



HAL
open science

Grooming

Jean-Claude Bermond, David Coudert

► **To cite this version:**

Jean-Claude Bermond, David Coudert. Grooming. Charles J. Colbourn and Jeffrey H. Dinitz. Handbook of Combinatorial Designs (2nd edition), 42, Chapman & Hall- CRC Press, pp.494-496, 2006, Discrete mathematics and Applications, 1584885068. inria-00429215

HAL Id: inria-00429215

<https://inria.hal.science/inria-00429215>

Submitted on 1 Nov 2009

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

The CRC Handbook of Combinatorial Designs

Edited by

Charles J. Colbourn

*Department of Computer Science and Engineering
Arizona State University*

Jeffrey H. Dinitz

*Department of Mathematics and Statistics
University of Vermont*

AUTHOR PREPARATION VERSION

27 February 2006

1 Grooming

JEAN-CLAUDE BERMOND AND DAVID COUDERT

1.1 Definitions and Examples

- 1.1 Remark** Traffic grooming in networks refers to group low rate traffic into higher speed streams (containers) so as to minimize the equipment cost [11, 7, 13, 12, 8, 9]. There are many variants according to the type of network considered, the constraints used and the parameters one wants to optimize which give rise to a lot of interesting design problems (graph decompositions).

To fix ideas, suppose that we have an optical network represented by a directed graph G (in many cases a symmetric one) on n vertices, for example a unidirectional ring \vec{C}_n or a bidirectional ring C_n^* . We are given also a traffic matrix, that is a family of connection requests represented by a multi-digraph I (the number of arcs from i to j corresponding to the number of requests from i to j). An interesting case is when there is exactly one request from i to j ; then $I = K_n^*$. Satisfying a request from i to j consists in finding a route (dipath) in G and assigning it a wavelength. The grooming factor, g , means that a request uses only $1/g$ of the bandwidth available on a wavelength along its route. Said otherwise, for each arc e of G and for each wavelength w , there are at most g dipaths with wavelength w which contain the arc e .

During the 90's, a lot of research has concentrated in minimizing the number of wavelengths used in the network. In the mean time, the bandwidth of each wavelength (> 10 Gbit/s), the number of wavelengths per fiber (> 100) and the number of fibers per cable (> 100) have exploded, thus reporting the operational cost of a wavelength into the terminal equipment cost: filters, optical cross-connect, add/drop multiplexer (ADM),.... For example, in a node of an optical network we place an ADM only for those wavelengths carrying a request which has to be added or dropped in this node. So nowadays the objective is to minimize the total number of ADMs in the network, which is a challenging issue.

- 1.2 Definition** [5] Grooming problem: Given a digraph G (network), a digraph I (set of requests) and a grooming factor g , find for each arc $r \in I$ a path $P(r)$ in G , and a partition of the arcs of I into subgraphs I_w , $1 \leq w \leq W$, such that $\forall e \in E(G)$, $\text{load}(I_w, e) = |\{P(r); r \in E(I_w); e \in P(r)\}| \leq g$. The objective is to minimize $\sum_{w=1}^W |V(I_w)|$, and this minimum is denoted by $A(G, I, g)$.
- 1.3 Definition** TT_n is a transitive tournament on n vertices, that is the digraph with arcs $\{(i, j) \mid 1 \leq i < j \leq n\}$. We denote $\{a, b, c\}$ the TT_3 with arcs $\{a, b\}$, $\{b, c\}$, and $\{a, c\}$.
- 1.4 Remark** When $G = P_n^*$, the shortest path from i to j is unique, and we can split the requests into two classes, those with $i < j$ and those with $i > j$. Therefore the grooming problem for P_n^* can be reduced to two distinct problems on \vec{P}_n . In particular we have $A(P_n^*, K_n^*, g) = 2A(\vec{P}_n, TT_n, g)$.
- 1.5 Example** $A(\vec{P}_7, TT_7, 2) = 20$, and the partition consists of 6 subgraphs, the 5 TT_5 $\{2,4,5\}$, $\{3,4,6\}$, $\{1,5,6\}$, $\{2,6,7\}$, and $\{1,4,7\}$, plus the union of two TT_3 $\{1,2,3\} + \{3,5,7\}$.

- 1.6 Theorem** [1] When n is odd, $A(\vec{P}_n, TT_n, 2) = \lceil (11n^2 - 8n - 3)/24 \rceil$; When n is even, $A(\vec{P}_n, K_n, 2) = (11n^2 - 4n)/24 + \varepsilon(n)$, where $\varepsilon(n) = 1/2$ when $n \equiv 2, 6 \pmod{12}$, $\varepsilon(n) = 1/3$ when $n \equiv 4 \pmod{12}$, $\varepsilon(n) = 5/6$ when $n \equiv 10 \pmod{12}$, and $\varepsilon(n) = 0$ when $n \equiv 0, 8 \pmod{12}$.
- 1.7 Remark** In a unidirectional cycle \vec{C}_n , the path from i to j is unique. Wlog we can assign the same wavelength to the two requests (i, j) and (j, i) , then the two associated paths contain each arc of \vec{C}_n . Therefore the load condition becomes $|E(I_w)| \leq g$ and the grooming problem becomes:
- 1.8 Definition** [5] Grooming problem for $G = \vec{C}_n$: given n and g , find a partition of I into subgraphs B_w , $1 \leq w \leq W$, such that $|E(B_w)| \leq g$, which minimizes $\sum_{w=1}^W |V(B_w)|$. The minimum value is $A(\vec{C}_n, I, g)$.
- 1.9 Remark** The partition of Definition 1.2 is obtained by associating to each B_w of the partition of Definition 1.8 its symmetric digraph B_w^* and letting $I_w = B_w^*$.
- 1.10 Example** $A(\vec{C}_4, K_4^*, 3) = 7$, using a partition of K_4 consisting of the $K_3 \{1, 2, 3\}$ and the $K_{1,3}$ with edges $\{1, 4\}$, $\{2, 4\}$, and $\{3, 4\}$. $A(\vec{C}_7, K_7^*, 3) = 21$ using a $(7, 3, 1)$ design (steiner triple system) and $A(\vec{C}_{13}, K_{13}^*, 6) = 52$ using a $(13, 4, 1)$ design.
- 1.11 Theorem** [2] $A(\vec{C}_n, K_n^*, 3) = n(n-1)/2 + \varepsilon_3(n)$, where $\varepsilon_3(n) = 0$ when $n \equiv 1, 3 \pmod{6}$, $\varepsilon_3(n) = 2$ when $n \equiv 5 \pmod{6}$, $\varepsilon_3(n) = \lceil n/4 \rceil + 1$ when $n \equiv 8 \pmod{12}$, and $\varepsilon_3(n) = \lceil n/4 \rceil$ otherwise.
- 1.12 Theorem** [10] $A(\vec{C}_n, K_n^*, 4) = n(n-1)/2$.
- 1.13 Theorem** [4] $A(\vec{C}_n, K_n^*, 5) = 4 \lfloor n(n-1)/10 \rfloor + \varepsilon_5(n)$, where $\varepsilon_5(n) = 0$ when $n \equiv 0, 1 \pmod{5}$, $n \neq 5$, $\varepsilon_5(5) = 1$, $\varepsilon_5(n) = 2$ when $n \equiv 2, 4 \pmod{5}$, $n \neq 7$, $\varepsilon_5(7) = 3$, $\varepsilon_5(n) = 3$ when $n \equiv 3 \pmod{5}$, $n \neq 8$, and $\varepsilon_5(8) = 4$.
- 1.14 Theorem** [3]
 When $n \equiv 0 \pmod{3}$, then $A(\vec{C}_n, K_n^*, 6) = \lceil n(3n-1)/9 \rceil + \varepsilon_6(n)$, where $\varepsilon_6(n) = 1$ when $n \equiv 18, 27 \pmod{36}$, and $\varepsilon_6(n) = 0$ otherwise, except for $n \in \{9, 12\}$ and some possible exceptions when $n \leq 2580$.
 When $n \equiv 1 \pmod{3}$, $A(\vec{C}_n, K_n^*, 6) = \lceil n(n-1)/3 \rceil + \varepsilon_6(n)$, where $\varepsilon_6(n) = 2$ when $n \equiv 7, 10 \pmod{12}$, and 0 otherwise, except for $A(\vec{C}_7, K_7^*, 6) = 17$, $A(\vec{C}_{10}, K_{10}^*, 6) = 34$, and $A(\vec{C}_{19}, K_{19}^*, 6) = 119$.
 When $n \equiv 2 \pmod{3}$, then $A(\vec{C}_n, K_n^*, 6) = (n^2 + 2)/3$, except possibly for $n = 17$.
- 1.15 Remark** In another grooming problem (see [6]), the requests can be routed via different pipes. Each pipe contains at most g requests, and the objective is to minimize the total number of pipes (as equipments are placed only at the terminal nodes of the pipe). Thus, given a digraph I (requests) and a grooming factor g , the problem consists in finding a virtual multi-digraph H and, for each arc $r \in I$, a path $P(r)$ in H such that $\forall e \in E(H)$, $\text{load}(I, e) \leq g$. The objective is to minimize $|E(H)|$, and the minimum is denoted by $T(I, g)$.
- 1.16 Example** When $I = K_4^*$ and $C = 2$, then $H = C_4^*$. Requests $(i, i+1)$ (resp. $(i, i-1)$) are routed via arc $(i, i+1)$ (resp. $(i, i-1)$), requests $(1, 3)$ and $(3, 1)$ are routed clockwise, and $(2, 4)$ and $(4, 2)$ counterclockwise.
- 1.17 Remark** For $C = 2$ the problem can be reduced to a partition of K_n^* or $(K_n - e)^*$ in TT_3 (See Directed Design or Mendelsohn's Designs). For $C = 3$ the result follows

from the existence of a $PBD(n, \{3, 4, 5\})$ for $n \neq 6, 8$ (see chapter PBD).

1.18 Theorem [6] $T(K_n^*, 2) = \lceil 2n(n-1)/3 \rceil$ and $T(K_n^*, 3) = n(n-1)/2$.

1.2 See Also

§???	Directed designs.
§???	Graph decompositions
§???	Mendelsohn designs

References

- [1] J.-C. BERMOND, L. BRAUD, AND D. COUDERT, *Traffic grooming on the path*, in SIROCCO, vol. 3499 of LNCS, 2005, pp. 34–48. [cited on pages]
- [2] J.-C. BERMOND AND S. CEROI, *Minimizing SONET ADMs in unidirectional WDM ring with grooming ratio 3*, Networks, 41 (2003), pp. 83–86. [cited on pages]
- [3] J.-C. BERMOND, C. COLBOURN, D. COUDERT, G. GE, A. LING, AND X. MUÑOZ, *Traffic grooming in unidirectional WDM rings with grooming ratio $C = 6$* , SIAM Journal on Discrete Mathematics, 19 (2005), pp. 523–542. [cited on pages]
- [4] J.-C. BERMOND, C. COLBOURN, A. LING, AND M.-L. YU, *Grooming in unidirectional rings : $k_4 - e$ designs*, Discrete Mathematics, Lindner’s Volume, 284 (2004), pp. 57–62. [cited on pages]
- [5] J.-C. BERMOND, D. COUDERT, AND X. MUÑOZ, *Traffic grooming in unidirectional WDM ring networks: The all-to-all unitary case*, in IFIP ONDM, 2003, pp. 1135–1153. [cited on pages]
- [6] J.-C. BERMOND, O. DERIVOYRE, S. PÉRENNES, AND M. SYSKA, *Groupage par tubes*, in Conference ALGOTEL, May 2003, pp. 169–174. [cited on pages]
- [7] R. DUTTA AND N. ROUSKAS, *Traffic grooming in WDM networks: Past and future*, IEEE Network, 16 (2002), pp. 46–56. [cited on pages]
- [8] M. FLAMMINI, L. MOSCARDELLI, M. SHALOM, AND S. ZAKS, *Approximating the traffic grooming problem*, in ISAAC, vol. 3827 of LNCS, 2005, pp. 915–924. [cited on pages]
- [9] O. GOLDSCHMIDT, D. HOCHBAUM, A. LEVIN, AND E. OLINICK, *The SONET edge-partition problem*, Networks, 41 (2003), pp. 13–23. [cited on pages]
- [10] J. HU, *Optimal traffic grooming for wavelength-division-multiplexing rings with all-to-all uniform traffic*, OSA Journal of Optical Networks, 1 (2002), pp. 32–42. [cited on pages]
- [11] E. MODIANO AND P. LIN, *Traffic grooming in WDM networks*, IEEE Communications Magazine, 39 (2001), pp. 124–129. [cited on pages]
- [12] P.-J. WAN, G. CALINESCU, L. LIU, AND O. FRIEDER, *Grooming of arbitrary traffic in SONET/WDM BLSRs*, IEEE Journal of Selected Areas in Communications, 18 (2000), pp. 1995–2003. [cited on pages]
- [13] K. ZHU AND B. MUKHERJEE, *A review of traffic grooming in WDM optical networks: Architectures and challenges*, Optical Networks Magazine, 4 (2003), pp. 55–64. [cited on pages]