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There are arbitrarily large minimal 2-pinning configurations

Xavier Goaoc∗  Hyo-Sil Kim†  Jung-Gun Lim‡

Let ℓ be a line and C a collection of disjoint convex sets in \( \mathbb{R}^d \). We say that ℓ is a transversal to C if it intersects each of its members and that C pins ℓ if, in addition, no arbitrarily small perturbation of ℓ is a transversal to C. We say that C k-pins ℓ if for any k-flat \( \Pi \) containing ℓ, \( \Pi \cap C \) pins ℓ in \( \Pi \) (so pinning and d-pinning are the same in \( \mathbb{R}^d \)). A minimal k-pinning configuration is a pair \( (\ell, C) \) where C k-pins ℓ but no proper subset of C does.

When C consists of disjoint balls, all its transversal that meet the balls in the same order make up a connected set in line space [1]. So, by continuity, C pins ℓ if and only if no other transversal to C realizes the same order as ℓ. Since two lines span a space of dimension at most 3, this implies that C pins ℓ if and only if it 3-pins it. As a consequence, all minimal 3-pinning configurations of disjoint balls in \( \mathbb{R}^d \) have size at most \( 2d - 1 \) [1, 2]. Here, we prove that the situation is different for 2-pinning:

Theorem 1. A minimal 2-pinning configuration of size \( n \geq 6 \) in \( \mathbb{R}^d \) exists if and only if \( n \) is even.

For simplicity we assume that the objects are smooth, e.g. balls. Our proof is essentially combinatorial:

1. Sign sequence. Let ℓ be an oriented line in \( \mathbb{R}^2 \) tangent to \( O_1, \ldots, O_n \) in that order. Its sign sequence is a word in \( \{- , + \}^n \) where the \( i^{th} \) letter is + if and only if \( O_i \) is on the right of ℓ.

2. Encoding. We now return to 3-space, assume that C 2-pins ℓ and choose some arbitrary orientation on ℓ. The tangent planes to the objects of C at their contact point with ℓ partition the space into \( n \) sectors. For all planes containing ℓ lying in the same sector, the sign sequence of the associated planar configuration is the same (up to exchanging − and +). We denote by \( \sigma_0, \ldots, \sigma_{n-1} \) the sign sequences obtained successively as we go through all sectors around ℓ.

3. Translating geometric properties. The family \( (\sigma_0, \ldots, \sigma_{n-1}) \) corresponds to a geometric configuration \( (C, \ell) \) if and only if (1) there exists a permutation \( \pi \) of \( \{1, \ldots, n\} \) such that \( \sigma_i \) differs from \( \sigma_{i-1[n]} \) from the inversion of its \( \pi(i)^{th} \) letter. Then, observe that C 2-pins ℓ if and only if (2) every \( \sigma_i \) contains an alternating triple. Last, notice that no proper subset of C 2-pins ℓ if and only if (3) for any \( 1 \leq i \leq n \) there is some \( t \) such that \( \sigma_t \) loses all alternating triples if its \( i^{th} \) letter is deleted.

4. Wrapping up. For odd \( n \), rules (1)–(3) are incompatible. For even \( n \), the sequence defined by

\[
\sigma_0 = + - + n - 2 \quad \text{and} \quad \pi = (4, 1, 6, 3, 8, 5, \ldots, n, n - 3, 2, n - 1)
\]

satisfies rules (1)–(3) and thus corresponds to a minimal 2-pinning configuration.

References


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