Accurately Detecting Symmetries of 3D Shapes

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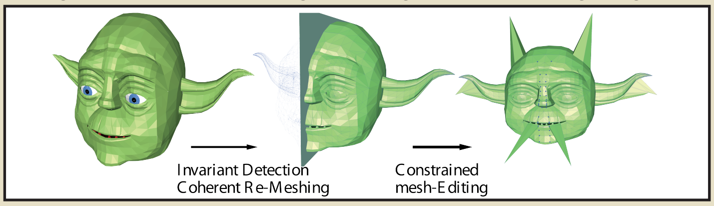
Goals and Motivations

Many shapes and geometrical models exhibit symmetries. These symmetries are sometimes explicitly represented in the model. However, symmetry information is often lost in automatic translations between file formats.

Using symmetries, one can, for example, manipulate models through coherent re-meshing or intelligent mesh editing programs.

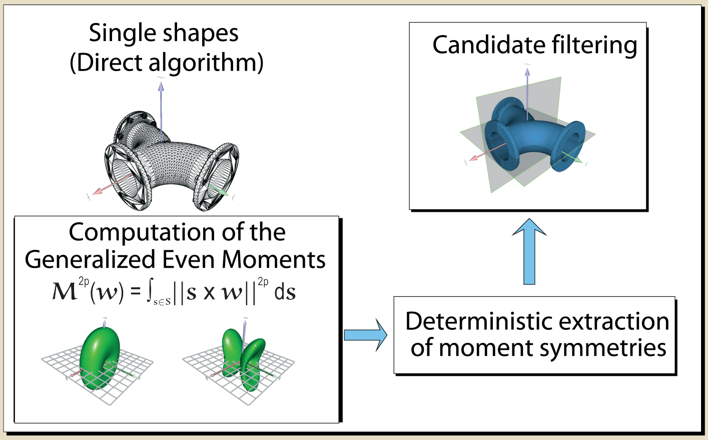


Symmetry detection enables coherent re-meshing of the object and can be used to constrain its modifications.

We present two algorithms that automatically retrieve symmetries in a 3D model through the use of an intermediate set of functions: **The Generalized Even Moments**

Overview

A. Symmetries of single shapes: Direct Algorithm



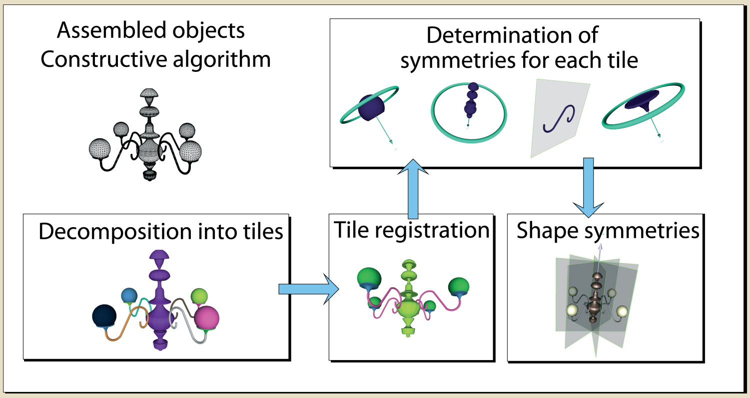
Overview of direct algorithm. In this exemple, we found two mirror-symmetries and one 2-fold rotational symmetry

We prove that any symmetry of a shape is also a symmetry of all its moment functions. The presented algorithm relies on this property and is divided in three main steps:

- **Computation of the Generalized Even Moments:** This computation is made easy by the fact that these functions decompose onto a finite number of spherical harmonics.
- **Deterministic extraction of moment symmetries:** Potential axes of symmetries are computed analytically and symmetry parameters are found by using spherical harmonic coefficients.
- **Candidate filtering:** Candidate symmetries have to be checked on the shape. This is done by computing a symmetry measure.

B. Symmetries of Assembled objects: Constructive algorithm

By considering an object as made of several sub-parts, we present an algorithm that pushes the accuracy issues down to a smaller scale in the object.



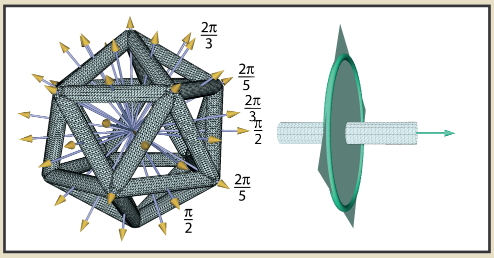
Overview of constructive algorithm. All the chandelier symmetries are correctly detected, i.e. four mirror-symmetries and one 4-fold symmetry.

The constructive algorithm proceeds in four steps:

- **Decomposition into tiles:** A tile is defined as a maximal set of edge-connected polygons.
- **Tile Registration:** We use a new shape descriptor, based on the various moment functions to easily detect congruent tiles. We use these informations to attach a local frame to each tile.
- **Tile Symmetries:** We use the direct algorithm (described above) on each tile independently.
- **Shape Symmetries:** We explore all one-to-one mappings between congruent tiles and progressively restrict allowable symmetries of the whole shape.

Results and Applications

Our algorithm finds symmetries in fairly complex models. Regarding accuracy, both algorithms compute the axes of the symmetries with a maximum error of 10^{-4} radians, independently of shape complexity.



Complex model (46,800 polygons) which has the same group of symmetries as the icosahedron. The constructive algorithm was capable of retrieving all 46 distinct axes of rotational-symmetries using the one rotational and one mirror symmetries of each tile (at right). Note the presence of 3-fold and 5-fold symmetries.

Other potential applications include:

- Model compression
- Consistent texture-mapping
- Automatic instantiation

*ARTIS is a research team in the GRAVIR/IMAG laboratory, a joint unit of CNRS, INRIA, INPG and UJF