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A TWO-SAMPLE TEST FOR COMPARISON OF LONG MEMORY PARAMETERS

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Mots clés :

1. Processus
2. Statistique mathématique

1 Introduction

We propose a procedure for testing the null hypothesis $d_1 = d_2$ that long memory parameters $d_i \in [0, 1/2)$ of two samples of length n , taken from respective stationary processes X_i , $i = 1, 2$, are equal, against the alternative $d_1 > d_2$. A natural extension of one-sample test developed in [1] about unknown long memory parameter d is to consider test statistic, T_n of the form

$$T_n = \frac{V_1/S_{11,q}}{V_2/S_{22,q}} + \frac{V_2/S_{22,q}}{V_1/S_{11,q}}, \quad (1)$$

where $V_i/S_{ii,q}$ is computed from sample $(X_i(1), \dots, X_i(n))$ ($i = 1, 2$). Here, V_i is the empirical variance of partial sums of X_i

$$V_i = n^{-2} \sum_{k=1}^n \left(\sum_{t=1}^k (X_i(t) - \bar{X}_i) \right)^2 - n^{-3} \left(\sum_{k=1}^n \sum_{t=1}^k (X_i(t) - \bar{X}_i) \right)^2. \quad (2)$$

$S_{ii,q}$ is the Newey-West or HAC estimator of the long-run variance of X_i (see [3])

$$S_{ij,q} = \sum_{h=-q}^q \left(1 - \frac{|h|}{q+1} \right) \hat{\gamma}_{ij}(h), \quad (3)$$

where $\gamma_{ij}(h)$ are the empirical cross covariances of samples $(X_1(1), \dots, X_1(n))$ and $(X_2(1), \dots, X_2(n))$.

From [1], one easily derives the asymptotic null distribution T of the statistic T_n under the condition that the two samples are independent. It is also easy to show that

for $d_1 \neq d_2$, one of the ratios in (1) tends to infinity and the other one to zero, meaning that the test is consistent against the alternative $d_1 \neq d_2$.

However, independence of the two samples is too restrictive and may be unrealistic in many applications. In order to eliminate the eventual dependence between samples, a modification \tilde{T}_n of (1) is proposed, which uses residual observations $(\tilde{X}_1(1), \dots, \tilde{X}_1(n))$, obtained by regressing partial sums of X_1 on partial sums of X_2 .

$$\tilde{X}_1(t) = X_1(t) - (S_{12,q}/S_{22,q})X_2(t), \quad t = 1, \dots, n. \quad (4)$$

The statistics \tilde{T}_n is defined as follows

$$\tilde{T}_n = \frac{\tilde{V}_1/\tilde{S}_{11,q}}{V_2/S_{22,q}} + \frac{V_2/S_{22,q}}{\tilde{V}_1/\tilde{S}_{11,q}}. \quad (5)$$

$\tilde{V}_1, \tilde{S}_{11,q}$ are the statistics in (2), (3), respectively, where $X_1(t), t = 1, \dots, n$ is replaced by $\tilde{X}_1(t), t = 1, \dots, n$.

2 Asymptotic properties of \tilde{T}_n

Consider the class of bivariate linear models $((X_1(t), X_2(t)), t \in \mathbf{Z})$ given by

$$X_i(t) = \sum_{k=0}^{\infty} \psi_{i1}(k)\xi_1(t-k) + \sum_{k=0}^{\infty} \psi_{i2}(k)\xi_2(t-k), \quad i = 1, 2, \quad (6)$$

where

- $\psi_{ij}(k) \sim |k|^{d_{ij}-1}$ ($k \rightarrow \infty$), and $d_{ij} \in (0, 1/2)$.
- $(\xi_1(t), \xi_2(t)), t \in \mathbf{Z}$ is a bivariate (weak) white noise with nondegenerate covariance matrix

$$\begin{pmatrix} 1 & \rho_\xi \\ \rho_\xi & 1 \end{pmatrix}. \quad (7)$$

Let $q = q_n \rightarrow \infty$ and $n/q_n \rightarrow \infty$ as $n \rightarrow \infty$.

Under these conditions, the statistic \tilde{T}_n satisfies the following convergence results :

- (i) If $d_1 = d_2 = d$ then

$$\tilde{T}_n \xrightarrow{\text{law}} \tilde{T}, \quad (8)$$

where the distribution of \tilde{T} only depends on $(d1, d2)$.

- (ii) If $d_1 > d_2$ then

$$\tilde{T}_n \xrightarrow{p} \infty. \quad (9)$$

3 Testing procedure

Let $t_\alpha(d)$ denote the upper α -quantile of the r.v. \tilde{T} defined in (8) when $\rho = 0$), viz.

$$\alpha = P(\tilde{T} > t_\alpha(d)), \quad \alpha \in (0, 1). \quad (10)$$

Let

$$\hat{d} = (\hat{d}_1 + \hat{d}_2)/2, \quad (11)$$

where \hat{d}_i is an estimator of d_i satisfying

$$\hat{d}_i - d_i = o_p(1/\log n) \quad (i = 1, 2). \quad (12)$$

Similarly to [1], it can be proved that the quantile function $t_\alpha(d)$ is continuous in $d \in [0, 1/2)$ for any $\alpha \in (0, 1)$. Therefore, the estimated quantile $t_\alpha(\hat{d}) \rightarrow_p t_\alpha(d)$ as $n \rightarrow \infty$ and the asymptotic level of the tests associated to the critical regions in (13) is preserved by replacing $t_\alpha(d)$ by $t_\alpha(\hat{d})$.

Testing the equality of the memory parameters in the case of possibly dependent samples. We wish to test the null hypothesis $d_1 = d_2$ against the alternative $d_1 > d_2$ in the general case when X_1 and X_2 are possibly dependent. The decision rule at α -level of significance is the following: we reject the null hypothesis when

$$\tilde{T}_n > t_\alpha(\hat{d}). \quad (13)$$

The consistency of this test is ensured by the result given in section 2.

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