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DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Information Propagation Speed in Bidirectional Vehicular Delay Tolerant Networks

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Abstract: This research report analyses information propagation speed in bidirectional vehicular delay tolerant networks such as roads or highways. The analysis provided in this report shows that phase transition occurs concerning information propagation speed, with respect to vehicle density in each direction of the highway. Under a certain threshold, information propagates on average at vehicle speed, while above this threshold, information propagates faster. This report gives the exact expression of this threshold and of the average propagation speed in this context.

Key-words: Ad hoc, DTN, vehicular network, information, propagation speed

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Vitesse de Propagation de l'Information dans les Réseaux DTN Véhiculaires Bidirectionnels

Résumé : Ce rapport étudie la vitesse de propagation de l'information dans les réseaux DTN véhiculaires bidirectionnels, tels des automobiles sur une une autoroute. L'analyse présentée dans ce rapport montre qu'une transition de phase se produit concernant la vitesse de propagation de l'information, par rapport à la densité des véhicules dans chaque direction. Sous une certaine borne, l'information se propage à la vitesse des véhicules, alors qu'au dessus de cette borne, l'information se propage plus vite. Ce rapport donne l'expression exacte de cette borne et de la vitesse moyenne de propagation de l'information dans ce contexte.

Mots-clés : Ad hoc, DTN, réseau véhiculaire, information, vitesse de propagation

1 Introduction

This research report analyses information propagation speed in bidirectional vehicular delay tolerant networks such as roads or highways. The analysis provided in this report shows that phase transition occurs concerning information propagation speed, with respect to vehicle density in each direction of the highway. Under a certain threshold, information propagates on average at vehicle speed, while above this threshold, information propagates faster. This report gives the exact expression of this threshold and of the average propagation speed.

2 Analysis

In the following we consider a bidirectional vehicular network, such as a highway, where vehicles move in two opposite directions (say east and west respectively) at speed v, and where eastbound vehicle density is Poisson with intensity λ_e , while westbound vehicle density is Poisson with intensity λ_w . We furthermore consider that radio propagation speed (including store and forward timings) is denoted v_r , and that the radio range of each transmission in each direction is of length R.

The main result presented in this report is that, concerning information propagation speed in such an environment, a phase transition occurs when $\lambda_e R$ and $\lambda_w R$ coincide on the curve $y = xe^{-x}$:

$$\lambda_e R e^{-\lambda_e R} = \lambda_w R e^{-\lambda_w R}.$$
(1)

Figure 1 shows the corresponding threshold curve for R = 1 (in the following, we will always consider the case R = 1, without loss of generality). We show in this report that below this threshold, average information propagation speed is blocked to vehicle speed, while above the curve, information propagates faster on average.

2.1 Proof Outline

We will first consider cases where radio propagation speed v_r is infinite. We start by proving in Section 2.2 that westbound cluster length L_w : $f_w(\theta) = \mathbf{E}(e^{-\theta L_w})$:

$$f_w(\theta) = e^{-\lambda_w - \theta} + f_w(\theta) \int_0^1 \lambda_w e^{-(\lambda_w + \theta)x} dx$$
(2)

$$= e^{-\lambda_w} + f_w(\theta) \frac{\lambda_w}{\lambda_w + \theta} (1 - e^{-\lambda_w - \theta}) .$$
(3)

Thus

$$f_w(\theta) = \frac{(\lambda_w + \theta)e^{-\lambda_w - \theta}}{\theta + \lambda_w e^{-\lambda_w - \theta}} .$$
(4)

The exponential tail of the distribution of L_w is given by θ_w where $-\theta_w$ is the root of the denominator, namely:

$$\theta_w e^{-\theta_w} = \lambda_w e^{-\lambda_w} \ . \tag{5}$$

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Figure 1: Information propagation threshold with respect to (λ_e, λ_w) . Below the curve, average information propagation speed is blocked to vehicle speed, while above the curve, information propagates faster on average.

Then assume that the gap until the next eastbound cars (counted from the previous car radio range) is G_e with $G_e > 0$. We show in Section 2.4 that $P(G_e > x) = O(e^{-\lambda_e x})$.

Now let $T_w(x)$ be the time needed to meet a west westbound cluster long enough to fill the gap x (*i.e.* larger than x). We show in Section 2.3 that

$$\mathbf{E}(T_w(x)) = O(\frac{1}{vP(L_w > x)}) = O(e^{\theta_w x}) .$$
(6)

Therefore the average time T_w to get a bridge over all possible gaps is

$$\mathbf{E}(T_w) = \int_0^\infty \mathbf{E}(T_w(x))e^{-x\lambda_e}dx = \frac{1}{2v}\int_0^\infty O(\exp((\theta_w - \lambda_e)x))dx \ . \tag{7}$$

The threshold with respect to (λ_w, λ_e) where $\mathbf{E}(T_w)$ diverges is clearly when we have

$$\theta_w = \lambda_e \tag{8}$$

or in other words, since $\theta_w e^{-\theta_w} = \lambda_w e^{-\lambda_w}$, when we have

$$\lambda_w e^{-\lambda_w} = \lambda_e e^{-\lambda_e} \ . \tag{9}$$

2.2 Cluster Distribution

Let us define a westbound cluster as a set of consecutive westbound cars such that this set forms a connected component of the westbound network. We define the length of a cluster as the length of the road between the first and the last car, plus one radio range (counted from the last car).

The length of the cluster L_w is counted from the first car. It satisfies

- $L_w = 1$, with probability $e^{-\lambda_w}$ when the first car has no car behind within the radio range;
- $L_w = g_w + L_w$, with probability $1 e^{-\lambda_w}$ where g_w is the gap with the next car and $g_w < 1$.

Translating this in term of Laplace transforms yields equation (2). Note that by the way we get

$$\mathbf{E}(L_w) = -f'_w(0) = \frac{e^{\lambda_w} - 1}{\lambda_w} .$$
(10)

The asymptotics on $P(L_w > x)$ is given by inverse Laplace

$$P(L_w > x) = -\frac{1}{2i\pi} \int_{-\varepsilon - i\infty}^{-\varepsilon + i\infty} \frac{f_w(\theta)}{\theta} e^{\theta x} d\theta$$
(11)

$$= -\frac{1}{2i\pi} \int_{-\varepsilon - i\infty}^{-\varepsilon + i\infty} \frac{(\lambda_w + \theta)e^{-\lambda_w}}{(\theta e^{\theta} + \lambda_w e^{-\lambda_w})\theta} e^{\theta x} d\theta .$$
(12)

For some $\varepsilon > 0$ small enough. The function $\frac{f_w(\theta)}{\theta}$ has two simple poles at $\theta = -\theta_w$ and $\theta = -\lambda_w$ and is absolutely integrable elsewhere. The residus theorem kills the pole at $-\lambda_w$, therefore for some $\varepsilon > 0$:

$$P(L_w > x) = \frac{(\lambda_w - \theta_w)e^{\theta_w - \lambda_w}}{(1 - \theta_w)\theta_w}e^{-\theta_w x} + O(e^{-(\theta_w + \varepsilon)x}) .$$
(13)

RR n° 7266

2.3 Road Length to Bridge a Gap

Let's assume we want to fill a gap of length x. In this case we want to know the average length of westbound road until the first cluster that has a length greater than x - 1. Let $f_w(\theta, x)$ be the Laplace transform of the cluster length smaller than x: $f_w(\theta, x) = \mathbf{E}(1_{(L_w < x)}e^{-\theta L_w})$. We have $f_w(0) = P(L_w < x)$. The Laplace transform of the road length $B_w(x)$ to bridge a gap of length xstarting from the beginning of an arbitrary cluster is

$$\beta_w(\theta, x) = \mathbf{E}(e^{-\theta B_w(x)}) = \frac{P(L_w > x - 1)}{1 - \frac{\lambda_w}{\lambda_w + \theta} f_w(\theta, x - 1)} .$$
(14)

Therefore the average

$$\mathbf{E}(B_w(x)) = -\frac{\partial}{\partial \theta} \beta_w(0, x) = -\left(\frac{\partial}{\partial \theta} f_w(0, x-1) - \frac{1}{\lambda_w} f_w(0, x-1)\right) \frac{1}{P(L_w > x-1)}$$
$$= \left(\frac{e^{\lambda_w}}{\lambda_w} + O(e^{-(x-1)\theta_w})\right) \frac{1}{P(L_w > x-1)}$$
(16)

$$= \left(1 + O(e^{-\varepsilon x})\right) \frac{e^{\lambda_w}}{\lambda_w} \frac{(1 - \theta_w)\theta_w}{(\lambda_w - \theta_w)e^{\theta_w - \lambda_w}} e^{(x-1)\theta_w}$$
(17)

2.4 Gap Distribution

Let's call G_e a gap made of a westbound cluster length L_w^* without eastbound cars, plus an average distance between two eastbound cars.

The Laplace transform of the westbound cluster length without eastbound cars is

$$\mathbf{E}(e^{-\theta L_w^*}) = \frac{f_w(\theta + \lambda_e)}{f_w(\lambda_w)} .$$
(18)

Therefore the distribution of G_e satisfies

$$\mathbf{E}(e^{-\theta G_e}) = \frac{f_w(\theta + \lambda_e)}{f_w(\lambda_e)} \frac{\lambda_e}{\lambda_e + \theta} .$$
(19)

In passing we have

$$\mathbf{E}(G_e) = -\frac{f'_w(\lambda_e)}{f_w(\lambda_e)} + \frac{1}{\lambda_e} , \qquad (20)$$

and the density of probability $p_e(x)$ of G_e at point x:

$$p_e(x) = \frac{\lambda_e}{f_w(\lambda_e)} e^{-\lambda_e x} (1 + O(e^{-\varepsilon x})) .$$
(21)

2.5 Average Time to Bridge

We have indeed:

$$2v\mathbf{E}(T_w) = \mathbf{E}(L_w^*) + \frac{1}{\lambda_e} + \int_0^\infty \mathbf{E}(B_w(x))p_e(x)dx , \qquad (22)$$

and this quantity converges when $\lambda_e > \theta_w$ and diverges when $\lambda_e < \theta_w$.

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2.6 Distance Travelled in Bridging

We know the probability that an eastbound car is not bridged to the next eastbound car, $f_w(\lambda_e)$. We know the average gap length when the car is not bridged $\mathbf{E}(G_e)$. We also know what is the unconditional gap length $\frac{1}{\lambda_e}$. Therefore the gap \overline{G}_e to the next car, given the node is bridged satisfies the following:

$$f_w(\lambda_e)\mathbf{E}(G_e) + (1 - f_w(\lambda_e))\mathbf{E}(\bar{G}_e) = \frac{1}{\lambda_e} , \qquad (23)$$

which gives $\mathbf{E}(\bar{G}_e) = \frac{1}{\lambda_e} - \frac{f'_w(\lambda_e)}{1 - f_w(\lambda_e)}$.

Thus the distance C_e travelled in bridging (beyond the first gap and extended to the next cluster eventually bridged) satisfies

$$\mathbf{E}(C_e) = (1 - f_w(\lambda_e)) \left(\mathbf{E}(\bar{G}_e) + \mathbf{E}(C_e) \right)$$
(24)

$$= \frac{1}{\lambda_e} \frac{1 - f_w(\lambda_e)}{f_w(\lambda_e)} - \frac{f'_w(\lambda_e)}{f_w(\lambda_e)} .$$
(25)

The total distance D_e travelled including the first gap satisfies $\mathbf{E}(D_e) = \mathbf{E}(G_e) + \mathbf{E}(C_e)$, which remains finite.

2.7 Average Information Propagation Speed

Since bridging operations are independent, the average propagation speed s is exactly:

$$s = \frac{\mathbf{E}(T_w)v + \mathbf{E}(D_e)}{\mathbf{E}(T_w)} = v + \frac{\mathbf{E}(D_e)}{\mathbf{E}(T_w)} .$$
(26)

2.8 Finite Radio Propagation Speed

If the radio propagation speed (including store and forward timings) is finite and constant, equal to v_r , then the average information propagation speed becomes:

$$s = \frac{\mathbf{E}(T_w)v + \mathbf{E}(D_e)}{\mathbf{E}(T_w) + \frac{1}{v_r}\mathbf{E}(D_e)} .$$
(27)

However, the finite speed v_r impacts the random variables G_e and T_w . The main impact is that to fill an eastbound gap of length x one needs a westbound cluster of length at least $\frac{x}{1-\gamma}$ with $\gamma = \frac{v}{v_r}$, otherwise the message will be off the

gap when arriving at the end of the cluster. Thus $\mathbf{E}(T_w(x)) = O(e^{\frac{\theta_w}{1-\gamma}x})$. The gap length is also modified since we must consider $(1-\gamma)L^*$:

The gap length is also modified since we must consider
$$(1 - \gamma)L_w$$
:

$$\mathbf{E}(e^{-\theta G_e}) = \frac{f_w((1-\gamma)(\theta+\lambda_e))}{f_w((1-\gamma)\lambda_e)} \frac{\lambda_e}{\lambda_e+\theta} .$$
(28)

But this does not change the exponential term $e^{-\lambda_e x}$ in the asymptotic expression of $p_e(x)$. Therefore the threshold condition becomes

$$\lambda_w e^{-\lambda_w} = (1 - \gamma)\lambda_e e^{-(1 - \gamma)\lambda_e} . \tag{29}$$

This corresponds to a dilatation by a factor $\frac{1}{1-\gamma}$ of the horizontal axis in the diagram shown in Fig. 1. Notice that in this case, the diagram loses its symmetry with respect to λ_e versus λ_w , as shown in Fig. 2.

Notice that when v_r decreases towards v the threshold limit tends to infinity.



Figure 2: Threshold zones for (λ_e, λ_w) for various $\gamma = \frac{v}{v_r}$, $\gamma = 0$ red, $\gamma = 0.1$ blue, $\gamma = 0.5$ green, $\gamma = 0.8$ yellow.

3 Conclusion

This research report has shown that in bidirectional vehicular DTNs, under a certain vehicle density threshold, information propagates on average at vehicle speed, while above this threshold, information propagates faster. This report gave the exact expression of this threshold and of the average information propagation speed in this context, through modeling and analysis.

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