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Benchmarking the (1+1)-CMA-ES on the BBOB-2009 Function Testbed

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ABSTRACT

The (1+1)-CMA-ES is an adaptive stochastic algorithm for the optimization of objective functions defined on a continuous search space in a black-box scenario. In this paper, an independent restart version of the (1+1)-CMA-ES is implemented and benchmarked on the BBOB-2009 noise-free testbed. The maximum number of function evaluations per run is set to 10^4 times the search space dimension. The algorithm solves 23, 13 and 12 of 24 functions in dimension 2, 10 and 40, respectively.

Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Evolutionary computation, CMA-ES

1. INTRODUCTION

Soon after the introduction of stochastic algorithms for optimizing functions [2], it was recognized that adaptive algorithms where the sampling distribution is adapted during the course of the optimization are necessary for optimizing efficiently. One pioneer work in this direction has been carried out by Schumer and Steiglitz in 1968 [11]. The algorithm they proposed is known in the field of evolutionary algorithms as the (1+1)-Evolution Strategy (ES) with one-fifth success rule and was independently discovered by others [10, 3]. In the (1+1)-ES with one-fifth success rule algorithm, the search distribution is spherical and one single

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parameter of the distribution, the step-length (or step-size), is adapted based on the observed probability of success, defined as the probability to sample a solution better than the current solution.

Recently, the (1+1)-CMA-ES has been proposed as an extension of the (1+1)-ES with one-fifth success rule where not only a step-size but also a covariance matrix associated to the search distribution is adapted [8]. The covariance matrix adaptation (CMA) mechanism of the (1+1)-CMA-ES follows the principles introduced in [7].

In this paper, we introduce the (1+1)-CMA-ES with independent restarts and benchmark it on the BBOB-2009 noise-free function testbed.

2. THE (1+1)-CMA-ES WITH INDEPENDENT RESTARTS

In this section we describe the (1+1)-CMA-ES with independent restarts.

2.1 The (1+1)-CMA-ES

We start by describing the original (1+1)-CMA-ES as proposed in [8]. We consider an objective function $f : \mathbb{R}^D \mapsto \mathbb{R}, \mathbf{x} \mapsto f(\mathbf{x})$ to be minimized. The algorithm is given in Table 1 with time index n and in Table 2 without time index. In the following we describe the iteration n of the algorithm following Table 1. A candidate solution $\tilde{\mathbf{x}}_n$ is sampled by perturbing the current solution \mathbf{x}_n by adding a gaussian vector with mean vector $\mathbf{0}$ and covariance matrix \mathbf{C}_n and scaled by the step-size σ_n (Line 5)¹. This candidate solution is accepted if and only if $f(\tilde{\mathbf{x}}_n) \leq f(\mathbf{x}_n)$ (Line 9). The step-size is adapted (Lines 6 and 7) using the averaged success rate p_{n+1}^{succ} : it is increased if the success rate is strictly larger than the target probability $p_{\text{target}}^{\text{succ}}$, and decreased if it is strictly smaller, according to the equation in Line 7². If $f(\tilde{\mathbf{x}}_n) \leq f(\mathbf{x}_n)$, the covariance matrix is adapted by adding to a multiple of \mathbf{C}_n the rank-one update matrix $\mathbf{p}_{n+1}\mathbf{p}_{n+1}^T$ (Line 11) where the evolution path \mathbf{p}_{n+1} was updated in Line 10. If $f(\tilde{\mathbf{x}}_n) > f(\mathbf{x}_n)$, the covariance matrix is not changed, i.e., \mathbf{C}_{n+1} equals \mathbf{C}_n . The evolution path is also not updated (Line 14). The default parameters for the (1+1)-CMA-ES are given in Table 3.

¹If \mathbf{A}_n satisfies $\mathbf{A}_n\mathbf{A}_n^T = \mathbf{C}_n$ and \mathbf{z} follows a multivariate normal distribution with mean vector $\mathbf{0}$ and covariance matrix \mathbf{I} , then $\mathbf{A}_n\mathbf{z}$ is a multivariate normal distribution with mean vector $\mathbf{0}$ and covariance matrix \mathbf{C}_n .

²For $c_p = 1$, $p_{\text{target}}^{\text{succ}} = 1/5$ one obtains the 1/5 success rule from [9] and benchmarked in [1] where the increase factor corresponds to $\exp(1/d)$.

Table 3: Default parameters for the (1+1)-CMA-ES

Step-size control:

$$d = 1 + \frac{D}{2}, p_{\text{target}}^{\text{succ}} = \frac{2}{11}, c_p = \frac{1}{12}$$

Covariance matrix adaptation:

$$c_c = \frac{2}{D+2}, c_{\text{cov}} = \frac{2}{D^2+6}, p^{\text{thresh}} = 0.44$$

2.2 The independent-restart (1+1)-CMA-ES

We have implemented an independent-restart version of the (1+1)-CMA-ES: for each start the initial solution \mathbf{x}_0 is sampled uniformly in $[-4, 4]^D$, the step-size σ_0 is initialized at 2 and the initial covariance matrix is the identity. After reaching a stopping criteria (described in the next section) the algorithm is (re-)initialized and restarted. This process is iterated. Whenever the overall number of function evaluations reaches $10^4 D$ or an objective function value below the target function value is reached the algorithm is stopped.

2.3 Termination criteria

A single run of the (1+1)-CMA-ES is terminated when one of the following condition is satisfied:

MaxIter = $100 + \lceil 1000D\sqrt{D} \rceil$ is the maximal number of iterations and function evaluations in each run of the (1+1)-CMA-ES

TolHistFun = 10^{-12} : the range of the best function values during the last $10 + 30D$ iterations is smaller than **TolHistFun**

TolX = 10^{-12} : all components of \mathbf{p}_n and all square roots of diagonal components of \mathbf{C}_n , multiplied by σ_n/σ_0 , are smaller than **TolX**

TolUpX = 10^8 : σ_n/σ_0 multiplied by the square root of the largest diagonal element of \mathbf{C}_n is larger than **TolUpX**

ConditionCov: the condition number of \mathbf{C}_n exceeds 10^{14} .

NoEffectAxis: \mathbf{x}_n remains numerically constant when adding $0.1\sigma_n\sqrt{l^t}\mathbf{v}^t$, where l^t is the $1 + (t \bmod D)$ -largest eigenvalue of \mathbf{C}_n and \mathbf{v}^t is the corresponding normalized eigenvector.

NoEffectCoor: any element of \mathbf{x}_n remains numerically constant when adding $0.2\sigma_n l^t$, where elements of l^t are the square root of the diagonal elements of \mathbf{C}_n .

3. PARAMETER TUNING

No parameter tuning has been conducted. The maximum number of iterations **MaxIter** has been set to prevent the possibility of excessively long runs. The other termination criteria were left to their standard values. The same parameter setting is used on all functions and therefore the crafting effort [5] computes to CrE = 0.

4. RESULTS

Results from experiments according to [5] on the benchmark functions given in [4, 6] are presented in Figures 1 and 2 and in Table 4.

In 5-D all the unimodal functions are solved and in 20-D and 40-D, the f_6 (Attractive sector) and f_7 (Step-ellipsoid)

functions are not solved. However, by decreasing the learning rate c_{cov} by a factor of 3, we have observed that the f_6 function could be solved. The decreased learning rate would increase the running time on ill-conditioned problems roughly by a factor of two. Among the multimodal functions, the (1+1)-CMA-ES can solve the f_{21} and f_{22} functions quite efficiently.

Compared to the (1+1)-ES with one-fifth success rule benchmarked in [1], the performance is greatly improved on all ill-conditioned problems that cannot be solved without the covariance matrix adaptation mechanism. The overall number of multimodal functions solved is the same for both algorithms, while CMA-ES is often faster. The (1+1)-CMA-ES is outperformed by the one-fifth success rule without CMA mechanism on the Attractive Sector function.

5. CPU TIMING EXPERIMENTS

For the timing experiment the (1+1)-CMA-ES restart was run with a maximum of $10^5 \times D$ function evaluations and restarted until 30 seconds has passed (according to Figure 2 in [5]). The experiments have been conducted with an Intel Pentium 4 CPU 3.80 GHz under Linux with Matlab R2007a. The time per function evaluation was 8.2; 8.3; 8.3; 8.7; 11; 20 times 10^{-4} seconds in dimensions 2; 3; 5; 10; 20; 40 respectively.

6. CONCLUSION

We have benchmarked the (1+1)-CMA-ES. The algorithm solves 23, 21, 16, 13, 12 and 12 of 24 functions in dimension 2, 3, 5, 10, 20 and 40, respectively, with a budget of $10^4 D$ function evaluations. The algorithm solves all unimodal functions in 5-D and all but 2 in 20 and 40-D. The good performances on the unimodal functions are to be expected from the adaptation mechanism of the algorithm inherited from the original CMA-ES [7]. Besides two exceptions the performance is poor on multi-modal functions due to the absence of a population in the algorithm. The results are expected to generalize well due to the invariance properties of the algorithm, namely invariance to order-preserving transformations of the function value and rotational invariance.

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7. REFERENCES

- [1] A. Auger. Benchmarking the (1+1) Evolution Strategy with One-Fifth Success Rule on the BBOB-2009 Function Testbed. In *Workshop Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2009)*. ACM Press, 2009.
- [2] S. H. Brooks. A discussion of random methods for seeking maxima. *Operations Research*, 6:244– 251, 1958.
- [3] L. Devroye. The compound random search. In *International Symposium on Systems Engineering and Analysis*, pages 195–110. Purdue University, 1972.

Table 1: Pseudo-code for the (1+1)-CMA-ES with time dependent notations

Algorithm: (1+1)-CMA-ES	
1.	Initialize $\mathbf{x}_0, \sigma_0, \mathbf{C}_0 = \mathbf{I}, \mathbf{p}_0 = \mathbf{0}, p_0^{\text{succ}} = p_{\text{target}}^{\text{succ}}, n = 1$
2. repeat	
3.	determine \mathbf{A}_n such that $\mathbf{C}_n = \mathbf{A}_n \mathbf{A}_n^T$
4.	$\mathbf{z}_n \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ standard normal distribution
5.	$\tilde{\mathbf{x}}_n = \mathbf{x}_n + \sigma_n \mathbf{A}_n \mathbf{z}_n$ new solution
6.	$p_{n+1}^{\text{succ}} = (1 - c_p)p_n^{\text{succ}} + c_p \mathbf{1}_{\{f(\tilde{\mathbf{x}}_n) \leq f(\mathbf{x}_n)\}}$ averaged success rate
7.	$\sigma_{n+1} = \sigma_n \exp\left(\frac{1}{d} \frac{p_{n+1}^{\text{succ}} - p_{\text{target}}^{\text{succ}}}{1 - p_{\text{target}}^{\text{succ}}}\right)$ update step-size
8.	if $f(\tilde{\mathbf{x}}_n) \leq f(\mathbf{x}_n)$ then
9.	$\mathbf{x}_{n+1} = \tilde{\mathbf{x}}_n$
10.	$\mathbf{p}_{n+1} = (1 - c_c)\mathbf{p}_n + \mathbf{1}_{\{p_{n+1}^{\text{succ}} < p^{\text{thresh}}\}} \sqrt{c_c(2 - c_c)} \mathbf{A}_n \mathbf{z}_n$ update evolution path
11.	$\mathbf{C}_{n+1} = \left(1 - c_{\text{cov}} + c_{\text{cov}} \mathbf{1}_{\{p_{n+1}^{\text{succ}} > p^{\text{thresh}}\}} c_c(2 - c_c)\right) \mathbf{C}_n + c_{\text{cov}} \mathbf{p}_{n+1} \mathbf{p}_{n+1}^T$ update covariance matrix
12.	else
13.	$\mathbf{x}_{n+1} = \mathbf{x}_n$
14.	$\mathbf{p}_{n+1} = \mathbf{p}_n$
15.	$\mathbf{C}_{n+1} = \mathbf{C}_n$
16.	$n = n + 1$
17. until	stopping criterion is met

Table 2: Pseudo-code for the (1+1)-CMA-ES without time dependent notations

Algorithm: (1+1)-CMA-ES	
1.	Initialize $\mathbf{x}, \sigma, \mathbf{C} = \mathbf{I}, \mathbf{p} = \mathbf{0}, p^{\text{succ}} = p_{\text{target}}^{\text{succ}}$
2. repeat	
3.	determine \mathbf{A} such that $\mathbf{C} = \mathbf{A} \mathbf{A}^T$
4.	$\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ standard normal distribution
5.	$\tilde{\mathbf{x}} = \mathbf{x} + \sigma \mathbf{A} \mathbf{z}$ new solution
6.	$p^{\text{succ}} \leftarrow (1 - c_p)p^{\text{succ}} + c_p \mathbf{1}_{\{f(\tilde{\mathbf{x}}) \leq f(\mathbf{x})\}}$ averaged success rate
7.	$\sigma \leftarrow \sigma \exp\left(\frac{1}{d} \frac{p^{\text{succ}} - p_{\text{target}}^{\text{succ}}}{1 - p_{\text{target}}^{\text{succ}}}\right)$ update step-size
8.	if $f(\tilde{\mathbf{x}}) \leq f(\mathbf{x})$ then
9.	$\mathbf{x} \leftarrow \tilde{\mathbf{x}}$
10.	$\mathbf{p} \leftarrow (1 - c_c)\mathbf{p} + \mathbf{1}_{\{p^{\text{succ}} < p^{\text{thresh}}\}} \sqrt{c_c(2 - c_c)} \mathbf{A} \mathbf{z}$ update evolution path
11.	$\mathbf{C} \leftarrow \left(1 - c_{\text{cov}} + c_{\text{cov}} \mathbf{1}_{\{p^{\text{succ}} > p^{\text{thresh}}\}} c_c(2 - c_c)\right) \mathbf{C} + c_{\text{cov}} \mathbf{p} \mathbf{p}^T$ update covariance matrix
12. until	stopping criterion is met

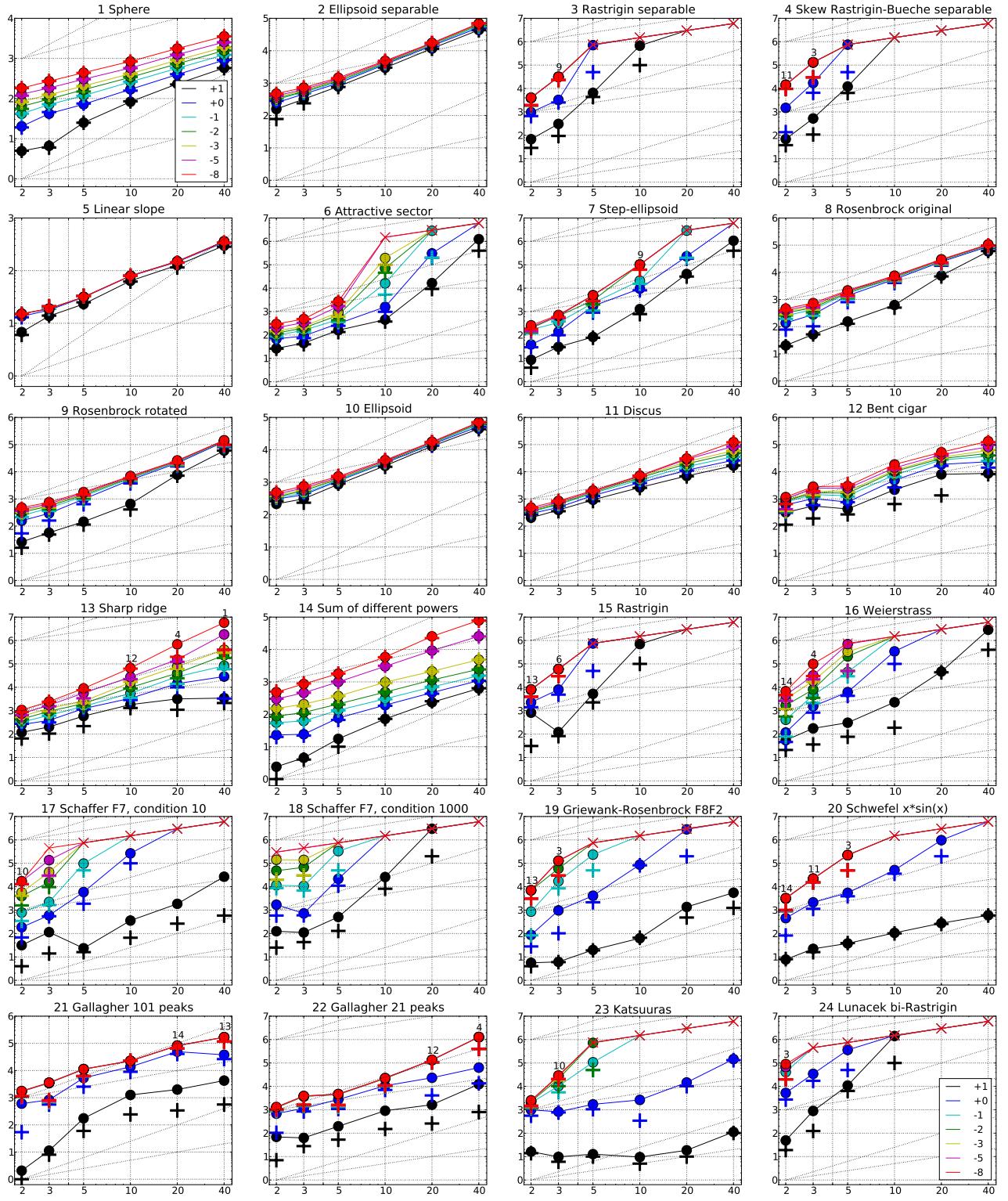


Figure 1: Expected Running Time (ERT, ●) to reach $f_{\text{opt}} + \Delta f$ and median number of function evaluations of successful trials (+), shown for $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ (the exponent is given in the legend of f_1 and f_{24}) versus dimension in log-log presentation. The $\text{ERT}(\Delta f)$ equals to $\#\text{FEs}(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\text{opt}} + \Delta f$ was surpassed during the trial. The $\#\text{FEs}(\Delta f)$ are the total number of function evaluations while $f_{\text{opt}} + \Delta f$ was not surpassed during the trial from all respective trials (successful and unsuccessful), and f_{opt} denotes the optimal function value. Crosses (×) indicate the total number of function evaluations $\#\text{FEs}(-\infty)$. Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.

Δf	f1 in 5-D, N=15, mFE=512	f1 in 20-D, N=15, mFE=1863	f2 in 5-D, N=15, mFE=1659	f2 in 20-D, N=15, mFE=18577
	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.5e1 2.0e1 3.0e1 2.5e1	15 2.3e2 2.2e2 2.4e2 2.3e2	15 1.2e4 1.1e4 1.2e4 1.2e4	15 1.2e4 1.1e4 1.2e4 1.2e4
1	15 7.2e1 6.6e1 7.8e1 7.2e1	15 4.0e2 3.9e2 4.1e2 4.0e2	15 1.4e4 1.4e4 1.5e4 1.4e4	15 1.4e4 1.4e4 1.5e4 1.4e4
le-1	15 1.2e2 1.1e2 1.3e2 1.2e2	15 5.7e2 5.5e2 5.8e2 5.7e2	15 1.5e4 1.5e4 1.5e4 1.5e4	15 1.5e4 1.5e4 1.5e4 1.5e4
le-3	15 2.1e2 2.0e2 2.1e2 2.1e2	15 9.1e2 8.9e2 9.2e2 9.1e2	15 1.6e4 1.6e4 1.6e4 1.6e4	15 1.6e4 1.6e4 1.6e4 1.6e4
le-5	15 3.0e2 2.9e2 3.1e2 3.0e2	15 1.2e3 1.2e3 1.3e3 1.2e3	15 1.7e4 1.7e4 1.7e4 1.7e4	15 1.7e4 1.7e4 1.7e4 1.7e4
le-8	15 4.4e2 4.3e2 4.5e2 4.4e2	15 1.8e3 1.7e3 1.8e3 1.8e3	15 1.7e4 1.7e4 1.8e4 1.7e4	15 1.7e4 1.7e4 1.8e4 1.7e4
	f3 in 5-D, N=15, mFE=50001	f3 in 20-D, N=15, mFE=200001	f4 in 5-D, N=15, mFE=50001	f4 in 20-D, N=15, mFE=200001
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 6.5e3 4.9e3 8.3e3 6.5e3	0 92e+0 62e+0 11e+1 1.3e5	10 1.2e4 1.1e4 1.2e4 1.2e4	10 1.4e+1 12e+1 18e+1 1.4e5
1	1 7.1e5 3.4e5 >7e5 5.0e4	.	.	.
le-1	0 30e-1 20e-1 40e-1 1.6e4	.	.	.
le-3
le-5
le-8
	f5 in 5-D, N=15, mFE=51	f5 in 20-D, N=15, mFE=262	f6 in 5-D, N=15, mFE=6322	f6 in 20-D, N=15, mFE=200001
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.3e1 2.0e1 2.6e1 2.3e1	15 1.3e2 1.2e2 1.4e2 1.3e2	10 1.6e4 1.1e4 2.2e4 1.6e4	10 1.5e2 1.3e2 1.8e2 1.5e2
1	15 3.1e1 2.8e1 3.4e1 3.1e1	15 1.5e2 1.4e2 1.6e2 1.5e2	1 3.1e5 2.1e5 5.0e5 1.3e5	1 3.1e5 2.1e5 5.0e5 1.3e5
le-1	15 3.2e1 2.8e1 3.5e1 3.2e1	15 1.5e2 1.4e2 1.6e2 1.5e2	1 2.8e6 1.3e6 >3e6 2.0e5	1 2.8e6 1.3e6 >3e6 2.0e5
le-3	15 3.2e1 2.8e1 3.5e1 3.2e1	15 1.5e2 1.4e2 1.6e2 1.5e2	0 13e-1 10e-2 26e-1 1.0e5	0 13e-1 10e-2 26e-1 1.0e5
le-5	15 3.2e1 2.8e1 3.5e1 3.2e1	15 1.5e2 1.4e2 1.6e2 1.5e2	.	.
le-8	15 3.2e1 2.8e1 3.5e1 3.2e1	15 1.5e2 1.4e2 1.6e2 1.5e2	.	.
	f7 in 5-D, N=15, mFE=17387	f7 in 20-D, N=15, mFE=200001	f8 in 5-D, N=15, mFE=7794	f8 in 20-D, N=15, mFE=57125
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 8.2e1 6.6e1 9.9e1 8.2e1	15 4.0e4 3.4e4 4.7e4 4.0e4	10 1.5e2 1.3e2 1.8e2 1.5e2	10 7.6e3 6.8e3 8.3e3 7.6e3
1	15 1.7e3 8.3e2 2.7e3 1.7e3	10 2.3e5 1.7e5 3.3e5 1.3e5	1 1.4e4 2.0e4 2.9e4 2.4e4	1 1.4e4 2.0e4 2.9e4 2.4e4
le-1	15 2.3e3 1.4e3 3.3e3 2.3e3	1 2.9e6 1.4e6 >3e6 2.0e5	1 1.7e4 2.2e4 3.2e4 2.7e4	1 1.7e4 2.2e4 3.2e4 2.7e4
le-3	15 4.9e3 3.4e3 6.5e3 4.9e3	0 92e-2 52e-2 16e-1 1.1e5	1 1.9e3 1.4e3 2.5e3 1.9e3	1 2.8e4 2.3e4 3.3e4 2.8e4
le-5	15 4.9e3 3.4e3 6.6e3 4.9e3	.	1 2.0e3 1.5e3 2.6e3 2.0e3	1 2.9e4 2.5e4 3.4e4 2.9e4
le-8	15 5.0e3 3.4e3 6.6e3 5.0e3	.	1 2.7e3 2.2e3 3.2e3 2.7e3	1 3.0e4 2.5e4 3.5e4 3.0e4
	f9 in 5-D, N=15, mFE=5975	f9 in 20-D, N=15, mFE=48036	f10 in 5-D, N=15, mFE=1730	f10 in 20-D, N=15, mFE=20132
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 1.5e2 1.1e2 1.8e2 1.5e2	15 7.8e3 6.5e3 9.2e3 7.8e3	10 1.3e4 1.2e4 1.3e4 1.3e4	10 1.3e4 1.2e4 1.3e4 1.3e4
1	15 9.8e2 6.4e2 1.3e3 9.8e2	15 2.1e4 1.7e4 2.4e4 2.1e4	1 1.5e4 1.4e4 1.5e4 1.5e4	1 1.5e4 1.4e4 1.5e4 1.5e4
le-1	15 1.3e3 9.1e2 1.6e3 1.3e3	15 2.3e4 1.9e4 2.6e4 2.3e4	1 1.6e4 1.5e4 1.6e4 1.6e4	1 1.6e4 1.5e4 1.6e4 1.6e4
le-3	15 1.5e3 1.2e3 1.9e3 1.5e3	15 2.4e4 2.1e4 2.8e4 2.4e4	1 1.7e4 1.6e4 1.7e4 1.7e4	1 1.7e4 1.6e4 1.7e4 1.7e4
le-5	15 1.6e3 1.3e3 2.0e3 1.6e3	15 2.5e4 2.2e4 2.9e4 2.5e4	1 1.7e4 1.7e4 1.7e4 1.7e4	1 1.7e4 1.7e4 1.7e4 1.7e4
le-8	15 1.8e3 1.4e3 2.2e3 1.8e3	15 2.6e4 2.3e4 3.0e4 2.6e4	1 1.8e4 1.7e4 1.8e4 1.8e4	1 1.8e4 1.7e4 1.8e4 1.8e4
	f11 in 5-D, N=15, mFE=2491	f11 in 20-D, N=15, mFE=33103	f12 in 5-D, N=15, mFE=5855	f12 in 20-D, N=15, mFE=79051
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 9.3e2 3.2e2 1.0e3 9.3e2	15 7.0e3 6.7e3 7.3e3 7.0e3	10 4.0e3 4.9e3 1.1e4 4.0e3	10 8.0e3 4.9e3 1.1e4 8.0e3
1	15 1.3e3 1.3e3 1.4e3 1.3e3	15 1.1e4 1.1e4 1.2e4 1.1e4	1 1.4e4 2.0e4 2.9e4 2.4e4	1 1.4e4 2.0e4 2.9e4 2.4e4
le-1	15 1.6e3 1.6e3 1.7e3 1.6e3	15 1.5e4 1.5e4 1.6e4 1.5e4	1 1.7e4 2.2e4 3.2e4 2.7e4	1 1.7e4 2.2e4 3.2e4 2.7e4
le-3	15 1.8e3 1.7e3 1.8e3 1.8e3	15 2.4e4 2.3e4 2.4e4 2.4e4	1 1.9e3 1.6e3 2.2e3 1.9e3	1 2.8e4 2.3e4 3.3e4 2.8e4
le-5	15 1.9e3 1.8e3 1.9e3 1.9e3	15 2.9e4 2.8e4 2.9e4 2.9e4	1 2.0e3 1.5e3 2.1e3 1.0e3	1 2.9e4 2.5e4 3.4e4 2.9e4
le-8	15 2.0e3 1.9e3 2.1e3 2.0e3	15 3.1e4 3.0e4 3.1e4 3.1e4	1 2.3e3 1.5e3 1.6e3 1.5e3	1 3.0e4 2.5e4 3.5e4 3.0e4
	f13 in 5-D, N=15, mFE=20761	f13 in 20-D, N=15, mFE=200001	f14 in 5-D, N=15, mFE=2081	f14 in 20-D, N=15, mFE=49317
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 5.9e2 4.1e2 7.9e2 5.9e2	15 3.2e3 2.2e3 4.2e3 3.2e3	10 4.3e2 5.4e2 4.3e2 4.3e2	10 8.0e3 4.9e3 1.1e4 8.0e3
1	15 1.2e3 1.1e3 1.4e3 1.2e3	15 1.4e4 9.3e3 2.0e4 1.4e4	1 5.7e2 5.9e2 9.7e2 7.7e2	1 5.9e4 1.3e4 2.4e4 1.9e4
le-1	15 1.5e3 1.4e3 1.7e3 1.5e3	15 2.8e4 1.9e4 3.8e4 2.8e4	1 1.5e3 1.3e3 1.5e3 1.3e3	1 1.7e4 2.0e4 3.4e4 2.7e4
le-3	15 2.5e3 2.1e3 2.9e3 2.5e3	14 7.8e4 5.8e4 1.0e5 7.0e4	1 1.9e3 1.6e3 2.2e3 1.2e3	1 2.1e3 2.0e3 2.2e3 2.1e3
le-5	15 5.7e3 4.1e3 7.3e3 5.7e3	12 1.5e5 1.1e5 2.1e5 1.1e5	1 2.1e3 2.0e3 2.2e3 2.1e3	1 2.1e3 2.0e3 2.2e3 2.1e3
le-8	15 8.9e3 7.0e3 1.1e4 8.9e3	4 6.9e5 4.2e5 1.4e6 1.6e5	1 2.5e4 3.6e4 4.9e4 4.3e4	1 2.5e4 3.6e4 4.9e4 4.3e4
	f15 in 5-D, N=15, mFE=50001	f15 in 20-D, N=15, mFE=200001	f16 in 5-D, N=15, mFE=50001	f16 in 20-D, N=15, mFE=200001
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 5.3e3 3.2e3 7.3e3 5.3e3	0 85e+0 70e+0 10e+1 1.0e5	10 3.1e2 4.5e2 4.6e2 3.1e2	10 4.7e4 3.5e4 5.9e4 4.7e4
1	1 7.4e5 3.7e5 >7e5 5.0e4	.	1 5.6e3 3.9e3 8.4e3 6.2e3	1 5.6e2 3.6e2 5.0e2 4.6e2
le-1	0 30e-1 20e-1 50e-1 3.2e4	.	1 1.3e-1 3.6e-1 8.6e-1 5.7e-1	1 1.3e-1 3.6e-1 8.6e-1 5.7e-1
le-3	.	.	1 2.3e-2 3.3e-2 6.2e-2 2.2e-2	1 2.3e-2 3.3e-2 6.2e-2 2.2e-2
le-5	.	.	1 5.0e-5 3.3e-5 6.2e-5 2.2e-4	1 5.0e-5 3.3e-5 6.2e-5 2.2e-4
le-8	.	.	1 7.0e-8 3.4e-8 5.2e-8 2.2e-7	1 7.0e-8 3.4e-8 5.2e-8 2.2e-7
	f17 in 5-D, N=15, mFE=50001	f17 in 20-D, N=15, mFE=200001	f18 in 5-D, N=15, mFE=50001	f18 in 20-D, N=15, mFE=200001
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.3e1 1.7e1 3.0e1 2.3e1	15 1.9e3 2.7e2 3.5e3 1.9e3	10 3.1e6 4.3e4 6.9e2 5.1e2	10 3.0e6 1.5e6 >3e6 2.0e5
1	15 5.9e3 2.7e3 9.4e3 5.9e3	0 49e-1 31e-1 65e-1 8.9e4	1 3.2e4 1.6e4 2.8e4 2.0e4	0 16e+0 10e+0 19e+0 8.9e4
le-1	6 9.7e4 6.6e4 1.7e5 3.6e4	.	1 2.3e5 1.8e5 >7e5 5.0e4	.
le-3	0 17e-2 20e-3 33e-2 2.2e4	.	1 3.7e-2 9.8e-3 12e-1 2.5e4	.
le-5	.	.	1 5.0e-5 3.3e-5 6.2e-5 2.2e-4	.
le-8	.	.	1 7.0e-8 3.4e-8 5.2e-8 2.2e-7	.
	f19 in 5-D, N=15, mFE=50001	f19 in 20-D, N=15, mFE=200001	f20 in 5-D, N=15, mFE=50001	f20 in 20-D, N=15, mFE=200001
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 2.0e1 1.6e1 2.4e1 2.0e1	15 1.4e3 4.9e2 2.3e3 1.4e3	10 3.8e1 4.1e1 4.4e1 3.8e1	10 2.8e2 2.6e2 2.9e2 2.8e2
1	15 4.1e3 2.4e3 5.9e3 4.1e3	1 2.8e6 1.3e6 >3e6 2.0e5	1 5.5e3 3.7e3 7.3e3 5.5e3	1 9.7e5 5.8e5 2.9e6 1.9e5
le-1	3 2.4e5 1.4e5 7.0e5 5.0e4	0 18e-1 10e-1 30e-1 8.9e4	1 2.2e5 1.3e5 7.0e5 3.4e4	1 2.1e5 9.9e4 2.9e6 1.9e4
le-3	0 19e-2 86e-3 27e-2 2.2e4	.	1 2.2e5 1.3e5 7.1e5 3.4e4	1 2.3e5 9.9e4 2.9e6 1.9e4
le-5	.	.	1 5.0e-5 3.3e-5 6.2e-5 2.2e-4	.
le-8	.	.	1 7.0e-8 3.4e-8 5.2e-8 2.2e-7	.
	f21 in 5-D, N=15, mFE=34808	f21 in 20-D, N=15, mFE=200001	f22 in 5-D, N=15, mFE=20086	f22 in 20-D, N=15, mFE=200001
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 1.7e2 7.5e1 2.9e2 1.7e2	15 2.0e3 1.0e3 3.0e3 2.0e3	10 2.0e2 4.6e1 3.3e2 2.0e2	10 1.6e3 4.9e2 3.0e3 1.6e3
1	15 5.3e3 3.1e3 7.8e3 5.3e3	15 4.9e4 3.6e3 6.3e4 4.9e4	1 2.8e3 1.6e3 3.9e3 2.8e3	1 2.3e4 1.3e4 3.3e4 2.3e4
le-1	15 1.1e4 7.2e3 1.5e4 1.1e4	14 8.2e4 5.8e4 1.1e5 6.7e4	1 4.4e3 2.7e3 6.3e3 4.4e3	1 12.3e5 9.6e4 1.8e5 9.3e4
le-3	15 1.1e4 7.5e3 1.5e4 1.1e4	14 8.2e4 5.9e4 1.1e5 6.8e4	1 2.3e5 9.9e4 2.9e6 1.9e4	1 12.3e5 9.9e4 2.9e6 1.9e4
le-5	15 1.1e4 7.4e4 1.5e4 1.1e4	14 8.2e4 5.9e4 1.1e5 6.8e4	1 4.6e3 3.0e3 6.5e3 4.6e3	1 12.3e5 9.9e4 2.9e6 1.9e4
le-8	15 1.1e4 7.5e3 1.5e4 1.1e4	14 8.3e4 6.1e4 1.1e5 6.8e4	1 4.7e3 3.0e3 6.6e3 4.7e3	1 12.4e5 1.0e5 1.8e5 9.5e4
	f23 in 5-D, N=15, mFE=50001	f23 in 20-D, N=15, mFE=200001	f24 in 5-D, N=15, mFE=50001	f24 in 20-D, N=15, mFE=200001
Δf	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}	# ERT 10% 90% RT _{succ}
10	15 1.3e1 9.3e0 1.6e1 1.3e1	15 1.9e1 1.2e1 2.6e1 1.9e1	10 1.4e4 7.7e3 1.4e4 1.4e4	10 90e+0 70e+0 11e+1 8.9e4
1	15 1.7e3 1.2e3 2.2e3 1.7e3	15 1.5e4 1.0e4 1.9e4 1.5e4	1 2 3.6e5 1.8e5 >7e5 4.6e4	.
le-1	6 1.1e5 7.5e4 1.8e5 4.5e4	0 37e-2 24e-2 53e-2 7.1e4	1 0 39e-1 82e-2 69e-1 2.2e4	.
le-3	0 11e-2 44e-3 29e-2 2.5e4	.	.	.
le-5
le-8

Table 4: Shown are, for a given target difference to the optimal function value Δf : the number of successful trials (#); the expected running time to surpass $f_{\text{opt}} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value (RT_{succ}). If $f_{\text{opt}} + \Delta f$ was never reached, figures in

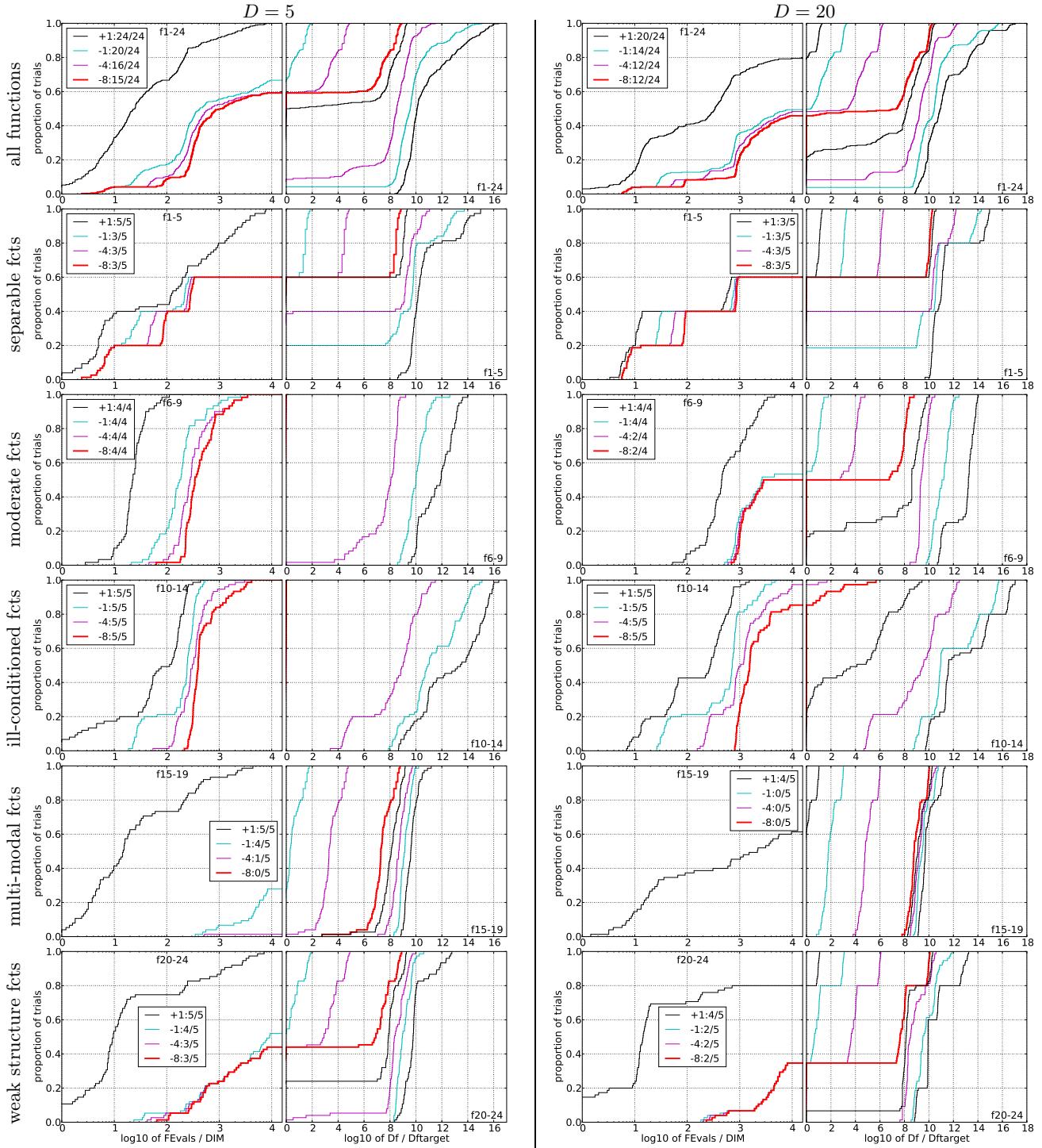


Figure 2: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus Δf (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D , to fall below $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10D, 100D \dots$ function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: separable functions; third row: misc. moderate functions; fourth row: ill-conditioned functions; fifth row: multi-modal functions with adequate structure; last row: multi-modal functions with weak structure. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and **DIM** denote search space dimension, and Δf and Df denote the difference to the optimal function value.

- [4] S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Presentation of the noiseless functions. Technical Report 2009/20, Research Center PPE, 2009.
- [5] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2009: Experimental setup. Technical Report RR-6828, INRIA, 2009.
- [6] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noiseless functions definitions. Technical Report RR-6829, INRIA, 2009.
- [7] N. Hansen and A. Ostermeier. Completely derandomized self-adaptation in evolution strategies. *Evolutionary Computation*, 9(2):159–195, 2001.
- [8] C. Igel, T. Suttorp, and N. Hansen. A computational efficient covariance matrix update and a (1+1)-CMA for Evolution Strategies. In *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2006)*, pages 453–460. ACM Press, 2006.
- [9] S. Kern, S. Müller, N. Hansen, D. Büche, J. Ocenasek, and P. Koumoutsakos. Learning Probability Distributions in Continuous Evolutionary Algorithms - A Comparative Review. *Natural Computing*, 3(1):77–112, 2004.
- [10] I. Rechenberg. *Evolutionsstrategie*. Friedrich Frommann Verlag (Günther Holzboog KG), Stuttgart, 1973.
- [11] M. Schumer and K. Steiglitz. Adaptive step size random search. *Automatic Control, IEEE Transactions on*, 13:270–276, 1968.