

Inverse EEG source problems and approximation

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Inverse EEG (electroencephalography) Problem:

From measurements by electrodes of the electric potential u on the scalp, recover a distribution of m pointwise dipolar current sources C_k with moments p_k located in the brain (modeling the presence of epileptic foci).

Model: The head Ω is modeled as a set of 3 spherical or ellipsoidal nested regions $\Omega_i \subset \mathbb{R}^3$, $i = 0, 1, 2$ (brain, skull, scalp), separated by interfaces S_i (with $S_2 = \partial\Omega$) and with piecewise constant conductivity σ , $\sigma|_{\Omega_i} = \sigma_i > 0$.

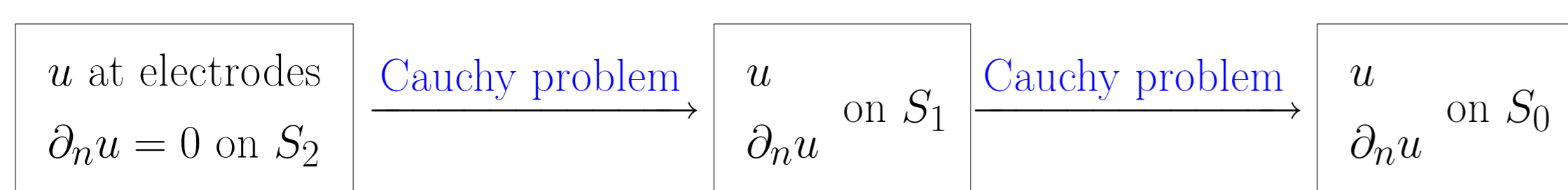


$$\text{Macroscopic model + quasi-static approximation of Maxwell equations} \rightarrow \text{Spatial behavior of } u \text{ in } \Omega [1] : (P) \begin{cases} \operatorname{div}(\sigma \nabla u) = \sum_{k=1}^m p_k \cdot \nabla \delta_{C_k} & \text{in } \Omega \\ u = g \text{ and } \partial_n u = \phi & \text{on } \partial\Omega \end{cases}$$

where g and ϕ denote the given potential and current flux on the scalp (or approximate interpolation of these quantities).

The resolution of this inverse problem can be divided into 3 main steps:

1. Data propagation (Cortical mapping step):



Since $C_k \in \Omega_0$, the function u is harmonic in the outer layers Ω_1 and Ω_2 , where boundary conditions are given by the continuity relations

$$[u]_i = [\sigma \partial_n u]_i = 0 \text{ on } S_i,$$

$[]_i$ denotes the jump across the surface S_i .

Based on these boundary conditions, data propagation can be achieved by using boundary element methods [2], or by using robust harmonic approximation techniques and expansions on appropriate basis [3].

2. Anti-harmonic projection (Signal space separation):

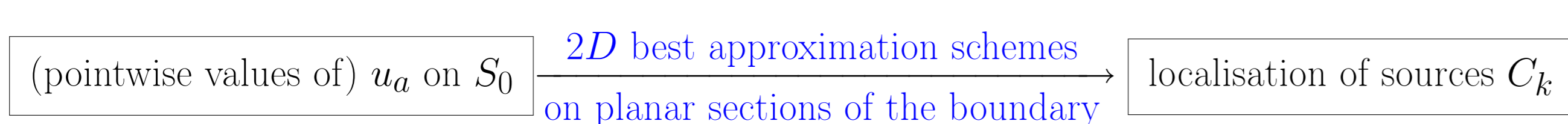
From these data on S_0 , the solution u to equation (P) in Ω_0 :

$$\begin{cases} \Delta u = \frac{1}{\sigma_0} \sum_{k=1}^m p_k \cdot \nabla \delta_{C_k} & \text{in } \Omega_0 \\ u \text{ and } \partial_n u & \text{given on } S_0 \end{cases}$$

assumes the form: $u(x) = h(x) + \sum_{k=1}^m \frac{\langle p_k, x - C_k \rangle}{4\pi \|x - C_k\|^3} = h(x) + u_a(x)$, h harmonic function in Ω_0 .

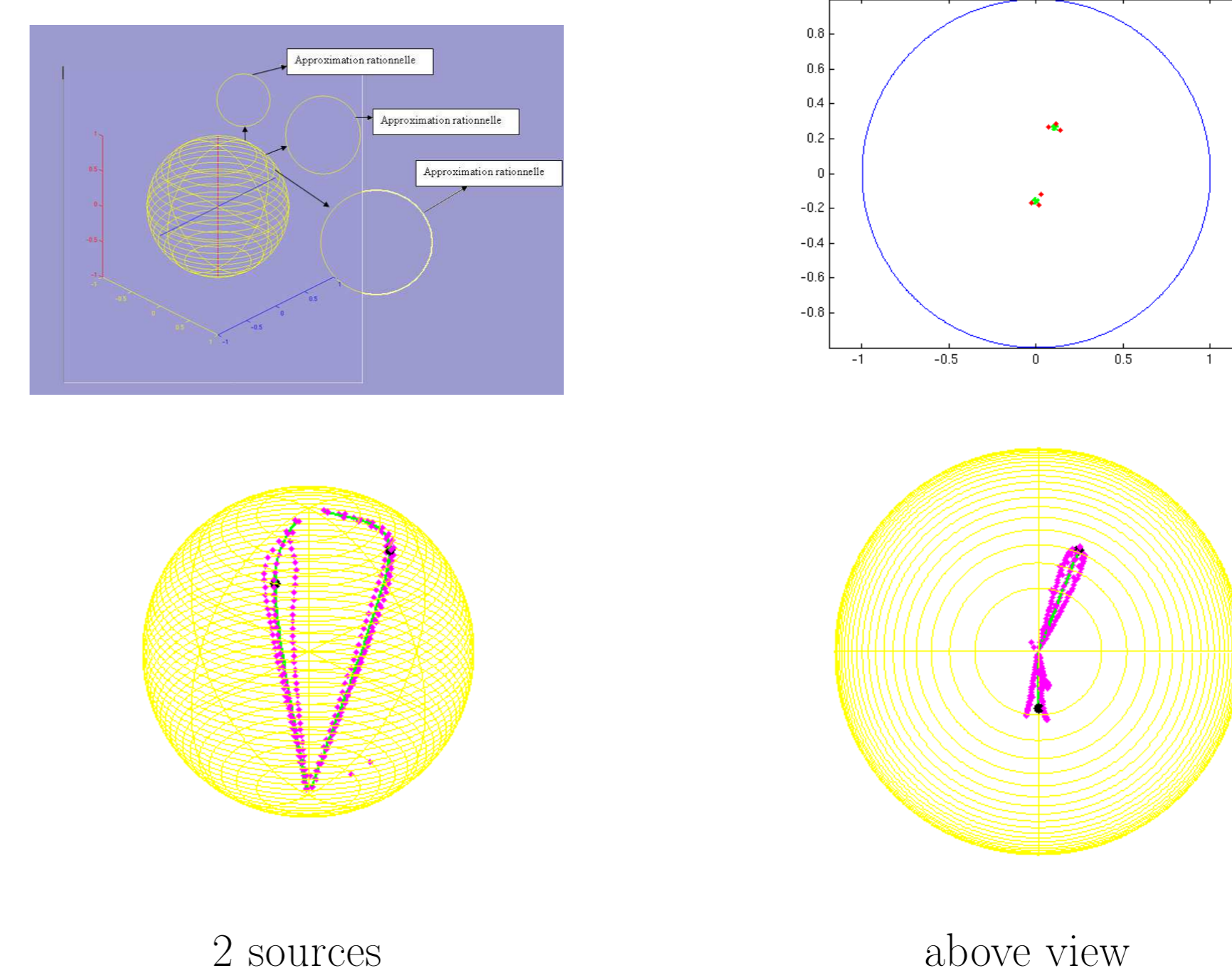
In order to recover the C_k inside Ω_0 , the knowledge of the singular function u_a is required on S_0 . This can be deduced from available boundary data by expanding u on bases of spherical or ellipsoidal harmonics [3, 4].

3. Best rational approximation on planar sections (Source localization):



- Slice Ω_0 along a family of planes Π_p : $\Pi_p \cap S_0 = \Gamma_p$ (circles or ellipses).
- From pointwise values of the singular part u_a on Γ_p , approximate $f_p = (P_- u_a)^2$ the square of the anti-analytic projection on Γ_p [3, 5]. Ellipses may then preliminary be mapped by a conformal rational transformation onto circles.
- f_p is a meromorphic function, analytic outside $D_p = \Pi_p \cap \Omega_0$ with singularities ζ_k, p inside D_p which are strongly and explicitly linked with the sources C_k .
- Approximate the ζ_k, p by $\tilde{\zeta}_k, p$ the poles of the best L^2 or L^∞ rational approximation to f_p on Γ_p (degree $\geq m$ for a sphere, $2m$ for an ellipsoid), the poles $\tilde{\zeta}_k, p$ accumulate to the singularities ζ_k, p , [5].
- Varying p , this allows us to approximately locate the m sources C_k in Ω_0 .

Illustrations (Spherical model):



The error estimate is around 10^{-2} .

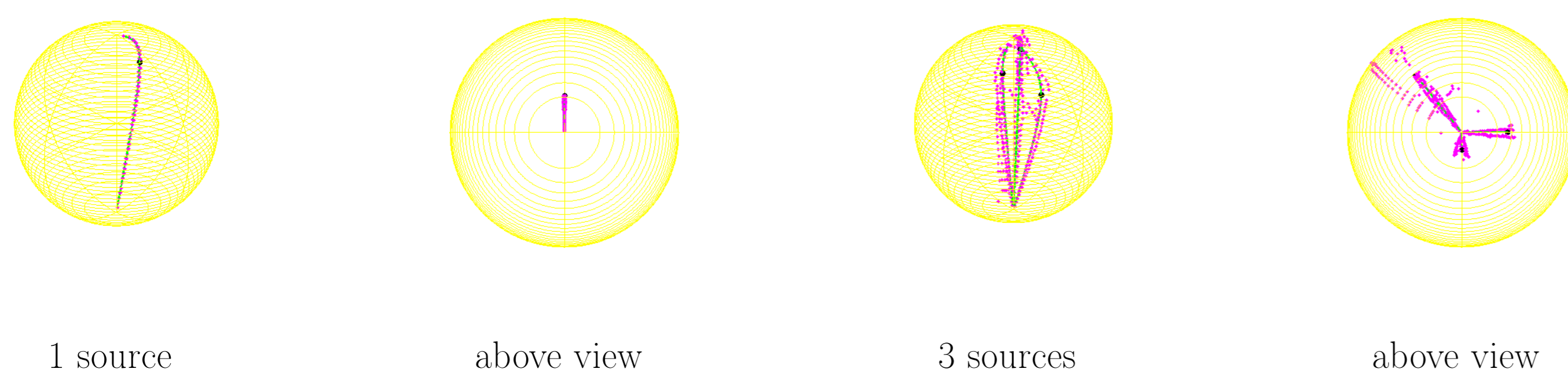
Numerical results:

Spherical model:

In this case the relation between $\zeta_{k,p}$ and C_k when the slice p varies is the following:

$-(\zeta_k, p)$ are aligned together and also with the complex coordinates ζ_k of C_k .

$|\zeta_k, p|$ is maximum at $\zeta_k, p = \zeta_k$ (the k^{th} source's section).



The error estimate is around 10^{-4} for the 1 source case and around 10^{-2} for the 3 sources case.

Ellipsoidal model:

In each slice p we construct a real valued polynomial $q_{k,p}(\zeta)$ whose coefficients are related to the conformal rational transformation from ellipse to circle, $q_{k,p}(\zeta)$ is minimum for $\zeta = \zeta_k$, [3].

These numerical results represent exact sources \otimes and approximated ones \diamond .

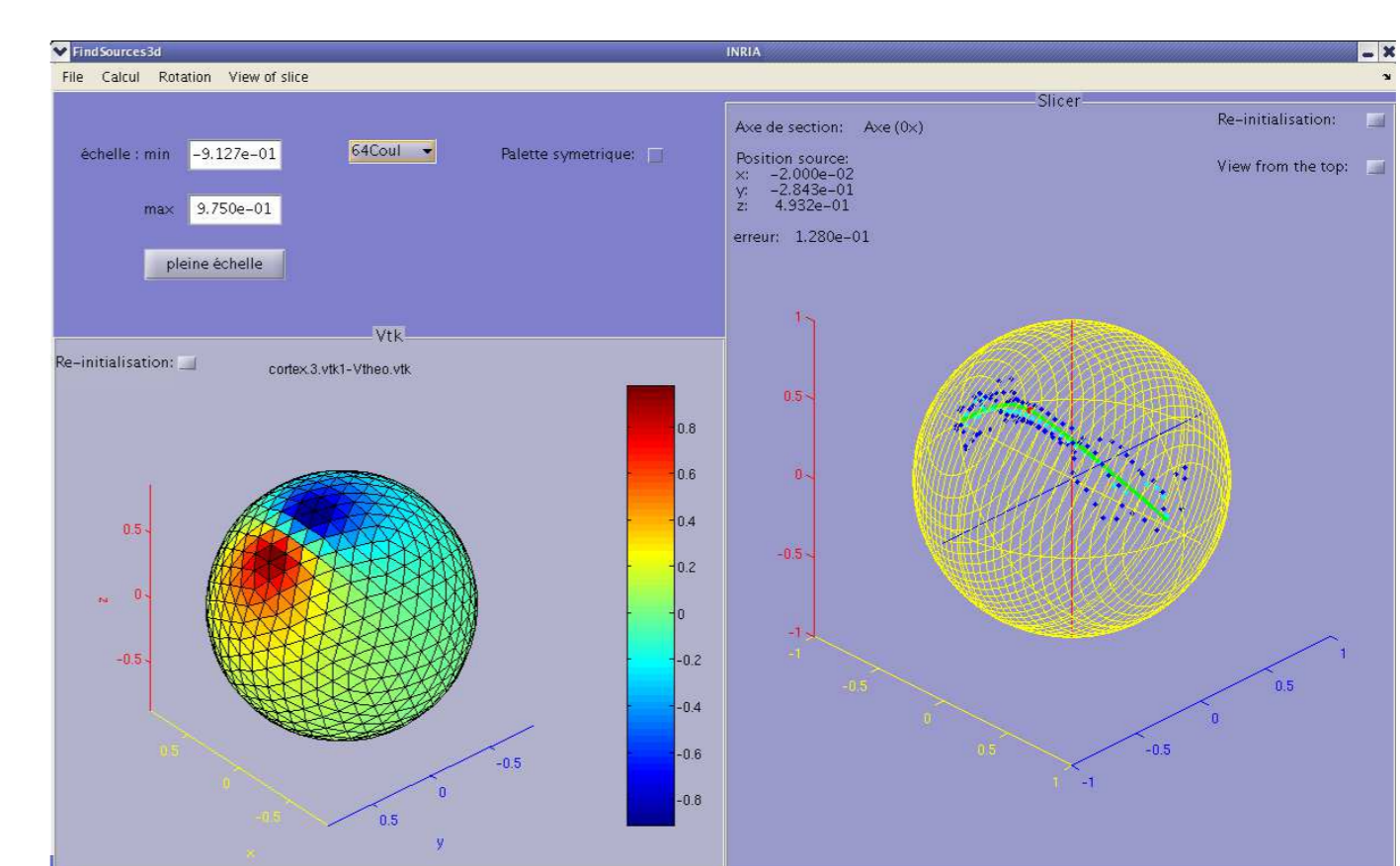


The error estimate is around 10^{-4} for the 1 source case and around 10^{-1} for the 2 sources case.

Perspectives:

- Recover the moments p_k , improve sources estimation (slicing along different directions).
- Software development (FindSources3D).
- More realistic geometries.
- The inverse MEG source problem.

FindSources3D :



(view of u on S_2 then of C_1 into Ω_0)

References:

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