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▶ To cite this version:

Jérémie Chalopin, Victor Chepoi, Nicolas Nisse, Yann Vaxès. Cop and robber games when the robber can hide and ride. 8th French Combinatorial Conference, Jun 2010, Orsay, France. inria-00482117

HAL Id: inria-00482117 https://inria.hal.science/inria-00482117

Submitted on 9 May 2010

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Cop and robber games when the robber can hide and ride

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In the classical cop and robber game, two players, the cop \mathcal{C} and the robber \mathcal{R} , move alternatively along edges of a finite graph G=(V,E). The cop captures the robber if both players are on the same vertex at the same moment of time. A graph G is called $cop\ win$ if the cop always captures the robber after a finite number of steps. Nowakowski, Winkler (1983) and Quilliot (1983) characterized the cop-win graphs as dismantlable graphs. In this talk, we will characterize in a similar way the class $\mathcal{CWFR}(s,s')$ of cop-win graphs in the game in which the cop and the robber move at different speeds s' and s, $s' \leq s$. We also establish some connections between cop-win graphs for this game with s' < s and Gromov's hyperbolicity. In the particular case s' = 1 and s = 2, we prove that the class of cop-win graphs is exactly the well-known class of dually chordal graphs. We show that all classes $\mathcal{CWFR}(s,1)$, $s \geq 3$, coincide and we provide a structural characterization of these graphs. We also investigate several dismantling schemes necessary or sufficient for the cop-win graphs (which we call k-winnable and denote by $\mathcal{CWW}(k)$) in the game in which the robber is visible only every k moves for a fixed integer k > 1. We characterize the graphs which are k-winnable for any value of k.

We present now our main results (for the full version, see [1]). A graph G = (V, E) is (s, s')-dismantlable if its vertices can be ordered v_1, \ldots, v_n so that for each $v_i, 1 \leq i < n$, there exists v_j with j > i, such that $N_s(v_i, G \setminus \{v_j\}) \cap \{v_i, v_{i+1}, \ldots, v_n\} \subseteq N_{s'}(v_j)$. A graph G is dually chordal if its clique hypergraph is a hypertree. Dually chordal graphs are exactly the graphs G = (V, E) admitting a maximum neighborhood ordering.

Theorem 1. For any $s, s' \in \mathbb{N} \cup \{\infty\}$, $s' \leq s$, a graph G = (V, E) belongs to $\mathcal{CWFR}(s, s')$ iff G is (s, s')-dismantlable.

Theorem 2. For a graph G = (V, E), the following conditions are equivalent: (i) $G \in \mathcal{CWFR}(2)$; (ii) G is (2, 1)-dismantlable; (iii) G is dually chordal.

A graph G is a big brother graph if its blocks can be ordered B_1, \ldots, B_r such that B_1 is dominated and, for any i > 1, the block B_i is a leaf in the block-decomposition of $\bigcup_{j \leq i} B_j$ and is dominated by the articulation point connecting B_i to $\bigcup_{j < i} B_j$. A graph G is a big two-brother graph if G can be represented as a union of its subgraphs G_1, \ldots, G_r labeled in such a way that G_1 has a dominating vertex, and for any i > 1, either the subgraph G_i intersects $\bigcup_{j < i} G_j$ in two adjacent vertices x_i, y_i belonging to a common subgraph $G_j, j < i$, so that y_i dominates G_i , or G_i has a dominating vertex y_i and intersects $\bigcup_{j < i} G_j$ in a single vertex x_i (that may coincide with y_i).

Theorem 3. For a graph G = (V, E) the following conditions are equivalent: (i) $G \in \mathcal{CWFR}(3)$; (i') G is (3,1)-dismantlable; (ii) $G \in \mathcal{CWFR}(\infty)$; (ii') G is $(\infty,1)$ -dismantlable; (iii) G is a big brother graph. In particular, the classes of graphs $\mathcal{CWFR}(s)$, $s \geq 3$, coincide.

Theorem 4. A graph G = (V, E) is k-winnable for all $k \ge 1$ if and only if G is a big two-brother graph.

A graph G = (V, E) is k-bidismantlable if its vertices can be ordered v_1, \ldots, v_n in such a way that for each vertex $v_i, 1 \leq i < m$, there exist two adjacent or coinciding vertices x, y with $y = v_j, x = v_\ell$ and $j, \ell > i$ such that $N_k(v_i, G \setminus \{x, y\}) \cap X_i \subseteq N_1(y)$, where $X_i := \{v_i, v_{i+1}, \ldots, v_m\}$.

Theorem 5. Any graph G = (V, E) of $\mathcal{CWW}(2)$ is 2-bidismantlable, however there exist 2-bidismantlable graphs G with $G \notin \mathcal{CWW}(2)$. For any odd integer $k \geq 3$, if a graph G is k-bidismantlable, then $G \in \mathcal{CWW}(k)$.

[1] J. Chalopin, V. Chepoi, N. Nisse, Y. Vaxès, Cop and robber games when the robber can hide and ride, arXiv:1001.4457.