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There are arbitrarily large minimal 2-pinning configurations

Xavier Goaoc∗ Hyo-Sil Kim† Jung-Gun Lim‡

Let ℓ be a line and C a collection of disjoint convex sets in ℜ^d. We say that ℓ is a transversal to C if it intersects each of its members and that C pins ℓ if, in addition, no arbitrarily small perturbation of ℓ is a transversal to C. We say that C k-pins ℓ if for any k-flat II containing ℓ, II ∩ C pins ℓ in II (so pinning and d-pinning are the same in ℜ^d). A minimal k-pinning configuration is a pair (ℓ, C) where C k-pins ℓ but no proper subset of C does.

When C consists of disjoint balls, all its transversal that meet the balls in the same order make up a connected set in line space [1]. So, by continuity, C pins ℓ if and only if no other transversal to C realizes the same order as ℓ. Since two lines span a space of dimension at most 3, this implies that C pins ℓ if and only if it 3-pins it. As a consequence, all minimal 3-pinning configurations of disjoint balls in ℜ^d have size at most 2d − 1 [1, 2]. Here, we prove that the situation is different for 2-pinning:

Theorem 1. A minimal 2-pinning configuration of size n ≥ 6 in ℜ^d exists if and only if n is even.

For simplicity we assume that the objects are smooth, e.g. balls. Our proof is essentially combinatorial:

1. Sign sequence. Let ℓ be an oriented line in ℜ^2 tangent to O_1, . . . , O_n in that order. Its sign sequence is a word in {−, +}^n where the ith letter is + if and only if O_i is on the right of ℓ.

2. Encoding. We now return to 3-space, assume that C 2-pins ℓ and choose some arbitrary orientation on ℓ. The tangent planes to the objects of C at their contact point with ℓ partition the space into n sectors. For all planes containing ℓ lying in the same sector, the sign sequence of the associated planar configuration is the same (up to exchanging − and +). We denote by σ_0, . . . , σ_{n−1} the sign sequences obtained successively as we go through all sectors around ℓ.

3. Translating geometric properties. The family (σ_0, . . . , σ_{n−1}) corresponds to a geometric configuration (C, ℓ) if and only if (1) there exists a permutation π of {1, . . . , n} such that σ_i differs from σ_{i−1}[n] from the inversion of its π(i)th letter. Then, observe that C 2-pins ℓ if and only if (2) every σ_i contains an alternating triple. Last, notice that no proper subset of C 2-pins ℓ if and only if (3) for any 1 ≤ i ≤ n there is some t such that σ_t loses all alternating triples if its ith letter is deleted.

4. Wrapping up. For odd n, rules (1)−(3) are incompatible. For even n, the sequence defined by

σ_0 = + − + n−2 and π = (4, 1, 6, 3, 8, 5, . . . , n, n − 3, 2, n − 1)

satisfies rules (1)−(3) and thus corresponds to a minimal 2-pinning configuration.

References


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