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A New Phase Transitions for Local Delays in MANETs

François Baccelli (*INRIA/ENS*) and Bartłomiej Błaszczyszyn (*INRIA/ENS*)

***Abstract*: We study a slotted version of the Aloha Medium Access (MAC) protocol in a Mobile Ad-hoc Network (MANET). Our model features transmitters randomly located in the Euclidean plane, according to a Poisson point process and a set of receivers representing the next-hop from every transmitter. We concentrate on the so-called outage scenario, where a successful transmission requires a Signal-to-Interference-and-Noise (SINR) larger than some threshold. We analyze the local delays in such a network, namely the number of times slots required for nodes to transmit a packet to their prescribed next-hop receivers. The analysis depends very much on the receiver scenario and on the variability of the fading. In most cases, each node has finite-mean geometric random delay and thus a positive next hop throughput. However, the spatial (or large population) averaging of these individual finite mean-delays leads to infinite values in several practical cases, including the Rayleigh fading and positive thermal noise case. In some cases it exhibits an interesting phase transition phenomenon where the spatial average is finite when certain model parameters (receiver distance, thermal noise, Aloha medium access probability) are below a threshold and infinite above. To the best of our knowledge, this phenomenon, which we propose to call the wireless contention phase transition, has not been discussed in the literature. We comment on the relationships between the above facts and the heavy tails found in the so-called "RESTART" algorithm. We argue that the spatial average of the mean local delays is infinite primarily because of the outage logic, where one transmits full packets at time slots when the receiver is covered at the required SINR and where one wastes all the other time slots. This results in the "RESTART" mechanism, which in turn explains why we have infinite spatial average. Adaptive coding offers another nice way of breaking the outage/RESTART logic. We show examples where the average delays are finite in the adaptive coding case, whereas they are infinite in the outage case.**

***Index Terms*—mobile ad-hoc network, slotted Aloha, transmission delay, Poisson point process, SINR, stochastic geometry, phase transition, RESTART algorithm, heavy tails.**

I. Introduction

Aloha is one of the most common examples of a multiple access protocol [4, 12]. The classical approach to Aloha adopts simplified packet collision models in which simultaneous transmissions are never successful. This makes this classical approach not well adapted to a wireless MANET scenario, where it is the SINRs at different receiver locations which determine the set of successful transmissions. The present paper contributes to the study of Spatial Aloha, a variant of Aloha well adapted to MANETs. More precisely, it bears on the mathematical analysis of Spatial Aloha in the context of large Mobile Ad hoc Network (MANETs) with randomly located nodes. It focuses on the SINR coverage scheme, where each transmission requires that the receiver be covered by the transmitter with a minimum SINR. The present paper uses the stochastic geometry approach proposed in [2, 3] where the *space-time density of successful transmissions* was evaluated and optimized. The present paper identifies a potential weakness of this SINR coverage scheme: in most practical cases, and in particular as soon as thermal noise is bounded from below by a positive constant, the mean delay for a typical node to transmit a packet is infinite e.g. in the Rayleigh fading case.

A. Main Paper Contributions

The main *modeling advances* of the present paper are two-fold:

- We add a *time dimension* to the existing spatial analysis. Time is slotted, and we hence focus on slotted Aloha. We assume that the geographical locations of the MANET nodes remain unchanged over time and that only the variables modeling the MAC status (allowed to transmit or delayed) and the channel characteristics (such as fading and thermal noise) vary over time. In other words, we consider a full separation of the time scale of node mobility on one side and the time scale of MAC and physical layer on the other side, which makes sense in many practical situations, where the former is much larger than the later. This is in contrast with what happens in delay tolerant networks (DTNs) ; see e.g. [7, 8].
- We propose some more *realistic receiver models* than the fixed-distance receiver model introduced in [2, 3]; the new models are inspired by the fact that the routing schemes typically choose, as next-hop, the closest possible receiver among some common set of potential receivers.

The main theoretical advances of the paper bear on the analysis of the *local delays* in such MANETs; the local delay of a node is the random numbers of slots required by this node to successfully transmit a packet to its next-hop node. We first perform a time analysis of these local delays given the location of the MANET nodes. This analysis shows that each node has finite-mean geometric conditional random delay and thus a positive throughput to its next-hop receiver.

However, the spatial irregularities of the network imply that these conditional throughputs vary from node to node; in a Poisson configuration, one can find nodes which have an arbitrarily small throughput and consequently an arbitrarily large delay. In order to capture the performance of the whole MANET, one usually considers its "typical node". The typical node statistics are spatial (or large population) averages of the individual node characteristics. Our analysis shows in several practical cases, including the Rayleigh fading and positive thermal noise case, that the local delay of the typical MANET node is heavy tailed and that its mean is infinite, which however does *not* imply that the mean throughput of the typical node is null. Moreover, in certain cases, the mean local delay of the typical node exhibit a phase transition phenomenon that we propose to call the *wireless contention phase transition*: it is finite when certain model parameters (as receiver distance, thermal noise, Aloha medium access probability) are below a threshold and infinite above.

On the theory side, we also comment on the connections between the heavy tailedness alluded to the local delay of the typical MANET node and the observations of [10, 11], where it was shown that a finite population ALOHA model with variable and unbounded size packets has power law transmission delays. Although the physical phenomena at hand are quite different here (our spatial MANET model has fixed-packet-sizes) and there, we use the so-called "RESTART algorithm phenomenon", which has recently received a lot of attention (see e.g. [1, 9]), to establish some links between our findings and the results of [10, 11]. More precisely, we argue that, in our spatial MANET model with fixed-packet-size Aloha MAC, the delay of the typical node is heavy tailed and can have infinite mean due to a "RESTART" phenomenon, where the *spatial irregularities in the MANET* play the same role as the packet size variability in [11] and [10].

The main practical contributions of the paper bear on ways to guarantee finite mean local delay of the typical node by *increasing diversity* in the MANET. The proposed solutions have the potential of breaking the RESTART rigidity, e.g. by increasing the variability of fading, by increasing mobility, by adding appropriate receivers or, finally, by using adaptive coding, which completely brakes out the outage/RESTART logic.

The paper is organized as follows. In the remaining part of this section we briefly present the related work. We present our MANET model in Section II. The mean local delays are introduced and evaluated in Section III. We analyze the phase-transition phenomenon in Section IV. Section V focuses on the ways to make the mean delays finite. We conclude our work in Section VI.

B. Related Work

As already mentioned, the present paper assumes a full time-scale separation for the mobility on one side and for the MAC and physical layer on the other side. This assumption makes a major difference between what is done in this paper and what is done in DTNs, where one leverages node mobility to contribute to the transport of packets. There is a large number of publications on the throughput in DTNs and we will not review the literature on the topic which is huge. Let us nevertheless stress that there are some interesting connections between the line of though started in ([7]), where it was first shown that mobility increases capacity and what is done in Section V-A2. We show in this section that mobility helps in a way which is quite different from that considered in [7]: mobility may in certain cases break dependence and hence mitigate the RESTART phenomenon, it may hence decrease the mean local delay of the typical node (or equivalently increase its throughput), even if one *does not use mobility to transport packets*.

Among recent papers, that we are aware of, on the time-space analysis of MANETs, we would quote [8] and [6]. The former focuses on node motion alone and assumes that nodes within transmission range can transmit packets instantaneously. The authors then study the speed at which some multicast information propagates on a Poisson MANET where nodes have independent motion (of the random walk or random way-point type). The latter focuses on a first passage percolation problem, however, the model used in [6] is the so called protocol model; its analysis significantly differs from that of our physical (SINR-based) model. In particular, there is no notion of local delay.

II. MANET MODEL WITH SLOTTED ALOHA

A. Space-Time Scenarios

In what follows we describe a few space-time models considered in this paper. In the most simple case this will consist in adding the time-dimension to the Poisson Bipolar model introduced and studied in [3]. We will go however beyond the simple receiver model proposed there. The idea consists in assuming that the geographical locations of the MANET nodes remain unchanged over time and that the MAC status (allowed to transmit or delayed) and other characteristics of each node (as fading and thermal noise) vary over time. Time is discrete with a sequence of *time slots*, $n = 0, 1, \ldots$, w.r.t. which all the nodes are perfectly synchronized.

More precisely, we assume that a snapshot of the MANET can be represented by a marked Poisson point process (P.p.p.) on the Euclidean plane, where the point process is homogeneous with intensity λ and represents the random locations of the nodes and where the multidimensional mark of a point/node carries information about its MAC status and other characteristics of the channels in the successive time slots. This marked Poisson p.p. is denoted by $\widetilde{\Phi} = \{(X_i, (e_i(n), y_i, \mathbf{F}_i(n), W_i(n) : n)\}$, where:

- $\Phi = \{X_i\}$ denotes the *locations of the potential transmitters* of the MANET; Φ is always assumed Poisson with positive and finite intensity λ;
- $e_i(n)$ is the *MAC decision* of point X_i of Φ at time n; we will always assume that, given Φ, the random variables $e_i(n)$ are i.i.d. in i and n; i.e. in space and time, with $\Pr\{\, e_i(n) = 1 \,\} = 1 - \Pr\{\, e_i(n) = 0 \,\} = p$.
- $y_i \in \mathbb{R}^2$ is the location of the *receiver* of the node i; a few scenario for the choice of the receivers are presented in Section II-B below.
- $\mathbf{F}_i(n) = \{F_i^j(n) : j\}$ is the *virtual power* emitted by node i (provided $e_i = 1$) towards receiver y_j at time n. By virtual power $F_i^j(n)$, we understand the product of the effective power of transmitter i and of the random fading from this node to receiver y_j. The random (vector valued) processes $\{F_i^j(n) : n\}$ are assumed to be i.i.d. in i and j given $\{X_i, y_i\}$, and we denote by F the generic marginal random variable $F_i^j(n)$. In the case of constant effective transmission power $1/\mu$ and Rayleigh fading (which is our default scenario), F is exponential with mean $1/\mu$ (see e.g. [14, p. 50 and 501]). However, one also consider also non exponential cases to analyze other types of fading such as e.g. Rician or Nakagami scenarios, or simply the case without fading (when $F \equiv 1/\mu$ is deterministic). Note that we do not have specified yet the dependence between the virtual powers $F_i^j(n)$ for different time slots n. This is done in what follows.
  - By the *fast fading* case we understand the scenario when the random variables $F_i^j(n)$ are i.i.d. in n (recall, that the default option is that they are also i.i.d. in i, j); [1]

[1] Note that our fast fading means that it remains constant over a slot duration and can be seen as i.i.d. over different time slots. This might not correspond to the terminology used in many papers of literature, where fast fading means that the channel conditions fluctuate much over a given time slot.

– The *slow fading* case is that where $F_i^j(n) \equiv F_i^j$, for all n.

- $W_i(n)$ represents the thermal noise at the receiver y_i at time n. The processes $\{W_i(n) : n\}$ are independent in i given $\{y_i\}$, with the generic marginal random variable denoted by W. For the time dependence, one can consider both $W_i(n)$ independent in n (*fast noise*) and constant $W_i(n) \equiv W_i$ (*slow noise*).

B. Receiver Models

We first recall the simple receiver model used in [3] and then propose a few possible extensions.

1) Bipolar Receiver Model: In this model the receivers are *external to the MANET p.p.* $\Phi = \{X_i\}$. We assume that, given Φ, the random variables $\{X_i - y_i\}$ are i.i.d random vectors with $|X_i - y_i| = r$; i.e. each *receiver is at distance* r from its transmitter. Note that the receivers of different MANET nodes are different (almost surely).

2) Nearest Receiver models: In practice, some routing algorithm specifies the receivers(s) (relay node(s)) of each given transmitter. In what follows focus on two *nearest receiver models* where each transmitter selects its receiver as close by as possible is some set of *potential receivers* Φ_0, common to all MANET nodes Φ; i.e. $y_i = Y_i^* = \arg\min_{Y_i \in \Phi_0, Y_i \neq X_i} \{|Y_i - X_i|\}$. Two incarnations are considered

a) Independent Poisson Nearest Receiver (IPNR) Model: In this model we assume that the potential receivers form some stationary P.p.p. Φ_0, of intensity λ_0, which is *independent* of (and in particular *external* to) the MANET Φ.

b) MANET Nearest Neighbor (MNN) Model: All the nodes of the MANET are considered as potential receivers; i.e. $\Phi_0 = \Phi$. [2]

C. Mean Path-loss Model

Below, we assume that the receiver of node i receives a power from the transmitter located at node j at time n which is equal to $F_i^j(n)/l(|X_j - y_i|)$, where $|\cdot|$ denotes the Euclidean distance on the plane and $l(\cdot)$ is the path loss function. An important special case consists in taking

$$l(u) = (Au)^\beta \quad \text{for } A > 0 \text{ and } \beta > 2. \tag{2.1}$$

Other possible choices of path-loss function avoiding the pole at $u = 0$ consist in taking e.g. $\max(1, l(u))$, $l(u+1)$, or $l(\max(u, u_0))$.

D. SINR Coverage

In this paper we mainly focus on the SINR coverage/outage scenario: we will say that transmitter $\{X_i\}$ *covers* (or *is successfully received by*) its receiver y_i at time slot n if

$$\text{SINR}_i(n) = \frac{F_i^i(n)/l(|X_i - y_i|)}{W_i(n) + I_i^1(n)} \geq T\,, \tag{2.2}$$

where $I_i^1(n) = \sum_{X_j \in \widetilde{\Phi}^1(n),\, j \neq i} F_j^i(n)/l(|X_j - y_i|)$ is the *interference* at receiver y_i at time n; i.e., the sum of the signal powers received by y_i at time n from all the nodes in $\Phi^1(n) = \{X_j \in \Phi : e_j(n) = 1\}$ except X_i. In mathematical terms, $I_i^1(n)$ is an instance of *Shot-Noise (SN)* field generated by of $\widetilde{\Phi}^1(n) \setminus \{X_i\}$.

Denote by $\delta_i(n)$ the indicator that (2.2) holds, namely, that location y_i is covered by transmitter X_i with the required quality at time n.

E. Typical MANET node

Let $\mathbf{P}^0$ denote the Palm distribution of the P.p.p. Φ (see e.g. [13]). Under this distribution, the MANET nodes are located at $\Phi \cup \{X_0 = 0\}$, where Φ is a copy of the stationary P.p.p. (cf. Slivnyak's theorem in [13]). Under $\mathbf{P}^0$, the other random objects/marks of the model, $(e_i(n), \mathbf{F}_i(n))$ and $W_i(n)$ as well as $y_i(n)$ in the Bipolar receiver model, are i.i.d. given $\Phi \cup \{X_0 = 0\}$, and have the same law as their original distribution. In the IPNR model, the potential receiver p.p. Φ_0 remains independent of $\Phi \cup \{X_0 = 0\}$ and Poisson-distributed; in the MNN model the receiver p.p. is still determined by the MANET configuration $\Phi \cup \{X_0 = 0\}$ (with the receiver of a node being its nearest neighbor). The node $X_0 = 0$, considered under $\mathbf{P}^0$, is called *the typical MANET node*.

III. Local Delay

The *local delay of the typical node is the number of time slots needed for node* $X_0 = 0$ *(considered under the Palm probability* $\mathbf{P}^0$ *with respect to* Φ*) to successfully transmit*:

$$\mathbf{L} = \mathbf{L}_0 = \inf\{n \geq 1 : \delta_0(n) = 1\}\,.$$

This random variable depends on the origin of time (here 1) but we focus on its law below, which does not depend on the chosen time origin.

The main objective of this paper is to study $\mathbf{E}^0[\mathbf{L}]$ under the full separation of time scales described in the introduction. Let $\mathcal{S}$ denote all the *static elements of the network model*: i.e. the elements which are random but which do not vary with time n. In all models, we have all locations $\Phi, \{y_i\} \in \mathcal{S}$. Moreover, in the slow fading model, we have $\{\mathbf{F}_i(n) = \mathbf{F}_i\} \in \mathcal{S}$ and similarly in the slow noise model, $\{W_i(n) = W_i\} \in \mathcal{S}$.

Given a realization of all the elements of $\mathcal{S}$, denote by

$$\pi_c(\mathcal{S}) = \mathbf{E}^0[e_0(1)\delta_0(1) \,|\, \mathcal{S}] \tag{3.1}$$

the conditional probability, given $\mathcal{S}$, that X_0 is authorized by the MAC to transmit and that this transmission is successful at time $n = 1$. Note that due to our time-homogeneity, this conditional probability does not depend on n. The following result allows us to express $\mathbf{E}^0[\mathbf{L}]$.

Lemma 3.1: *We have*

$$\mathbf{E}^0[\mathbf{L}_0] = \mathbf{E}^0\Big[\frac{1}{\pi_c(\mathcal{S})}\Big]\,. \tag{3.2}$$

Remark: One can interpret $\pi_c(\mathcal{S})$ as the *(temporal) rate of successful packet transmissions* (or the *throughput*) of node X_0 given all the static elements of the network. Its inverse $1/\pi_c(\mathcal{S})$ is the local delay of this node in this environment. In many cases, this throughput is a.s. positive (so that we will have a.s. finite delays) for all static environments. If this last condition holds true, by Campbell's formula, almost surely, all the nodes have finite mean delays and positive throughputs. However, the spatial irregularities of the network imply that this throughput varies from node to node, and in

[2] Both IPNR and MNN models require some additional specifications on what happens if two or more transmitters pick the same receiver and, the MNN model, what happens if the picked receiver is also transmitting. Our analysis applies to the situation when the SINR threshold $T > 1$ (cf. II-D), which excludes multiple receptions by a given receiver and simultaneous emission and reception.

a Poisson configuration, one can find nodes which have an arbitrarily small throughput (and consequently an arbitrarily large delay). The mean local delay of the typical node $\mathbf{E}^0[\mathbf{L}]$ is the spatial average of these individual mean local delays. A finite mean indicates that the fraction of nodes in bad shape (for throughput or delay) is in some sense not significant. In contrast, $\mathbf{E}^0[\mathbf{L}] = \infty$ indicates that an important fraction of the nodes are in a bad shape. This is why the finiteness of the mean local delay of the typical node is an important indicator of a good performance of the network.

Proof: (of Lemma 3.1) Since the elements that are not in $\mathcal{S}$ change over time in the i.i.d. manner given a realization of the elements of $\mathcal{S}$, the successive attempts of node X_0 to access to the channel and successfully transmit at time $n \geq 1$ are independent (Bernoulli) trials with probability of success $\pi_c(\mathcal{S})$. The local delay $\mathbf{L} = \mathbf{L}_0$ is then a geometric random variable (the number of trials until the first success in the sequence of Bernoulli trials) with parameter $\pi_c(\mathcal{S})$. Its (conditional) expectation (given $\mathcal{S}$) is known to be $\mathbf{E}^0[\mathbf{L} \,|\, \mathcal{S}] = 1/\pi_c(\mathcal{S})$. The result follows by integration with respect to the distribution of $\mathcal{S}$. ■

Example 3.2: In order to understand the reasons for which $\mathbf{E}^0[\mathbf{L}]$ may or may not be finite, consider the following two extremal situations. Suppose first that the whole network is independently re-sampled at each time slot (including node locations Φ, which is *not* our default option). Then $\mathcal{S}$ is empty (the σ-algebra generated by it is trivial) and the temporal rate of successful transmissions is equal to the space-time average rate $\pi_c(\mathcal{S}) = \mathbf{E}^0[e_0(1)\delta_0(1)] = p\,p_c$, where p_c is the space-time probability of coverage for the typical node in the corresponding receiver model. Consequently, in this case of extreme variability (w.r.t. time), we have $\mathbf{E}^0[\mathbf{L}] = 1/p_c < \infty$ provided $p_c > 0$, which holds true under very mild assumptions, e.g. for all considered receiver and path-loss models with fast or slow Rayleigh fading model and any noise model provided, $0 < p < 1$ (cf. [3] for the evaluation of p_c for the Poisson Bipolar model).

On the other hand, if nothing varies over time (including MAC status, which again is ruled out in our general assumptions), we have $\pi_c(\mathcal{S}) = e_0(1)\delta_0(1)$ (because the conditioning on $\mathcal{S}$ determines $e_0(1)\delta_0(1)$ in this case). In this case under very mild assumptions (e.g. if $p < 1$), this temporal rate $e_0(1)\delta_0(1)$ is zero with positive probability, making $\mathbf{E}^0[\mathbf{L}] = \infty$. Note that in this last case, some nodes in the MANET succeed in transmitting packets every time slot, whereas others never succeed. Having seen the above two extremal cases, it is not difficult to understand that the mean local delay of the typical node very much depends on how much the time-variability "averages out" the spatial irregularities of the distribution of nodes in the MANET.

Note that by Jensen's inequality,

$$\mathbf{E}^0[\mathbf{L}] \geq \frac{1}{\mathbf{E}^0[\pi_c(\mathcal{S})]} = \frac{1}{p_c}\,.$$

The inequality is in general strict and we may have $\mathbf{E}^0[\mathbf{L}] = \infty$ while $p_c > 0$.

In the remaining part of this section we will study several particular instances of space-time scenarios.

A. Local Delays in Poisson Bipolar Model

In the Poisson bipolar model, we assume a static repartition for the MANET nodes Φ and for their receivers $\{y_i\}$. The MAC variables $e_i(n)$ are i.i.d. in i and n. All other elements (fading and noise) have different time-scenarios.

1) Slow Fading and Noise Case: Let us first consider the situation where $\{\mathbf{F}_i\}$ and W are static.

Proposition 3.3: *Assume the Poisson Bipolar network model with slow fading and slow noise. If the distribution of F, W is such that $\Pr\{\, WTl(r) > F \,\} > 0$, then $\mathbf{E}^0[\mathbf{L}] = \infty$.*

Proof: We have

$$\begin{aligned}\pi_c(\mathcal{S}) &= p\mathbf{E}^0[e_0(1)\delta_0(1) \,|\, \mathcal{S}] \\ &= p\,\overset{0}{\Pr}\{F_0^0 \geq Tl(r)(W_0 + I_0^1(0)) \,|\, \mathcal{S}\} \\ &\leq p\mathbf{1}(F_0^0 \geq Tl(r)W)\,.\end{aligned}$$

The last indicator is equal to 0 with non-null probability for our assumptions. Using (3.2), we conclude that $\mathbf{E}^0[\mathbf{L}] = \infty$. ■

2) Fast Fading Case: The following auxiliary result is useful when studying fast Rayleigh fading.

Lemma 3.4: *Consider the Poisson shot-noise $I = \sum_{X_i \in \Phi} G_i/l(|X_i|)$, where Φ is some homogeneous Poisson p.p. with intensity α on $\mathbb{R}^2$, G_i are i.i.d. random variables with Laplace transform $\mathcal{L}_G(\xi)$ and $l(r)$ is any response function (in our case it is always some path-loss function). Denote by $\mathcal{L}_I(\xi \,|\, \Phi) = \mathbf{E}[e^{-\xi I} \,|\, \Phi]$ the conditional Laplace transform of I given Φ. Then*

$$\mathbf{E}\Big[\frac{1}{\mathcal{L}_I(\xi \,|\, \Phi)}\Big] = \exp\Big\{-2\pi\alpha \int_0^\infty v\Big(1 - \frac{1}{\mathcal{L}_G(\xi/l(v))}\Big)\,\mathrm{d}v\Big\}\,.$$

Proof: By the independence of G_i given Φ, we have

$$\begin{aligned}\mathbf{E}[e^{-\xi I} \,|\, \Phi] &= \prod_{X_i \in \Phi} \mathbf{E}[e^{-\xi \mathcal{L}_G(\xi/l(|X_i|))}] \\ &= \prod_{X_i \in \Phi} \mathcal{L}_G(\xi/l(|X_i|)) \\ &= \exp\Big\{\sum_{X_i \in \Phi} \log(\mathcal{L}_G(\xi/l(|X_i|)))\Big\}\,.\end{aligned}$$

Taking the inverse of the last expression and using the known formula for the Laplace transform of the Poisson p.p. (it can be derived from the formula for the Laplace functional of the Poisson p.p.; see e.g. [5], and was already used e.g. [3]). we obtain

$$\mathbf{E}\Big[\frac{1}{\mathcal{L}_I(\xi \,|\, \Phi)}\Big] = \exp\Big\{-\alpha \int_{\mathbb{R}^2} \Big(1 - e^{-\log(\mathcal{L}_G(\xi/l(|x|)))}\Big)\,\mathrm{d}x\Big\}\,. \tag{3.3}$$

Passing to polar coordinates completes the proof. ■

Coming back to local delays, let us consider now the situation where the random variables $\{\mathbf{F}_i(n)\}$ are i.i.d. in n. We consider only the Rayleigh fading case.

Proposition 3.5: *Assume the Poisson Bipolar network model with fast Rayleigh fading. In the case of fast thermal noise, we have*

$$\mathbf{E}^0[\mathbf{L}] = \frac{1}{p}\mathcal{D}_W(Tl(r)) \exp\Big\{2\pi p\lambda \int_0^\infty \frac{vTl(r)}{l(v) + (1-p)Tl(r)}\,\mathrm{d}v\Big\}\,,$$

where

- $\mathcal{D}_W(s) = \mathcal{D}_W^{slow}(s) = \mathcal{L}_W(-s)$ *for the slow noise case,*
- $\mathcal{D}_W(s) = \mathcal{D}_W^{fast}(s) = 1/\mathcal{L}_W(s)$ *for the fast noise case.*

Proof: In the fast Rayleigh fading case, we have $\pi_c(\mathcal{S}) = \Pr\{F \geq Tl(r)(W + I^1) \,|\, \Phi\}$ for the fast noise case and $\pi_c(\mathcal{S}) = \Pr\{F \geq Tl(r)(W + I^1) \,|\, \Phi, W\}$ for the slow noise model. Using the assumption on F, we obtain

$$\pi_c(\mathcal{S}) = \mathcal{L}_W(\mu Tl(r))\mathbf{E}[e^{-\mu Tl(r) I^1} \,|\, \Phi]$$

in the fast noise case and

$$\pi_c(\mathcal{S}) = e^{-\mu W Tl(r)}\mathbf{E}[e^{-\mu Tl(r) I^1} \,|\, \Phi]$$

for the slow noise case. The result then follows from (3.2) and Lemma 3.4 with $G = eF$. Note that in this case $\mathcal{L}_G(\xi) = 1 - p + p\mathcal{L}_F(\xi)$, which gives $\mathcal{L}_{eF}(\xi) = 1 - p + p\mu/(\mu + \xi)$. ■

B. Local Delay in the Nearest Receiver Models

In this section we study the IPNR and the MNN receiver models. We work out formulas for the mean local delay of the typical node under the following conditions: *fast Rayleigh fading* and fast or slow noise of arbitrary distribution.

Proposition 3.6: *Assume fast Rayleigh fading.*

- *In the IPNR model, we have*

$$\mathbf{E}^0[\mathbf{L}] = \frac{2\pi\lambda_0}{p}\int_0^\infty r e^{-\pi\lambda_0 r^2}\mathcal{D}_W(\mu Tl(r))\,\mathcal{D}_I^{INR}(\mu Tl(r))\mathrm{d}r \tag{3.4}$$

where

$$\mathcal{D}_I^{INR}(s) = \exp\left\{2\pi\lambda\int_0^\infty \frac{ps}{l(v) + (1-p)s} v\,\mathrm{d}v\right\} \tag{3.5}$$

and $\mathcal{D}_W(s)$ is as in Proposition 3.5.

- *In the MNN model, we have*

$$\mathbf{E}^0[\mathbf{L}] = \frac{2\pi\lambda}{p(1-p)} \times \int_0^\infty r e^{-\pi\lambda r^2}\mathcal{D}_W(\mu Tl(r))\,\mathcal{D}_I^{MNN}(r, \mu Tl(r))\,\mathrm{d}r\,, \tag{3.6}$$

where

$$\mathcal{D}_I^{MNN}(r,s) = \exp\left\{\lambda\pi\int_0^\infty \frac{ps}{l(v) + (1-p)s} v\,\mathrm{d}v + \lambda\int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{v>2r\cos\theta}\frac{ps}{l(v) + (1-p)s} v\,\mathrm{d}v\mathrm{d}\theta\right\} \tag{3.7}$$

and $\mathcal{D}_I(s)$ is as above (as in Proposition 3.5).

Proof: We condition on the location of the nearest neighbor $y_0 = Y_0^*$ of $X_0 = 0$ under $\mathbf{P}^0$. In the IPNR model the distance from the origin to Y_0^* is known to have the following distribution (under both the Palm and the stationary law): $\mathbf{P}\{|Y_0^*| > r\} = e^{-\pi\lambda_0 r^2}$. Due to the independence assumption, in this model, given the location of the receiver Y_0^*, the distribution of the MANET nodes Φ remains unchanged under $\mathbf{P}^0$. The remaining part of the proof follows the same lines as that of Proposition 3.5.

For the MNN model, recall from Section II-E that under $\mathbf{P}^0$, the nodes of $\Phi \setminus \{X_0\}$ are distributed as those of the homogeneous Poisson p.p. Thus the distance $|Y_0^* - X_0| = |Y_0^*|$ has the same distribution as in the IPNR model with $\lambda_0 = \lambda$. However, in the MNN model, given some particular location of $y_0 = Y_0^*$, one has to take the following fact into account: there are *no MANET nodes* (thus, in particular, no interferers) in $B_0(|y_0|)$. Consequently, under $\mathbf{P}^0$, given $Y_0^* = y_0$, the SN I_0^1 in (2.2) is no longer driven by the stationary Poisson p.p. of intensity λ_1, but as the SN of Φ^1 given that there are no nodes of Φ in $B_0(|y_0|)$. Note that the location y_0 at which we evaluate this last SN is on the boundary (and not in the center) of the empty ball. By the strong Markov property of Poisson p.p. (cf. [15]), the distribution of a Poisson p.p. given that $B_0(|y_0|)$ is empty is equal to the distribution of the (non-homogeneous) Poisson p.p. with intensity equal to 0 in $B_0(|y_0|)$ and λ_1 outside this ball. Putting these arguments together, and exploiting the rotation invariance of the picture conclude the proof. ■

Notice that the integrals in (3.5) and (3.7) are finite for any of the path-loss models suggested in Section II-C. However, the outer integrals (in r) in (3.4) and (3.6) may be infinite. In order to study this problem note first that we have the following bounds in the MNN model:

Remark 3.7: In the MANET receiver case, we have the bounds

$$\left(\mathcal{D}_I^{INR}(s)\right)^{1/2} \leq \mathcal{D}_I^{MNN}(r,s) \leq \mathcal{D}_I^{INR}(s)\,. \tag{3.8}$$

IV. Wireless contention phase transition for the local delay

In this section we will show that under quite natural assumptions them mean local delay of the typical node can be infinite. For some models, it can exhibit the following phase transition: $\mathbf{E}^0[\mathbf{L}] < \infty$ or $\mathbf{E}^0[\mathbf{L}] = \infty$ depending on the model parameters (as p, distance r to the receiver, or the mean fading $1/\mu$).

A. Bipolar Model

We begin with the simple Bipolar receiver model. Proposition 3.5 shows that in the fast fading and noise case, $\mathbf{E}^0[\mathbf{L}] < \infty$; indeed, $\int_0^\infty v/l(v)\,\mathrm{d}v < \infty$. However for the fast fading, slow noise case the finiteness of the mean local delay of the typical node depends on whether W has finite *exponential moments* of order $Tl(r)\mu$. This is a rather strong assumption concerning the tail distribution function of W. Often this moment is finite only for some sufficiently small value of $Tl(r)\mu$.

To see the wireless contention phase transition in this model consider the following example.

Example 4.1: Let us assume exponential noise with mean $1/\nu$. Then $\mathcal{L}_W(-\xi) = \nu/(\nu - \xi) < \infty$ provided $Tl(r)\mu < \nu$ and infinite for $Tl(r)\mu > \nu$. This means that in the corresponding Poisson Bipolar MANET with a Rayleigh fading, exponential noise, the mean local delay of the typical node is finite whenever $Tl(r) < \nu/\mu$ and infinite otherwise. Here are a few incarnations of this phase transition:

- For fixed mean transmission power μ^{-1} (we recall that a typical situation is that where fading has mean 1 and where μ^{-1} is actually the effective transmission power) and mean thermal noise ν^{-1}, there is a threshold on the distance r between transmitter and receiver below which mean local delay of the typical node is finite and above which they are infinite;
- For fixed mean thermal noise ν^{-1} and fixed distance r, there is a threshold on mean transmission power μ^{-1} *above* which the mean local delay of the typical node is finite and *below* which it is infinite;

- For fixed mean transmission power μ^{-1} and fixed distance r, there is a threshold on mean thermal noise power ν^{-1} *below* which the mean local delay of the typical node is finite and *above* which it is infinite.

The fact that all transmissions contend for the shared wireless channel may lead to infinite mean local delays of the typical node if the system is stressed by either of the phenomena listed above: too distant links, a too high thermal noise or a too transmission power.

Remark 4.2: *(RESTART)* There is a direct interpretation of the local delay of the typical node in terms of the so called *RESTART algorithm*: assume a file of random size B is to be transmitted over an error prone channel. Let $\{A_n\}_{n\geq 1}$ be the sequence of channel inter-failure times. If $A_1 > B$ (resp. $A_1 \leq B$), the transmission succeeds (resp. fails) at the first attempt. If the transmission fails at the first attempt, one has to restart the whole file transmission in the second attempt and so on. Let

$$N = \inf\{n \geq 1 \ s.t. \ A_n > B\}$$

be the first attempt where the file is successfully transmitted. In the classical RESTART scheme, the sequence $\{A_n\}_{n>0}$ is assumed to be i.i.d. and independent of B. It can then be proved (see [1]) that when B has infinite support and A_n is light tailed (say exponential), then N is heavy tailed. This observation comes as a surprise because one can get heavy tails (including infinite first moments) in situations where B and A_n are both light tailed.

Consider the fast fading, slow noise case (and ignore the interference for simplicity) Then the local delay of the typical node can be seen as an instance of this algorithm with the following identification: $A_n = F_0^0(n)e_0(n)$ and $B = TWl(r)$. In the next section, we will see other incarnation of the above RESTART algorithm in the nearest-receiver models with deterministic W, where the role of the unboundedness of B is played by the distance to the receiver; cf. Remark 4.7.

The above interpretation in terms of the RESTART scheme formally shows that the local delay of the typical node is heavy tailed and thus it is not surprising that in certain cases its mean is infinite. However, the physical phenomena at hand are quite different here and in the classical RESTART context, so let us now comment on what exactly this heavy tailedness means in our MANET context. Let $B_0(R)$ be the ball centered at 0 of radius R and let $\mathbf{L}_i$ be the number of time slots required by the node X_i to transmit a packet. The ergodic interpretation of the Palm probability implies that, for all m,

$$\mathbf{P}^0\{\,\mathbf{L}_0 > m\,\} = \lim_{R\to\infty} \frac{1}{\Phi(B_0(R))} \sum_{X_i \in B_0(R)} \mathbb{1}(\mathbf{L}_i > m),$$

where the last limit is in the almost sure sense. The fact that the distribution of $\mathbf{L}_0$ is heavy tailed under $\mathbf{P}^0$ means that the (discrete) law $\mathbf{P}^0\{\,\mathbf{L}_0 > m\,\}$ has no exponential moments. In view of the above ergodic interpretation, this is equivalent to saying that the asymptotic fraction of MANET modes which experience a local delay of more than m time slots decreases slowly with m (more slowly than any exponential function).

Finally, we remark that the fact that the *mean local delay of the typical node is infinite does not imply that the mean throughput of the typical node is null*. The last quantity boils down to the probability of success of node X_0 under $\mathbf{P}^0$, i.e., to p_c and, as already mentioned in Example 3.2, it is positive for all the considered models.

B. Nearest Receiver Models

In order to analyze mean local delays in these more complex receiver models will study separately the impact of the thermal noise and of the interference.

1) Noise Limited Networks: Consider first the IPNR and MNN models under the assumption that the interference is perfectly canceled (and that only noise has to be taken into account). In what follows we consider the fast noise scenario.

a) IPNR model: The following result follows from (3.4) with $\mathcal{D}_I(s) = 1$ and with $\mathcal{D}_W$ given in Proposition 3.5:

Corollary 4.3: *In the IPNR model with fast Rayleigh fading and fast noise, if interference is perfectly canceled, then*

$$\mathbf{E}^0[\mathbf{L}] = 2\pi\lambda_0 \int_0^\infty \frac{r\exp(-\pi\lambda_0 r^2)}{p\mathcal{L}_W(\mu l(r)T)}\mathrm{d}r.$$

Hence, for the simplified path loss function (2.1) $\mathbf{E}^0[\mathbf{L}] < \infty$ whenever

$$\mathcal{L}_W(\xi) \geq \eta \exp\left\{-\pi\lambda_0\left(\frac{\xi}{\mu T A^\beta}\right)^{2/\beta}\right\}\left(\xi^{2(1+\epsilon)/\beta}\right),\ \xi\to\infty, \tag{4.9}$$

for some positive constants ϵ and η, and whenever some natural local integrability conditions also hold. This condition requires that there be a sufficient probability mass of W in the neighborhood of 0. For instance, under any of the path-loss models suggested in Section II-C, this holds true for a thermal noise with a rational Laplace transform (e.g. Rayleigh).

The condition (4.9) is sharp in the sense that when $\mathcal{L}_W(\xi)$ is asymptotically smaller that the expression in the right-hand-side of (4.9) with $(1+\epsilon)$ replaced by $(1-\epsilon)$ for some positive constants ϵ and η, then $\mathbf{E}^0[\mathbf{L}] = \infty$. This is the case, e.g. when W is a positive constant.

b) MNN model: In the MNN model (with $0 < p < 1$) and the simplified path loss function (2.1), similar arguments show that the same threshold as above holds with

$$\mathcal{L}_W(\xi) \geq \eta \exp\left\{-\pi\lambda\left(\frac{\xi}{\mu T A^\beta}\right)^{2/\beta}\right\}\left(\xi^{2(1+\epsilon)/\beta}\right),\ \xi\to\infty, \tag{4.10}$$

implying that $\mathbf{E}^0[\mathbf{L}] < \infty$ and a similar converse statement.

2) Interference Limited Networks: In this section we assume and $W \equiv 0$.

a) IPNR model: In the IPNR case with the simplified path loss function (2.1), using the fact that

$$2\pi\int_0^\infty \frac{pTl(r)}{l(v)+(1-p)Tl(r)} v\,\mathrm{d}v = p(1-p)^{\frac{2}{\beta}-1}T^{\frac{2}{\beta}}K(\beta)r^2, \tag{4.11}$$

with

$$K(\beta) = \frac{2\pi\Gamma(2/\beta)\Gamma(1-2/\beta)}{\beta} = \frac{2\pi^2}{\beta\sin(2\pi/\beta)}. \tag{4.12}$$

Using the above observations we get the following result from (3.4) and (3.5):

Corollary 4.4: *In the IPNR model with $W = 0$, fast Rayleigh fading and the path loss function (2.1), we have*

$$\mathbf{E}^0[\mathbf{L}] = 2\pi\lambda_0\frac{1}{p}\int_0^\infty r\exp\left(-\pi\lambda_0 r^2 + \lambda\theta(p,T,\beta)r^2\right)\mathrm{d}r\,,$$

with

$$\theta(p,T,\beta) = \frac{p}{(1-p)^{1-\frac{2}{\beta}}}T^{\frac{2}{\beta}}K(\beta)\,. \tag{4.13}$$

Notice that $\theta(p,T,\beta)$ is increasing in p and in T. We hence get the following *incarnation of the wireless contention phase transition*:

- If $p \neq 0$ and $\lambda_0 \pi > \lambda\theta(p,T,\beta)$, then
$$\begin{aligned}\mathbf{E}^0[\mathbf{L}] &= \frac{1}{p}\frac{\pi\lambda_0}{\pi\lambda_0 - \lambda\theta(p,T,\beta)} \\ &= \frac{1}{p}\frac{\lambda_0}{\lambda_0 - \lambda\frac{2}{\beta}\Gamma(\frac{2}{\beta})\Gamma(1-\frac{2}{\beta})p(1-p)^{2/\beta-1}T^{2/\beta}} \\ &< \infty\,.\end{aligned}$$
- If either $\lambda_0\pi < \lambda\theta(p,T,\beta)$ or $p=0$, then $\mathbf{E}^0[\mathbf{L}] = \infty$.

Remark 4.5: *(Pole of the path-loss function)* The above phase transition is not linked to the pole of the simplified path loss function (2.1) used in the analysis. To show this, one can consider e.g. $l(u) = (\max(1,u))^4$ and evaluate explicitly the integral in the left-hand-side of (4.11) (we skip the details due to the lack of space) and conclude that that the mean delay is finite if $p \neq 0$ and
$$\lambda_0 > \lambda\hat{\theta} = \lambda\frac{\pi}{2}\frac{p}{\sqrt{1-p}}\sqrt{T}$$
and infinite if either $p=0$ or $\lambda_0 < \lambda\hat{\theta}$.

Remark 4.6: Here are a few comments on this phase transition.

- The fact that $p = 0$ ought to be avoided for having $\mathbf{E}^0[\mathbf{L}] < \infty$ is clear;
- The fact that intensity of potential transmitters λ_0 cannot be arbitrarily small when the other parameters are fixed is clear too as this implies that: (i) at any given time slot, the transmitters compete for too small set of receivers; (ii) targeted receivers are too far away from their transmitter.
- For T and β fixed, stability ($\mathbf{E}^0[\mathbf{L}] < \infty$) requires that receivers outnumber potential transmitters by a factor which grows like $p(1-p)^{2/\beta-1}$ when p varies; if this condition is not satisfied, this drives the system to instability because some receivers have too persistent interferers nearby (for instance, if $p=1$, a receiver may be very close from a persistent transmitter which will most often succeed, forbidding (or making less likely) the success of any other transmitter which has the very same receiver).

b) MNN model: Fix $a, r \geq 0$. For the path loss function (2.1) we have
$$\begin{aligned}&\int_{ar}^{\infty}\frac{pTl(r)}{l(v)+(1-p)Tl(r)}v\,\mathrm{d}v \\ &= \frac{1}{2}r^2p(1-p)^{\frac{2}{\beta}-1}T^{\frac{2}{\beta}}H(a,T(1-p),\beta/2)\,,\end{aligned}$$
with
$$H(a,w,b) = \int_{a^2w^{-1/b}}^{\infty}\frac{1}{1+u^b}\,\mathrm{d}u. \tag{4.14}$$
Let
$$J(w,b) = \int_{\theta=-\pi/2}^{\pi/2} H(2\cos(\theta),w,b)\,\mathrm{d}\theta\,. \tag{4.15}$$
From (3.6) and (3.7), we then get the same type of phase transitions as for the IPNR model above:

- If $p \neq 0$
$$\frac{p}{(1-p)^{1-\frac{2}{\beta}}}T^{\frac{2}{\beta}}\left(\frac{K(\beta)}{2} + J(T(1-p),\frac{\beta}{2})\right) < \pi, \tag{4.16}$$
then $\mathbf{E}^0[\mathbf{L}] < \infty$;
- If
$$\frac{p}{(1-p)^{1-\frac{2}{\beta}}}T^{\frac{2}{\beta}}\left(\frac{K(\beta)}{2} + J(T(1-p),\frac{\beta}{2})\right) > \pi, \tag{4.17}$$
then $\mathbf{E}^0[\mathbf{L}] = \infty$.

We can use the bounds of Remark 3.7 to get the following and simpler conditions:

- If $p \neq 0$ and $\theta(p,T,\beta) < \pi$ then $\mathbf{E}^0[\mathbf{L}] < \infty$.
- If $\theta(p,T,\beta) > 2\pi$ then $\mathbf{E}^0[\mathbf{L}] = \infty$.

Remark 4.7: *(RESTART, cont.)* We continue the analogy with the RESTART algorithm described in Remark 4.2. Consider the fast Rayleigh fading, with slow, constant noise $W = Const$ in the context of one of the nearest receiver models. Then the local delay time of a packet can be seen as an instance of this algorithm with the following identification: $A_n = F_0^0(n)e_0(n)$ and $B = TWl(\mathcal{D})$, where $\mathcal{D}$ is the (random) distance between the node where the packet is located and the target receiver. The support of $l(\mathcal{D})$ is unbounded (for instance, in the Poisson receiver model, for all the considered path-loss models, the density of $\mathcal{D}$ at $r > 0$ is $\exp(-\lambda_0\pi r^2)r$ for r large).

The interference limited case can be seen as an extension of the RESTART algorithm where the file size varies over time. More precisely, the model corresponding to e.g. MNN is that where at attempt n, the file size is $B_n = f(\Phi, C_n)$, where Φ is the Poisson p.p. and $\{C_n\}_{n>0}$ is an independent i.i.d. sequence (here C_n is the set of fading variables and MAC decisions at time n).

V. Finite Mean Delays and Diversity

As we saw in the last subsection, the existence of big void regions as found in Poisson configurations leads to the surprising property in the nearest receiver models, that the mean (time) local delay is finite everywhere but may have an infinite spatial average in rather classical scenarios. We describe below a few ways of getting finite mean spatial average of the mean local delay in fast fading scenarios. All the proposed methods rely on an increase of *diversity*: more variability in fading, more receivers, more mobility, more flexible (adaptive) coding schemes.

A. Heavy Tailed Fading

c) Weibull Fading: Assume the path loss function (2.1) and deterministic $W > 0$, Recall from the discussion after condition (4.9) (cf. also (4.10) that in this case the mean local delay of the typical node is infinite in the IPNR and MNN model (due to the noise constraint) if one has the (fast) Rayleigh fading. However if we assume that F is Weibull of shape parameter k i.e. $\mathbf{P}[F > x] = \exp(-(x/c)^k)$, for some c, with c and k positive constants, then the condition $k < 2/\beta$ is sufficient to have $\mathbf{E}^0[\mathbf{L}] < \infty$ in the noise limited scenario. Indeed then $\mathbf{P}(F > l(r)WT) = \exp\left\{-(l(r)TW/c)^k\right\} \geq \exp\left\{-(TW/c)^k(Ar)^{2-\epsilon}\right\}$, for $r \geq 1/A$, and some $\epsilon > 0$.

Therefore the finiteness of $\mathbf{E}^0[\mathbf{L}]$ (with canceled interference) follows from the fact that the integral

$$\int_{1/A}^{\infty} r \exp\left\{-\pi\lambda_0 r^2 + (TW/c)^k (Ar)^{2-\epsilon}\right\} \mathrm{d}r$$

is finite.

d) Lognormal Fading: Assume now F is lognormal with parameters (μ, σ), that is $\log(F)$ is $\mathcal{N}(\mu, \sigma^2)$ (Gaussian of mean μ and variance σ^2) and that W is constant. Using

$$\mathbf{P}(F > x) \sim \frac{1}{(\log(x) - \mu)/\sigma} \exp\left(-(\log(x) - \mu)^2/2\sigma^2\right),$$

when $x \to \infty$ and

$$\mathbf{P}(F > x) \geq \frac{(\log(x) - \mu)/\sigma}{1 + (\log(x) - \mu)^2/\sigma^2} \exp\left(-(\log(x) - \mu)^2/2\sigma^2\right)$$

for all $x > 0$ one can show that $\mathbf{E}^0[\mathbf{L}] < \infty$ in the noise-limited scenario for IPNR and MNN model with the path loss function (2.1) (we skip the details).

Let us conclude that a fading with heavier tails may be useful *in the noise limited scenario* in that the mean delay of the typical node may be infinite for the Rayleigh case and finite for this heavier tailed case.

1) Networks with an Additional Periodic Infrastructure: The second line of thoughts is based on the idea that extra receiver should be added to fill in big void regions. We assume again fast Rayleigh fading in conjunction with the "Poisson + periodic" independent receiver model. In this receiver model we assume that the pattern of potential receiver consists of Poisson p.p. and an additional periodic infrastructure. Since there is a receiver at distance at most, say, κ from every point, the closest receiver from the origin is at a distance at most κ and

$$\mathbf{E}^0[\mathbf{L}] = \int_0^{\kappa} \mathcal{D}_W(\mu T l(r))\, \mathcal{D}_I^{INR}(T l(r))\, D(\mathrm{d}r),$$

where $\mathcal{D}_W$, $\mathcal{D}_I^{INR}$ are as in Proposition 3.6 and $D(\cdot)$ is the distribution function of the distance from the origin to the nearest receiver in this model. This latter integral is obviously finite.

Notice that periodicity is not required here. The only important property is that each location of the plane has a node at a distance which is upper bounded by a constant.

2) High Mobility Networks: It was already mentioned in Example 3.2 that if one can assume that the whole network is independently re-sampled at each time slot (including node locations Φ, which is *not* our default option) — an assumption which can be justified when there is a high mobility of nodes — then $\mathbf{E}^0[\mathbf{L}] = 1/p_c < \infty$ provided $p_c > 0$. This observation can be refined in at least two ways:

- Assume the IPNR model, with fixed potential receives, and high mobility of MANETS nodes, i.e., with $\Phi = \Phi(n)$ i.i.d. re-sampled at each $n \geq 1$. Assume also fast noise and fading. Then one can easily argue that

$$\mathbf{E}^0[\mathbf{L}] = 2\pi\lambda_0 \int_{r>0} r \exp(-\pi\lambda_0 r^2) \frac{1}{p\mathcal{L}_W(\mu l(r)T)\mathcal{L}_{I^1}(\mu l(r)T)} \mathrm{d}r.$$

  The finiteness of the last integral can be assessed using arguments similar to those given above.

- Assume now the IPNR model, with i.i.d. potential receives $\Phi_0(n)$ and static MANET Φ. Assume also fast noise and fast fading. We found no closed form expression for the mean local delay of the typical node in this case, however using some convexity arguments it can be shown that it is smaller that the mean local delay of the typical node in the original IPNR model.

Remark: An important remark is in order. In the examples considered in this section, we perform (at least some part of) the space average *together* with the time average to get the mean local delay. This operation, which makes sense in the case of high mobility (of potential receivers, MANET nodes) more easily leads to a finite mean local delay of the typical node. In contrast, in the previous sections (case of static Φ and Φ_0) we perform the time average first and then the space average, and we get a different result, which can for instance be infinite.

3) Adaptive Coding and Shannon Local Delay: One may argue that if the mean delays are infinite in the previously considered models, it is primarily because of the *coverage logic*, where one transmits full packets at time slots when the receiver is covered at the required SINR and where one wastes all the other time slots. This results in a RESTART mechanism (cf. Remark 4.2 and 4.2), which in turn explains why we have heavy tails and infinite means. Adaptive coding offers the possibility of breaking the coverage/RESTART logic: it gives up with minimal requirements on SINR and it hence provides some non-null throughput at each time slot, where this throughput depends on the current value of the SINR, e.g. via Shannon's formula as briefly described in what follows.

Let $\mathcal{T}_0 = \log(1 + \mathrm{SINR}_0)$ be the bit rate obtained by the typical node X_0 under, Palm probability $\mathbf{P}^0$, at time slot 0, where $\mathrm{SINR}_0 = \mathrm{SINR}_0(0)$ is given by (2.2). It is natural to define the *Shannon local delay* of the typical node X_0 as

$$\mathbf{L}^{Sh} = \mathbf{L}_0^{Sh} = \frac{1}{p\mathbf{E}^0[\mathcal{T}_0 \,|\, \mathcal{S}]},$$

namely as the inverse of the time average of $\mathcal{T}_0$ given all the static elements (cf. Section III). This definition is the direct analogue of that of the local delay in the packet model. Observe that

$$\begin{aligned}
&\mathbf{E}^0[\log(1 + \mathrm{SINR}_0) \,|\, \mathcal{S}] \\
&\quad = \int_0^{\infty} \mathbf{P}^0\{\, \log(1 + \mathrm{SINR}_0) > t \,|\, \mathcal{S}\,\}\, \mathrm{d}t \\
&\quad = \int_0^{\infty} \mathbf{P}^0\{\, \mathrm{SINR}_0 > e^t - 1 \,|\, \mathcal{S}\}\, \mathrm{d}t \\
&\quad = \int_0^{\infty} \pi_c(e^t - 1|\mathcal{S})\mathrm{d}t
\end{aligned} \tag{5.18}$$

where $\pi_c(v|\mathcal{S})$ is as defined in (3.1) and where we made the dependence on $T = v$ explicit. Consequently, we obtain

$$\mathbf{E}^0[\mathbf{L}^{Sh}] = \mathbf{E}^0\left[\frac{1}{\int_0^{\infty} \pi_c(v \,|\, \mathcal{S})/(v+1)\, \mathrm{d}v}\right]. \tag{5.19}$$

We now show two examples where $\mathbf{E}^0[\mathbf{L}] = \infty$ but $\mathbf{E}^0[\mathbf{L}^{Sh}] < \infty$.

a) Bipolar Receiver model: Consider the slow noise, fast Rayleigh fading scenario. We saw in Remark 4.1 that in this case, for Poisson Bipolar, noise limited networks, a necessary condition for $\mathbf{E}^0[\mathbf{L}] < \infty$ is that the noise W has

finite exponential moment $\mathbf{E}[e^{\{WTl(r)\mu}] < \infty$. For the mean Shannon local delay of the typical node we have

$$\mathbf{E}^0[\mathbf{L}^{Sh}] = \mathbf{E}\left[\frac{W}{\int_0^\infty e^{-vl(r)\mu}/(v/W+1)\,\mathrm{d}v}\right],$$

in the noise limited case. It is easy to see that the last expression is finite provided $\mathbf{E}[W] < \infty$ (which is much less constraining than the finiteness of the exponential moment).

b) IPNR model: Consider now the IPNR model, with fast Rayleigh fading. Consider the interference limited case and with the path loss function (2.1) It follows from the discussion after Corollary 4.4, that if $\lambda_0\pi < \lambda\theta(p,T,\beta)$, where $\theta(\cdot)$ is given by (4.13), then $\mathbf{E}^0[\mathbf{L}] = \infty$. For the Shannon delay, in the interference limited case, we have

$$\mathbf{E}^0[\mathbf{L}^{Sh}] = 2\pi\lambda_0\frac{1}{p}\int_0^\infty re^{-\lambda_0\pi r^2}$$
$$\times\mathbf{E}^0\left[\left(\int_0^\infty \frac{1}{v+r^\beta}\exp\Big\{\sum_{X_i\neq X_0}\log\mathcal{L}_{eF}\big(\mu v/|X_i|^\beta\big)\Big\}\,\mathrm{d}v\right)^{-1}\right]\mathrm{d}r\,.$$

Using the inequalities

$$\begin{aligned}\int_0^\infty \frac{\exp\{\dots\}}{v+r^\beta}\,\mathrm{d}v &\geq \frac{1}{2r^\beta}\int_0^{r^\beta}\exp\{\dots\}\,\mathrm{d}v + \int_{r^\beta}^\infty\frac{\exp\{\dots\}}{2v}\,\mathrm{d}v\\ &\geq \min\Big(\frac{1}{2r^\beta},1\Big)\int_0^\infty\min\Big(\frac{1}{2v},1\Big)\exp\{\dots\}\,\mathrm{d}v\\ &\geq \min\Big(\frac{1}{2r^\beta},1\Big)\int_0^{1/2}\exp\{\dots\}\,\mathrm{d}v\,.\end{aligned}$$

Note that $\int_0^\infty 2r/(\min(r^{-\beta},2))e^{-\lambda_0\pi r^2}\,\mathrm{d}r < \infty$. Using Jensen's inequality, we get that for all X_i

$$\log\mathcal{L}_{eF}\big(\mu v/|X_i|^\beta\big) \geq -\mathbf{E}[eF]\mu v/|X_i|^\beta = -pv/|X_i|^\beta$$

for $|X_i| > \rho$, where $\rho > 0$ is some fixed constant. From this and from the inequality $\mathcal{L}_{eF} \geq 1-p$ for $|X_i| \leq \rho$, we conclude that $\mathbf{E}^0[\mathbf{L}^{Sh}] < \infty$ provided

$$\mathbf{E}^0\left[\frac{\exp\Big\{-\log(1-p)\Phi(\{X_i : |X_i| \leq \rho\})\Big\}}{\int_0^{1/2}\exp\Big\{-pv\sum_{|X_i|>\rho}|X_i|^{-\beta}\Big\}\,\mathrm{d}v}\right] < \infty\,.$$

Using the independence property of the Poisson p.p., the fact that the Poisson variable $\Phi(\{X_i : |X_i| \leq \rho\})$ has finite exponential moments, it remains to prove that

$$\mathbf{E}^0\left[\left(\int_0^{1/2} e^{-(pv\sum_{|X_i|>\rho}|X_i|^{-\beta})}\,\mathrm{d}v\right)^{-1}\right] = \mathbf{E}^0\Big[\frac{pJ}{1-e^{-pJ/2}}\Big] < \infty\,,$$

where $J = \sum_{|X_i|>\rho}|X_i|^{-\beta}$. Note that for J small, the expression under the expectation is close to 2, whereas for J bounded away from 0, we have

$$\begin{aligned}\mathbf{E}^0\Big[\frac{pJ}{1-e^{-pJ/2}}\mathbf{I}(J>\epsilon)\Big] &\leq (1-e^{-p\epsilon/2})p\mathbf{E}^0[J]\\ &= (1-e^{-p\epsilon/2})2p\pi\lambda\int_\rho^\infty t^{1-\beta}\,\mathrm{d}t,\end{aligned}$$

which is finite since $\beta > 2$. Note that the last inequality is essentially (modulo the problem of the pole of the simplified path loss function (2.1 at 0) equivalent to the finiteness of the mean of the shot-noise.

VI. Conclusion

In the present paper, we introduced a space-time scenario for describing the dynamics of a MANET using Spatial Aloha. This was used to analyze the law of the time to transmit a typical packet from a typical node to its next-hop node in such networks. This analysis was shown to lead to non trivial observations on the spatial variability of the local delays in such MANETs, when assuming that the time scale of the physical layer and the MAC layer is much smaller than that of mobility. In this case, *the local delay of the typical node has heavy tails and infinite mean values* in most standard scenarios, which however *does* not *imply that the mean throughput of the typical node is null*. In addition, a new kind of phase transition, related to the mean local delay of the typical node (being the spatial, large-population, average of mean delays experienced by individual nodes) was identified for the interference limited case; closed form expressions were also given for the thresholds separating the two phases in some computational cases. Various ways of guaranteeing that the network is in the phase where the spatial average of the mean delays is finite were discussed, some based on an increase of the variability (very high mobility, heavy tailed fading), some based on bounding the distance to the next-hop. It was also shown that adaptive coding offers fundamentally different performance compared to the coverage/outage scheme, allowing for finite spatial mean local delays in cases when the non-adaptive coverage/RESTART scheme gives infinite values. The discussion on how to reduce the mean value of local delays (and in particular how to move from an infinite to a finite mean value) opens many interesting research directions which are left for a companion paper: for instance, it would be useful to understand what classes of moderate mobility lead to such a decrease. The same question is natural for fading scenarios, or point processes representing additional receivers.

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