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Optimal Monotone Forwarding Policies in Delay Tolerant Mobile Ad Hoc Networks with Multiple Classes of Nodes

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***Abstract*—In this paper we describe a framework for the optimal control of delay tolerant mobile ad hoc networks where multiple classes of nodes co-exist. We specialize the description of the energy-delay tradeoffs as an optimization problem based on a fluid approximation. We then adopt two product forms to model message diffusion and show that optimal controls are of bang-bang type. Under this general framework, we analyze some specific cases of interest for applications.**

***Index Terms*—Delay tolerant networks, optimal control, Pontryagin maximum principle**

I. Introduction

Delay Tolerant Networks (DTNs) received recently the attention of the scientific community [1], [2]. In such networks, no continuous connectivity guarantee can be assumed. However, communication is still possible: in order to deliver messages to destination, in fact, the source can leverage the mobility of some of the nodes in order to relay the message [3], [4]. Due to lack of permanent connectivity, though, a central problem of DTNs is how to route messages towards the intended destination. In particular, mobile nodes which perform message forwarding rarely possess any *a priori* information on the encounter pattern. This is also known as the zero knowledge scenario [5], [6]. One intuitive solution is to disseminate multiple copies of the message in the network, increasing the probability that at least one of them will reach the destination node within a given time window [4].

The above scheme is referred to as epidemic forwarding [7], for it is similar to the spread of infectious diseases. Each time a message-carrying node encounters an uninfected node, it infects this node by passing on the message. Finally, the destination receives the message when it meets an infected node. In this paper, we also refer to a more efficient variant of the plain epidemic routing, namely the two-hop routing protocol. The source transmits copies of its message to all mobiles it encounters, but the latter relay the message only if they meet the destination [8].

In the paper, we study the problem of optimal control of routing. To this respect, in view of the use of battery operated mobile terminals, a mobile DTN depends on its overall energy budget. The energy budget has to accommodate the cost of energy expended on message forwarding. Intuitively, the higher the number of message copies, the smaller the message delay. This gain comes at a price of higher energy expenditure. More precisely, a finite energy cost accrues every time a message is transmitted and received. In the following we will assume that the energy expenditure is linear in the number of released copies; this is a viable approximation especially for sparse networks where the impact of interference and collisions can be neglected.

Control of forwarding schemes has been addressed in the DTNs literature before, see for example, [9], [6], [5], [10], [11], [12], [13]. The work in [14] employs stochastic approximation to avoid the explicit estimation of network parameters. The performance of the two-hop forwarding protocol along with the effect of the timers have been evaluated in [12]; the framework proposed there allows for performance optimization by choosing the average timer duration.

In a previous work [13], we have provided a general framework for the optimal control of a broad variety of monotone relay strategies for the case of a single homogeneous class of mobiles. There, we addressed the

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general optimal control of a dynamical system which is ruled by fluid equations of the kind $\dot{X} = u(t)g(X(t), t)$, where u is a control variable whose natural interpretation is the forwarding probability of DTN nodes. The optimal control was proved there to be of threshold type.

Another central issue in modeling DTNs is the heterogeneity of mobile nodes contacts. The work [15], in particular, showed evidence of how the presence of community structures in human contacts lead to different relative intermeeting intensities between groups of mobiles. The detection of local community structures – and the relative intermeeting patterns – is an important step to design efficient routing strategies.

However, once the estimation of relative intermeeting intensities has been performed, the problem of leveraging such information in order to optimize message forwarding remains a challenging problem in the DTN field. Very few papers in fact addressed this aspect from the modeling perspective. One such work is [16], where the central focus is the general model of multi-class DTNs, where nodes may migrate from one class to another. We use the term m–dimensional, shortly mD, to indicate a DTN with m classes.

We addressed in the past the control of specific instances of multidimensional DTNs; in particular, in [17], we addressed the problem of joint activation and forwarding: the system studied there turned out to be a two-class one. Furthermore, [18] and [19] proposed the optimal control of coding for DTNs under two-hop routing of a DTN comprising N relays; the system studied there is N dimensional, since each node may carry only one frame of the original file.

Motivated by findings in [17] and [18], [19], we address the general problem of how to control a DTN where several classes of mobiles co-exist. We cover in this work explicitly the situation where each one of such classes interacts with other classes of mobiles at different contact rates.

Our goal here is to control transmission policies in such a way to maximize the probability of successful delivery of the message by some time τ, given the total energy budget.

We leverage fluid approximations of the mD system dynamics, focusing on a pair of product type forms that regulate forwarding; we then use tools from control theory to characterize in closed-form dynamic optimal policies.

II. The model

Consider a network that contains $N + 2$ mobile nodes: 1 source node s, N potential relay nodes and 1 special node d which acts as destination. We assume that two nodes are able to communicate when they are within reciprocal radio range, and communications are bidirectional. We also assume that contact intervals are sufficient to exchange all messages: thus we consider nodes *meeting times* only, i.e., time instants at which a pair of not-connected nodes fall within reciprocal radio range.

To this respect, we let the time between contacts of pairs of nodes to be exponentially distributed with given intensity; the validity of this model has been discussed in [20], and its accuracy has been shown for a number of mobility models (Random Walker, Random Direction, Random Waypoint).

In particular, let us assume that there exist M classes of relay nodes: class k, $1 \leq k \leq M$, contains N_k nodes, and $N = \sum N_k$. Also, if two nodes belong to classes i and j respectively, the time between the contacts of such two nodes will have intensity λ_{ij}. Similarly, we let λ_{id} be the intensity for the contact time of destination d and nodes of class i.

We assume that the source has a packet to send to the destination node and such a message is relevant during some time τ. We do not assume any feedback that allows the source or other mobiles to know whether the messages have made it successfully to the destination within the time τ.

In the following we restrict the analysis to a class of so called *monotone relay strategies*. A relay strategy is said to belong to this class if the number of nodes that contain the message does not decrease in time during the interval $[0, \tau]$. An infected node is a node that has received a message copy.

III. Problem statement

Let $X_i(t)$ be the number of nodes of class i that have the message at time t; hence, $X_f = N - \sum_{i=1}^{M} X_i$ is the number of nodes that are not infected. Also, $X = (X_1, X_2, \ldots, X_M)$.

We assume that, under a fluid approximation, the dynamics of the system are regulated by the following set of M coupled differential equations

$$\dot{X}_i = f_i(X_1, \ldots, X_M) = \sum_j u_{ji} \lambda_{ij} f_{ji}(X_j, X_i, X_f) \quad (1)$$

which hold for $i = 1, \ldots, M$.

Notice that u_{ji} has the meaning of a control and represents the probability by which a mobile of class j forwards a message to a mobile of class i when it has one, and u_{ji} in general depends on time. We assume that $u_{ji} \in \mathcal{U} \subseteq [u^{ji}_{\min}, 1]$, where $u^{ji}_{\min} > 0$ can be chosen arbitrarily small.

Furthermore, we let $f_{ji} > 0, i = 1, 2, \ldots, M$, i.e., we consider monotone forwarding policies whose evolution obeys (1). Our goal is to study the energy-delay tradeoff of such a system and to derive the form of the optimal control policies.

To this aim, we first write the fluid approximation for the cumulative distribution function (CDF) of the delay, denoted by $F_D(t) := P(T_d \leq t)$. It is based on a generalization of [13], [21]. Recall that λ_{jd} is the intensity by which nodes of class j meet the destination. We can write

$$F_D(t) = 1 - \exp\Big(- \sum_j \lambda_{jd} \int_{s=0}^{t} X_j(s) ds \Big). \qquad (2)$$

The controlled version reported in (2) derives from the separable differential equation in the form

$$\begin{aligned} \frac{d}{dt} F_D(t) &= \lim_{h \to 0} \frac{\mathbb{P}\,[T_d > t+h] - \mathbb{P}\,[T_d > t]}{h} \\ &= [1 - F_D(t)] \sum_j \lambda_{jd} X_j(s) \end{aligned}$$

Let $\mathcal{E}(t)$ be the energy consumed by the whole network for transmission during the time $[0, t]$. It is proportional to $X(t) - X(0)$ since we assume that the message is transmitted only to mobiles that do not have the message, and thus the number of transmissions of the message during $[0, t]$ plus the number of mobiles that had it at time zero equals to the number of mobiles that have it. We thus have $\mathcal{E}(t) = \varepsilon(X(t) - X(0))$, for some $\varepsilon > 0$, where ε represents the energy spent per contact to transmit and receive.

In view of (1), we want to solve the following optimization problem: we wish to maximize the delivery probability at time τ, i.e., maximize $D(\tau)$, and bound the energy expenditure at time τ.

Due to linearity of the energy expenditure in the number of messages, we can formulate the problem as:

Problem 3.1: Find control policy $\mathbf{u} = (u_1, u_2, \ldots, u_M)$ which solves:

$$\begin{aligned} \max_{\mathbf{u}} \sum_j \lambda_{jd} \int_{s=0}^{\tau} X_j(s) \\ \text{s.t.} \qquad \sum_j X_j(\tau) \leq \Psi \end{aligned} \qquad (3)$$

where $\Psi \geq 0$ is dictated by the constraint on energy.

In order to characterize optimal policies, we will consider in the following two reference product forms, i.e., policies of the type

a) $f_{ji}(X_j, X_i, X_f) = f_j(X_j)(N_i - X_i)$; in this case the message is forwarded from a class j node to a class i node only; furthermore, the N_i nodes belonging to class i are defined in advance and may be infected or not depending on the dynamics;

b) $f_{ji}(X_j, X_i, X_f) = f_j(X_j)(N - X_f)$; in this case the message is forwarded from a class j node to any node without the message, irrespective of the class it belongs to.

Also, for the sake of simplicity, we will consider in the following f_j to be regular enough wherever required.

IV. Structure of the Optimal control

In order to study the maximization problem formulated above, we introduce some notation. First, we resort to an unconstrained formulation using

$$J_\tau(u) := q(X(\tau)) + \int_0^{\tau} g(X)\, ds \qquad (4)$$

as a modified objective function, where from Problem (3.1) we have

$$q(X(\tau)) = \mu(\Psi - \sum_j X_j(\tau)) \quad \text{and} \quad g(X) = \sum \lambda_{jd} X_j$$

Remark 4.1: In what follows, we make two assumptions: first, we assume that the problem is feasible, that is under the control $u_{ij} = u^{ji}_{\min}$ for all i's and j's, the inequality $\sum_j X_j(\tau) \leq \Psi$ holds; second, in view of the monotonicity of the forwarding policies considered in the following, we exclude the trivial case that the control $u_{ij} = 1$ leads to $\sum_j X_j(\tau) < \Psi$. Here μ plays the role of a Lagrange multiplier, and we have $\mu > 0$ because $\sum_j X(\tau) \geq \Psi$. The optimal solution is attained for the value μ^* such that slackness is attained.

Note that the dependence on u is implicitly through X; we should further note that each component of u is upper bounded by 1 and lower bounded by some positive number. We assume that regularity conditions hold for the dynamics of the system, so that we can apply the Pontryagin's maximum principle. Let us introduce the Hamiltonian

$$H(X, u, p) = p(t)^T f(X, u) + g(X)$$

where p is the vector with components p_k. By the Maximum Principle, if an optimal solution exists, the optimal control u^* is such that

$$u^*(t) = \arg\max_{u \in \mathcal{U}} H(X, u, p)$$

where the associated Hamiltonian system is

$$\dot{X}_k = H_{p_k}(X, u, p) \qquad (5)$$

$$\dot{p}_k = -H_{X_k}(X, u, p) \qquad (6)$$

and the second equation is the adjoint equation that regulates multipliers (co-state variables) p_k. Furthermore, under the optimal control, we have M terminal conditions in the form $p(\tau) = [q(X(\tau))]_{X_k} = -\mu$.

Writing out the explicit expression for H,

$$H(X, u, p) = \sum_{i=1}^{M} \lambda_{id} X_i + p_i \sum_{j=1}^{M} u_{ji} \lambda_{ij} f_{ji}(X_j, X_i, X_f)$$

we have

$$\dot{p}_k = -\lambda_{kd} - \sum_{i=1}^{M} p_i \sum_{j=1}^{M} u_{ji} \lambda_{ij} \left[f_{ji}(X_j, X_i, X_f) \right]_{X_k}$$

In what follows, we will derive sufficient conditions under which the solution to the optimal control problem has a very simple form. In particular, we define [22]

Definition 4.1: A *bang-bang policy* is one where $u_{ij}(t)$ takes only extreme values, that is $u_{ij}(t) = 1$ or $u_{ij}(t) = u_{\min}^{ij}$ a.e. in $[0, \tau]$.

Notice that, in order to control forwarding policies, bang-bang policies are very convenient for implementation purposes since they rely only on the age of the message and on a set of *switching epochs*, where the control at each class switches from 1 to $u_{\min}^{ij}$ or *vice versa*. Notice that a bang-bang policy requires to monitor at least M^2 timers.

In some particular cases only one switch from 1 to $u_{\min}^{ij}$ per component is sufficient; in such cases the control policy is called a *threshold policy*.

In the following, without loss of generality, we will restrict to the case $u_{\min}^{ij} = u_{\min}$.

Type a) product forms

For systems of type a), the corresponding expression for the Hamiltonian is

$$H(X, u, p) = \sum_{i=1}^{M} \lambda_{id} X_i + p_i \sum_{j=1}^{M} u_{ji} \lambda_{ij} f_j(X_j)(N_i - X_i)$$

With respect to optimal policies for Problem 3.1 we obtain the following.

Theorem 4.1: Consider a system of type a), and assume $\lambda_{kd} > 0$ for $1 \le k \le M$. Then every optimal policy is a bang-bang policy.

Proof: Let us first consider the case when the p_i are non-zero almost everywhere. From (7), it easy to see that, since the optimal policy maximized the Hamiltonian, the optimal policy has to satisfy

$$u_{jk}(t) = \begin{cases} u_{\min} & \text{if } p_k(t) < 0 \\ 1 & \text{if } p_k(t) > 0 \end{cases} \qquad (7)$$

We need some further investigation for the singular arcs: in particular, we shall prove that multipliers p_k's cannot vanish over any interval I of positive measure $\mu(I) > 0$, and this guarantees that the control is actually bang-bang [22]. Let us suppose that $p_k|_I \equiv 0$. Then, from the dual equation we obtain

$$\dot{p}_k = -H_{X_k} = -\lambda_{kd} - \dot{f}_k(X_k) \sum_{i=1}^{M} p_i \, u_{ki} \lambda_{ik} (N_i - X_i)$$

$$+ p_k \sum_{j=1}^{M} u_{jk} \lambda_{kj} f_j(X_j)$$

$$= -\lambda_{kd} - \dot{f}_k(X_k) \sum_{i \in \mathcal{C}_k} p_i \, u_{ki} \lambda_{ik} (N_i - X_i), \qquad \forall t \in I$$

where $\mathcal{C}_k = \{1 \le i \le M | \; p_i|_I \not\equiv 0\}$. Notice that we must have $\mathcal{C}_k \neq \emptyset$, as otherwise $\dot{p}_k|_I \not\equiv 0$.

Since $\lambda_{kd} > 0$, and due to the monotonicity of X_k, for $X_k \in \mathrm{Im}[X_k|_I]$, we can write

$$[H_{X_k}]_{X_k} = -\ddot{f}_k(X_k) \sum_{i \in \mathcal{C}_k} p_i \, u_{ik} \lambda_{ik} (N_i - X_i) = 0, \; \forall t \in I$$

and due to the fact that $k \notin \mathcal{C}_k$, it follows that $\ddot{f}_k(X_k) \equiv 0$ over I. In turn, this implies that $f_k(X_k) = A_k X_k + B_k$ for some constants A_k and B_k. Thus, it follows that, for $j \in \mathcal{C}_k$

$$[H_{X_k}]_{X_j} = A_k p_j \, u_{jk} \lambda_{jk} = 0, \qquad \forall t \in I \qquad (8)$$

so that $A_k = 0$. But, since $\lambda_{kd} > 0$, from (8), it would be $p_k \neq 0$ over I, which is a contradiction. $\diamond$

Type b) product forms

Here we provide a characterization of a class of type b) product forms for which the optimal control policy is of bang-bang type. The proof is similar to the one reported for type a) product forms; however, it does require additional assumptions, namely $\dot{f}_k > 0$ for all ks, a.e., due to the presence of the X_f term in the product form.

Theorem 4.2: Consider a system of type b), and assume $\lambda_{kd} > 0$ and $\dot{f}_k > 0$ for all k's, a. e.. Then, every optimal policy is a bang-bang policy.

Proof: In the case of product forms of type b), the Hamiltonian can be written as

$$H(X,u,p) = \sum_{i=1}^{M} \lambda_{id} X_i + p_i \sum_{j=1}^{M} u_{ji}\lambda_{ij} f_j(X_j)(N - X_f)$$

so that the over non-singular arcs the control is indeed bang-bang, as in the case of the product forms of type a). The case of singular arcs needs separate discussion. As before, we need to prove that p_k cannot vanish over any interval I of positive measure $\mu(I) > 0$. In fact, let us assume that $p_k|_I \equiv 0$. Then, from the dual equation we obtain

$$\begin{aligned} \dot{p}_k &= -H_{X_k} \\ &= -\lambda_{kd} - \sum_{i=1}^{M} p_i \sum_{j=1}^{M} u_{ji}\lambda_{ij} \left[f_j(X_j)(N - \sum_{r=1}^{M} X_r) \right]_{X_k} \\ &= -\lambda_{kd} - \dot{f}_k(X_k)(N - \sum_r X_r) \sum_{i=1}^{M} p_i\, u_{ki}\lambda_{ik} \\ &- \sum_{i=1}^{M} p_i \sum_{j=1}^{M} u_{ji}\lambda_{ij} f_j(X_j) = 0 \qquad \forall t \in I \end{aligned} \tag{9}$$

Case 1: We can first assume that $\mathfrak{L} = \sum_{i=1}^{M} p_i\, u_{ki}\lambda_{ik} \neq 0$ for $t \in I$, hence, we can write

$$\frac{\partial^2 H}{\partial {X_k}^2} = -\ddot{f}_k(X_k)(N - X_f) \sum_{i \in \mathcal{C}_k} p_i\, u_{ik}\lambda_{ik} = 0,\ \forall t \in I$$

and it follows that $f_k(X_k) = A_k X_k + B_k$ for $X_k \in Im[X_k|_I] = \Im$. Hence, this means that

$$A_k = -\frac{\lambda_{kd} + \sum_{i=1}^{M} p_i \sum_{j=1}^{M} u_{ji}\lambda_{ij} f_j(X_j)}{(N - \sum_r X_r) \sum_{i=1}^{M} p_i\, u_{ki}\lambda_{ik}} \tag{10}$$

For the sake of notation, we let $A_k = N/D$. Then, we must have $[A_k]_{X_k} = 0$. Hence, we can simply derive (10) and obtain

$$\begin{aligned} \frac{\partial H}{\partial X_k} &= \frac{\frac{\partial N}{\partial X_k} D - N \frac{\partial D}{\partial X_k}}{D^2} = \frac{A_k \mathfrak{L} D + N \mathfrak{L}}{D^2} \\ &= \frac{A_k \mathfrak{L}}{D} + \frac{N}{D}\frac{\mathfrak{L}}{D} = \frac{2A_k\mathfrak{L}}{D} = 0 \end{aligned} \tag{11}$$

from which $A_k = 0$ follows. Hence, $\dot{f}_k(X_k) = 0$ for $X_k \in \Im$, contradicting our initial assumption.

Case 2: Let us now assume that $\mathfrak{L} = \sum_{i=1}^{M} p_i\, u_{ki}\lambda_{ik} = 0$ for $t \in I$: then it follows that

$$\sum_{j=1}^{M} f_j(X_j) \sum_{i=1}^{M} p_i\, u_{ji}\lambda_{ij} = -\lambda_{kd} \tag{12}$$

Now, in view of the monotonicity and continuity of the solutions, we can consider a suitable open set containing the state trajectory corresponding to the singular arc due to p_k: we can then differentiate (12)

$$\dot{f}_j(X_j) \sum_{i=1}^{M} p_i\, u_{ji}\lambda_{ij} = 0 \tag{13}$$

Thus, $\sum_{i=1}^{M} p_i\, u_{ji}\lambda_{ij} = 0$, for all j-th terms appearing in (13). But, this is not possible, since the left-hand term (12) would vanish, and $\lambda_{kd} > 0$ from the initial assumption. ⋄

Remark 4.2: We observe that the assumption that $\dot{f} > 0$ for type b) product is quite demanding: we have to exclude all forwarding policies that implement two-hop routing depending on the number of infected mobiles in a given class. To give an example, consider a forwarding policy that performs epidemic routing until a certain number of mobiles say X_0 have been infected, and then switches to two-hop routing. Thus, $f_j(X_j) = X_j$ for $0 \le X_j \le X_0$, while $f_j(X_j) = X_0$ for $0 \le X_j \le X_0$.

However, in the final example section, we will show that for the mD two-hop routing protocol obeying to type b) form not only the optimal control is in fact bang–bang but it is of threshold type.

A. Switching epochs

From the general characterization of optimal policies described above, it follows that the switching epochs of the forwarding controls to a given class from the remaining classes are synchronized, i.e.,

Corollary 4.1: If assumptions of Thm. 4.1 or Thm. 4.2 hold, then, under the optimal policy switching epochs are aligned, i.e., $u^*_{hi} = u^*_{ki}$ for $1 \le i, h, k \le M$.

The above corollary suggests that the optimal control of the forwarding policy only depends on the destination class and not on the source class. This in turn reduces the number of timers to be stored to M per message instead of M^2.

V. Example: multiclass Two-hops routing

In the following we look at the simplest possible case of a multi-dimensional system, i.e., an M–dimensional two-hop routing protocol where a source node meets relays with different rates; with the formalism introduced before, the source infects relays with rate λ_{si}, $i = 1, \ldots, M$. In turn, such infected nodes meet the destination node with given rate λ_{id}, $i = 1, \ldots, M$.

Here we want to find the optimal forwarding strategy to be implemented at the source node. For simplicity, let us refer to the type b) $M = 2$ case; the dynamics is represented by the following system

$$\dot{X}_1 = u_1 \lambda_{s1}(N - X_f)$$
$$\dot{X}_2 = u_2 \lambda_{s2}(N - X_f) \quad (14)$$

The corresponding Hamiltonian is

$$H(X, u, p) = \lambda_{1d} X_1 + \lambda_{2d} X_2 + p_1\, u_1\, \lambda_{s1}(N - X_1 - X_2) + p_2\, u_2\, \lambda_{s2}(N - X_1 - X_2)$$

from which the co-state dynamics is derived as

$$\dot{p}_1 = -H_{X_1} = -\lambda_{1d} + p_1\, u_1\, \lambda_{s1} + p_2\, u_2\, \lambda_{s2} \quad (15)$$
$$\dot{p}_2 = -H_{X_2} = -\lambda_{2d} + p_1\, u_1\, \lambda_{s1} + p_2\, u_2\, \lambda_{s2}$$

From the termination condition we know that $p_1(\tau) = p_2(\tau) = -\mu$.

Notice that the above dynamics can be easily generalized to the mD case for $m > 2$ and the dynamics of the multipliers as well. Assume that at least one lag i exists such that $\lambda_{id} > 0$: we observe that the mD two-hop routing with type b) has optimal solution that is of bang-bang type since indeed singular arcs cannot occur due to the form of (15). Notice that in the case of type a) form two-hop routing, we already know that the solution is of bang-bang type; the dynamics are obtained from (14) replacing N with N_i and X_f with X_i; the expression for the Hamiltonian follows similarly.

Overall, in the case of mD two hop routing we can state a stronger result. Recall that a threshold policy is a bang-bang policy that switches from 1 to $u_{\min}$ at most once in $[0, \tau]$ on each component.

Theorem 5.1: The optimal control for the mD two-hop routing is of threshold type.

Proof: In the case of a system of type a) the result follows immediately from the result given in [13] in the 1D case. In the case of type b) mD two-hop routing, assume that the slackness condition is attained. By contradiction, assume that component i of the corresponding optimal control switches once from $u_{\min}$ to 1 at some time t_s: $u_i(t) = 1$ for $0 \le t \le t_s$ and $u_j(t) = u_{\min}$ for $t_s < t \le \tau$ (if multiple such switches would occur we can restrict to a suitable interval and consider an appropriate number of nodes and terminal condition). Notice that not all the remaining controls can be equal to $u_{\min}$ during $[0, t_s]$, otherwise we would contradict again the 1D dimensional case (again changing terminal value and number of nodes according to the optimal policy at hand). Finally, we are left with the case when there exists another component j such that $u_j(t) = 1$ for $t \in [0, t_s]$. Consider

$$\ddot{J}_j = \lambda_{sj}\lambda_{jd}(N - X_f)u_{sj} \ge 0$$
$$\ddot{J}_i = \lambda_{si}\lambda_{id}(N - X_f)u_{si} \ge 0$$

where $J_i(t) = \int_0^t \lambda_{jd} X_j(u)du$. Note that at time 0 the optimality of $u_j(t) = 1$, $t \in [0, t_s]$ brings $\lambda_{sj}\lambda_{jd} \ge \lambda_{si}\lambda_{id}$. Since it also follows that $J_j(t_s) \ge J_i(t_s)$, then the optimal control is $u_{si}(t) = u_{\min}$ and $u_{sj}(t) = 1$ for $t \in [t_s, \tau]$, contradicting our initial assumption. $\diamond$

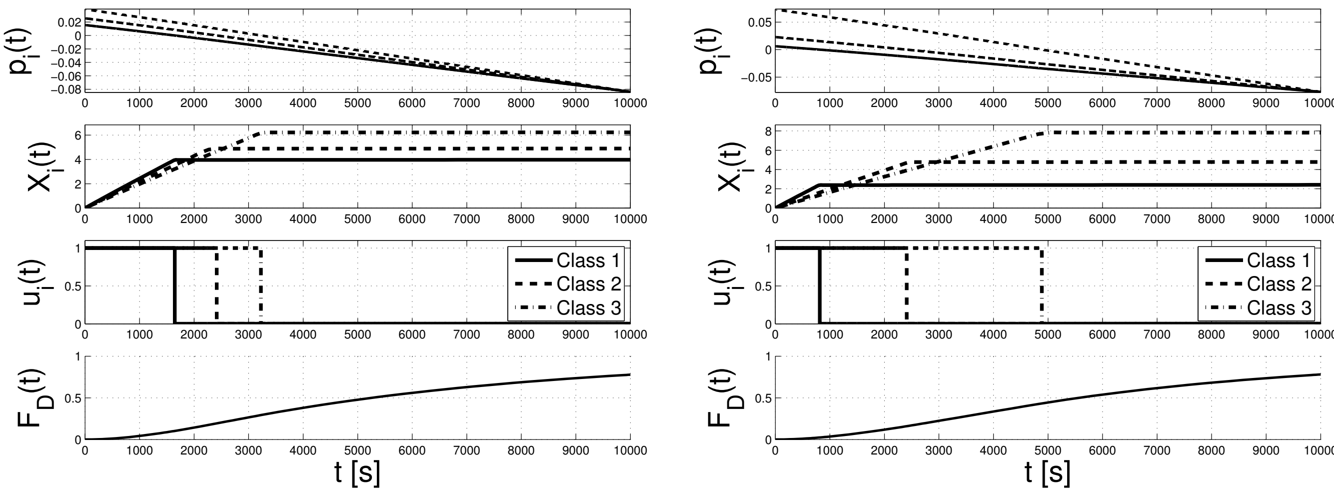
In order to determine the optimal control, we performed numerical evaluations that we present below; in particular, we report on the dynamics of the system in some reference cases for $M = 3$. The numerical integration procedure was performed as follows:

1) the dynamics of the p_is has been obtained integrating (15) backwards in $[0, \tau]$;
2) the control law u and the corresponding message delivery probability are derived accordingly;
3) finally, an interpolated version of the p_is are input to the forward integration of the ODE describing the state trajectory X;

Furthermore, we performed a binary search for the optimal multiplier μ that attains the slackness condition: we recall that this multiplier corresponds to the maximization of the original Lagrangian appearing in (4).

For the cases described in the following we employed the Matlab® ODE suite. Mobile nodes intermeeting intensities are dimensioned using the Random Waypoint (RWP) synthetic mobility model for which intermeeting intensities can be calculated [20]. The reference intermeeting intensity, for the following experiments, is $\lambda_0 = 4.46 \cdot 10^{-5}$; this value is experienced by RWP mobile nodes moving on a square playground of side $L = 2500$ m, speed $v = 3$ m/s and radio range 20 m. In particular, we considered terminal time $\tau = 10000$ s, $u_{\min} = 0.001$ and $N = 200$. The constraint is set to $\Psi = 15$.

Next we show how the optimal control behaves in some sample cases; recall that if the intermeeting intensities were the same, the optimal control would be a single threshold one, equal for all classes. Consider the numerical results reported in Fig 1a) and b): they describe two different settings – case 1 and case 2 in the following – for three classes of nodes under the



(a) Case 1: $\lambda_{1d} = 0.30\lambda_0$, $\lambda_{2d} = 0.33\lambda_0$ $\lambda_{3d} = 0.37\lambda_0$, $\lambda_{s1} = 0.37\lambda_0$, $\lambda_{s2} = 0.33\lambda_0$, $\lambda_{s3} = 0.30\lambda_0$,

(b) Case 2: $\lambda_{1d} = 0.2\lambda_0$, $\lambda_{2d} = 0.3\lambda_0$ $\lambda_{3d} = 0.5\lambda_0$, $\lambda_{s1} = 0.5\lambda_0$, $\lambda_{s2} = 0.3\lambda_0$, $\lambda_{s3} = 0.2\lambda_0$,

Fig. 1: Numerical evaluation of the optimal policy in the case of two-hop routing for $M = 3$ classes.

3D two hop routing of type b). The two upper graphs of each figure depict the behavior of the multipliers and the related threshold-type of control. The bottom two graphs describe the dynamics of the infected nodes and the CDF of the success probability. In Fig 1a) we reported the optimal control in the case when intensities λ_{si} and λ_{id} are similar for the three classes. It is seen there that the optimal control proves very sensible to such parameters, since a very small difference in the intermeeting intensities – namely $\lambda_{1d} = 0.30\lambda_0$, $\lambda_{2d} = 0.33\lambda_0$ $\lambda_{3d} = 0.37\lambda_0$, $\lambda_{s1} = 0.37\lambda_0$, $\lambda_{s2} = 0.33\lambda_0$, $\lambda_{s3} = 0.30\lambda_0$ – produce thresholds far apart on the scale of τ: $t_{s1} = 657.5$ s, $t_{s2} = 2472.5$ s and $t_{s3} = 4318s$, respectively. The success probability at terminal time is $F_D(\tau) = 0.7777$.

Now consider the setting of Fig 1b): here $\lambda_{1d} = 0.2\lambda_0$, $\lambda_{2d} = 0.3\lambda_0$ $\lambda_{3d} = 0.5\lambda_0$, $\lambda_{s1} = 0.5\lambda_0$, $\lambda_{s2} = 0.3\lambda_0$, $\lambda_{s3} = 0.2\lambda_0$. Notice that the cumulative rate at which the source and the destination encounter relays is the same in the two cases: it is not obvious what is the most favorable condition in terms of optimal forwarding probability for the same constraint. In fact, we obtain another triple of thresholds: $t_{s1} = 148$ s, $t_{s2} = 3593.5$ s and $t_{s3} = 6083s$, respectively. However, the success probability at terminal time is $F_D(\tau) = 0.7872$. The behavior of the control is understood: class 1 in both cases has the lowest intermeeting rate with the destination and the highest intermeeting rate with the source, whereas class 3 does the opposite and class 2 is the intermediate case. The control thus tries to infect nodes of class 1 such in a way to give them the possibility to meed the destination over the largest interval; shorter intervals would not permit delivery of message copies with large probability by nodes of class 1. Conversely, nodes of class 3 are infected actively for a large interval since they can be met at lower pace but meet the destination with high probability over relatively short intervals.

Intuitively it is optimal to spend more energy and over a larger time interval to give message copies to those relays able to deliver messages to the destination at faster rate. In our example, the heterogeneity of intermeeting intensities may be beneficial to increase the delivery probability, as it is seen in Fig 2. However, repeating the same numerical analysis of the optimal control for $\tau = 3000$, we obtain $F_D(\tau) = 0.2688$ for the case 1, and $F_D(\tau) = 0.2584$ for case 2. Hence, we observe that larger intermeeting heterogeneity is not always beneficial, showing a dependence on the deadline τ.

VI. CONCLUSIONS

In this paper we addressed the problem of controlling forwarding in delay tolerant networks where several classes of mobiles exist and a message is to be delivered to a destination node. Our model is based on fluid approximations that are common in the field of epidemic spreading: we addressed in particular the case of two product forms, able to capture cases of interest such as mD epidemic routing and mD two-hop routing. We described the structure of the optimal solution for the problem of time-limited delivery of a message under a constraint on the total energy expenditure in the network.

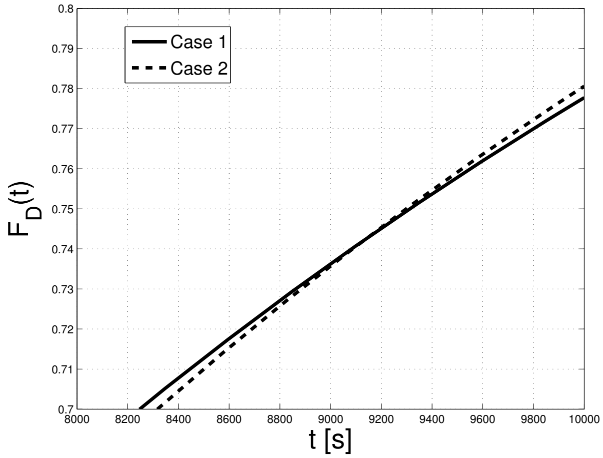


Fig. 2: Detail: comparison of the the delay CDF in the two cases.

The optimal solution is proved to be of bang-bang type. In particular, for the case of multi-class two-hop routing, the optimal control is of threshold type, and the procedure to derive threshold values was described numerically. These results provide novel insight and additional tools in order to study a central problem in delay tolerant networks, namely the effect of the heterogeneity of contact times, where the relative relevance of relays – or classes of relays – has to be accounted for in order to optimally control the delivery of a message.

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