar{T}_2}{h_1 (h_2 - h_{12})} \right\} \right) \end{aligned}$$

then

$$\begin{aligned} T_1(C) &:= \frac{1}{C} \frac{h_2 (h_1 - h_{21})}{h_1 h_2 - h_{12} h_{21}} - \frac{N_0 (h_2 - h_{21})}{h_1 h_2 - h_{12} h_{21}}, \\ T_2(C) &:= \frac{1}{C} \frac{h_1 (h_2 - h_{12})}{h_1 h_2 - h_{12} h_{21}} - \frac{N_0 (h_1 - h_{12})}{h_1 h_2 - h_{12} h_{21}}. \end{aligned} \quad (17)$$

(1i) if  $1/C \in I_{1i}$  where

$$I_{1i} = \left( \max \left\{ \frac{N_0 + h_{12}\bar{T}_1}{h_2}, \frac{(h_2 - h_{21})N_0 + (h_1h_2 - h_{12}h_{21})\bar{T}_1}{h_2(h_1 - h_{21})} \right\}, \frac{N_0 + h_{12}\bar{T}_1 + h_2\bar{T}_2}{h_2} \right)$$

then

$$T_1(C) := \bar{T}_1 \text{ and } T_2(C) := \frac{1}{C} - \frac{N_0 + h_{12}\bar{T}_1}{h_2},$$

(01) if  $1/C \in I_{01}$  where

$$I_{01} = \left[ \frac{N_0 + h_2\bar{T}_2}{h_2}, \frac{N_0 + h_{12}\bar{T}_2}{h_1} \right]$$

then

$$T_1(C) := 0 \text{ and } T_2(C) := \bar{T}_2,$$

(1i) if  $1/C \in I_{i1}$  where

$$I_{i1} = \left( \max \left\{ \frac{N_0 + h_{21}\bar{T}_2}{h_1}, \frac{(h_1 - h_{12})N_0 + (h_1h_2 - h_{12}h_{21})\bar{T}_2}{h_1(h_2 - h_{12})} \right\}, \frac{N_0 + h_{21}\bar{T}_2 + h_1\bar{T}_1}{h_1} \right)$$

then

$$T_1(C) := \frac{1}{C} - \frac{N_0 + h_{21}\bar{T}_2}{h_1} \text{ and } T_2(C) := \bar{T}_2,$$

(11) if  $1/C \in I_{11}$  where

$$I_{11} = \left[ \max \left\{ \frac{N_0 + h_{21}\bar{T}_2 + h_1\bar{T}_1}{h_1}, \frac{N_0 + h_{12}\bar{T}_1 + h_2\bar{T}_2}{h_2} \right\}, \infty \right)$$

then

$$T_1(C) := \bar{T}_1 \text{ and } T_2(C) := \bar{T}_2.$$

*Sketch of proof.* We just prove the case (ii). The other cases can be proven analogously.

Let  $(T_1, T_2)$  be an equilibrium where  $T_1 \in (0, \bar{T}_1)$  and  $T_2 \in (0, \bar{T}_2)$ . Then by (15) and (16),

$$T_1 = \frac{1}{C} - \frac{N_0 + h_{21}T_2}{h_1}, \quad (18)$$

$$T_2 = \frac{1}{C} - \frac{N_0 + h_{12}T_1}{h_2}, \quad (19)$$

$$\frac{N_0 + h_{21}T_2}{h_1} < \frac{1}{C} < \frac{N_0 + h_{21}T_2 + h_1\bar{T}_1}{h_1}, \quad (20)$$

and

$$\frac{N_0 + h_{12}T_1}{h_2} < \frac{1}{C} < \frac{N_0 + h_{12}T_1 + h_2\bar{T}_2}{h_2}. \quad (21)$$

Then (18) and (19) yield (17). Substituting (17) into (20) and (21) implies that  $1/C$  has to belong to  $I_{ii}$  and (ii) follows.

The optimal taxation level supplies the maximal point for the taxation authority payoff  $v_P(C) = (T_1(C) + T_2(C))C$ . By Theorem 6  $v_P(C)$  is increasing only in  $I_{11}$ , since it is decreasing in  $I_{ii}$  by the fact that  $h_1 + h_2 > h_{21} + h_{12}$ . Thus, we have the following result.

*Theorem 7:* The optimal taxation rate is given by

$$C = \min \left\{ \frac{h_1}{N_0 + h_{21}\bar{T}_2 + h_1\bar{T}_1}, \frac{h_2}{N_0 + h_{12}\bar{T}_1 + h_2\bar{T}_2} \right\}.$$

This taxation rate allows both the users to transmit all the information they intend to with the best rate, so their optimal strategies are  $T_1 = \bar{T}_1$  and  $T_2 = \bar{T}_2$ . The optimal taxation rate for the taxation authority is the one which allows the users to transmit with the best possible rates but for the maximal price both of them agree to pay. We would like to mention again that in reality the taxation authority might choose the taxation level slightly larger than the optimal value so that to encourage the economical use of the energy resource.

#### ACKNOWLEDGEMENT

We would like to acknowledge financial support of the ANR VERSO project ECOSCELLS and RFBR Grant no.09-01-00334-a.

#### REFERENCES

- [1] A. Al Daoud, T. Alpcan, S. Agarwal, and M. Alanyali, "A stackelberg game for pricing uplink power in wide-band cognitive radio networks", in Proc. of 47th IEEE Conf. on Decision and Control, Cancun, Mexico, December 2008.
- [2] E. Altman, K. Avrachenkov, A. Garnaev, "A jamming game in wireless networks with transmission cost". in *Proc. of NET-COOP 2007. Lecture Notes in Computer Science*, v.4465, pp.1-12, 2007.
- [3] L. Lai, and H. El Gamal, "The Water-Filling Game in Fading Multiple-Access Channels". *IEEE Transactions on Information Theory* 54(5): 2110-2122, 2008.
- [4] A. Galegov, A. Garnaev, "Competitive Homogeneous Production with Different Tax Systems" (In Russian). *Vestnik St Petersburg University, Seria 10, Applied Mathematics*, 2009, N 4: 33-47, 2009.