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OTIS-Based Multi-Hop Multi-OPS Lightwave Networks

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Abstract. Advances in optical technology, such as low loss Optical Passive Star couplers (OPS) and the possibility of building tunable optical transmitters and receivers have increased the interest for multiprocessor architectures based on lightwave networks because of the vast bandwidth available. Many research have been done at both technological and theoretical level. An essential effort has to be done in linking those results. In this paper we propose optical designs for two multi-OPS networks: the single-hop POPS network and the multi-hop stack-Kautz network; using the Optical Transpose Interconnecting System (OTIS) architecture, from the Optoelectronic Computing Group of UCSD. In order to achieve our result, we also provide the optical design of a generalization of the Kautz digraph, using OTIS.

1 Introduction

The advances in optical technology, such as micro-lenses, low energy loss optical passive star couplers (OPS) [14, 20], optical transmission lines allowing more than 1 G-bits per second with less power consumption than electrical wires [12] as well as tunable optical transmitters and receivers [8, 21, 23], have increased the interest for optical interconnection networks for multiprocessor systems [4] because of their large bandwidth.

Our work focuses on multi-OPS optical network topologies and design. Although many results exist in the literature at both technological and theoretical level, an essential effort has to be done in linking those results together. In this paper, we study the optical design of several graph-theoretical topologies using the Optical Transpose Interconnecting System (OTIS) architecture, from the Optoelectronic Computing Group of UCSD [19].

Optical systems are usually divided in two classes, according to the number of intermediate processors a message has to visit before delivery. In a *single-hop* network, the nodes communicate with each other in only one step [20]. Such topologies require either a large number of transceivers per node, or rapidly tunable transmitters and receivers. In a *multi-hop* topology [21], there is no direct path between all pairs of nodes: a communication should use intermediate nodes to reach the destination. This allows the use of a small number of statically tuned transmitters and receivers, but on the other hand the processing of the

information by the intermediate nodes causes a loss of speed. Moreover, OPS-based networks can be further classified according to the number of optical couplers used (See p. 209–233 of [4]), being single-OPS [8, 21] or multi-OPS [7, 9]. A great deal of research effort have been concentrated on single-hop *single-OPS* topologies [10, 22, 25]. However, *multi-OPS* networks seem more viable and cost-effective under current optical technology [9, 11].

Optical networks topologies can be studied using graph-theoretical tools, as graphs and digraphs play an important role in the analysis and synthesis of *point-to-point* networks [2, 24]. With respect to *one-to-many* networks, where messages sent by the processors can be broadcast to all outputs of the OPS couplers, they can be modeled by hypergraphs, a generalization of graphs, where edges may connect more than two nodes [1]. In this paper, we address implementation issues of three optical interconnection networks:

- Graphs of Imase and Itoh, a generalization of Kautz graphs allowing point-to-point communications [15].
- The single-hop multi-OPS *POPS* network, which allows one-to-many communications at every communication step [9].
- The multi-hop multi-OPS *stack-Kautz* network, which also allows one-to-many communications at every communication step [11].

Graphs of Imase and Itoh are very interesting for the realization of an interconnection network due to their large number of nodes, N, for small constant degree, d, and low diameter, $\lceil \log_d N \rceil$. The POPS network is based on complete graphs and on the concept of *groups* of processors. The stack-Kautz is based on the Kautz graphs [18] and on the stack-graphs, a useful model to manipulate multi-OPS networks [7].

This paper is organized as follows. We start by recalling, in Sec. 2, the results from the literature upon which we constructed ours. In particular, we present the Optical Transpose Interconnecting System (OTIS) architecture from [19], the optical passive star, and the single-hop multi-OPS Partitioned Optical Passive Star network (POPS) network from [9]. We also recall the stack-graphs from [7], the definition of the Kautz graph from [18], generalized with the graph of Imase and Itoh from [15], and close presenting the multi-hop multi-OPS stack-Kautz network from [11]. In Section 3, we propose the implementation of some building blocks using the OTIS architecture: first, the optical interconnections between a group of processors and its corresponding OPS couplers, and then an optical interconnection network having the topology of a graph of Imase and Itoh. Finally, Section 4 presents the application of these building blocks to the optical design of the POPS network and of the stack-Kautz network. We close the paper with some concluding remarks and directions for further research.

2 Preliminaries

In this section, we recall the basic features of the OTIS architecture and OPS couplers. We also show the model proposed in [7] for studying multi-OPS networks with stack-graphs. Then, we exemplify the use of this model on the POPS

network. Finally, we recall the definition of the Kautz graph, the Imase and Itoh's graph and of the stack-Kautz.

2.1 OTIS

The **Optical Transpose Interconnection System** (OTIS) architecture, was first proposed in [19]. $OTIS(G,T)$ is an optical system which allows point-to-point (1-to-1) communications from G groups of size T onto T groups of size G. This architecture connects the transmitter of processor (i,j), $0 \leq i \leq G-1$, $0 \leq j \leq T-1$, to the receiver of processor $(T-1-j, G-1-i)$.

Optical interconnections in the OTIS architecture are realized with two planes of lenses [5] in a free optical space as shown in Fig. 1.

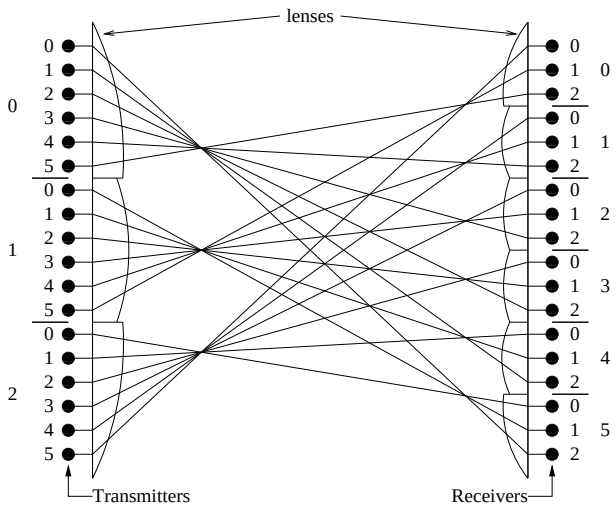

Fig. 1. $OTIS(3,6)$.

The OTIS architecture was used in [24] to realize interconnections networks such as hypercubes, 4-D meshes, mesh-of-trees and butterfly for multi-processors systems. It was shown that, in such electronic interconnection networks (in which connections are realized with electronic wires), a set of wires can be replaced with pairs of transmitters and receivers connected using the OTIS architecture. This is interesting in terms of speed, power consumption [12] and space reduction.

2.2 Optical passive stars

An **optical passive star coupler** is a single-hop one-to-many optical transmission device. An OPS(s,z) has s inputs and z outputs. In the case where s equals z, the OPS is said to be of degree s (see Fig. 2, left). When one of the input

processors sends a message through an OPS coupler, the s output processors have access to it. An OPS coupler is a **passive** optical system, i.e. it requires no power source. It is composed of an optical multiplexer followed by an optical fiber or a free optical space and a beam-splitter that divides the incoming light signal into s equal signals of a s-th of the incoming optical power. Note that only one optical beam has to be guided through the network (see Fig. 2, right). A practical realization of an OPS coupler using a hologram [14] at the outputs, is described in [6]. Throughout this paper, we will deal with **single-wavelength** OPS couplers of degree s. Consequently, only one processor can send an optical signal through it per time step.

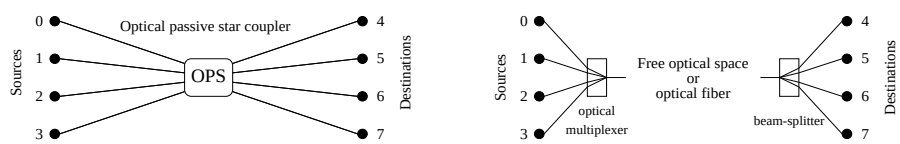

Fig. 2. A degree 4 optical passive star coupler.

2.3 Stack-graphs

We saw in the introduction that one-to-many networks (e.g., OPS based networks) are better modeled by hypergraphs. Let us define a special kind of directed hypergraphs, called *stack-graphs*, which have the property of being very easy to deal with. Informally, they can be obtained by piling up copies of a directed graph and subsequently viewing each stack of arcs as a hyperarc [7]. A formal definition is as follows.

Definition 1. *[7] Let $G = (V, A)$ be a directed graph. The* **stack-graph** $\varsigma(s, G) = (V_\varsigma, A_\varsigma)$ *is as follows, s being called* **stacking-factor** *of the stack-graph.*

1. *The set of nodes V_ς of $\varsigma(s, G)$ is $V_\varsigma = \{0, \cdots, s-1\} \times V$, $s \geq 1$.*
2. *Let π be the projection function defined from V_ς onto V such that $\pi((i, v)) = v$, for $0 \leq i < s$ and $v \in V$.*
3. *The set of hyperarcs A_ς of $\varsigma(s, G)$ is then $A_\varsigma \stackrel{def}{=} \{a_\varsigma = (\pi^{-1}(u), \pi^{-1}(v)) | (u, v) \in A\}$.*

Therefore, an OPS coupler of degree s can be modeled by a hyperarc linking two sets of s nodes, meaning that the nodes of one can only send messages through the hyperarc while the other set can only receive messages through the same hyperarc. Figure 3 shows an OPS coupler modeled by a hyperarc.

2.4 POPS

The **Partitioned Optical Passive Star network** $POPS(t, g)$, introduced in [9], is composed of $N = tg$ processors and g^2 OPS couplers of degree t. The

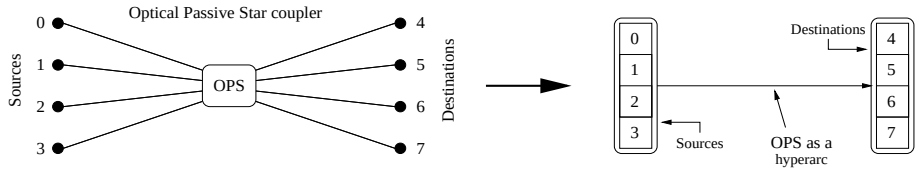

Fig. 3. Modeling an OPS by a hyperarc.

processors are divided into g groups of size t (see Fig. 4). Each OPS coupler is labeled by a pair of integers (i, j), $0 \leq i, j < g$. The input of the OPS (i, j) is connected to the i-th group of processors, and the output to the j-th group of processors. The POPS is a single-hop multi-OPS network.

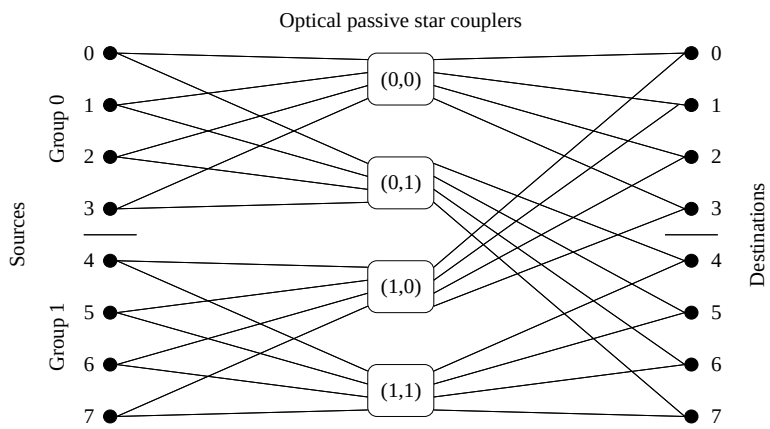

Fig. 4. Partitioned Optical Passive Star POPS(4,2) with 8 nodes.

Since an OPS coupler is modeled as a hyperarc, the POPS network $POPS(d, g)$ can be modeled as a stack-K_g^+ (or $\varsigma(d, K_g^+)$, for short) of stacking-factor d, where K_g^+ is the complete digraph with loops[1] and with g nodes and g^2 arcs (see Fig. 5), as proposed in [3].

2.5 Kautz graph

The Kautz graph was first defined in [18] as follows.

Definition 2. *[18] The* **directed Kautz graph** $KG(d, k)$ *of degree d and diameter k is the digraph defined as follows (see Fig. 6).*

1. *A vertex is labeled with a word of length k, $(x_1, \cdots, x_k)$, on the alphabet $\Sigma = \{0, \cdots, d\}$, $|\Sigma| = d + 1$, in which $x_i \neq x_{i+1}$, for $1 \leq i \leq k - 1$.*

[1] A loop is an arc from a node to itself.

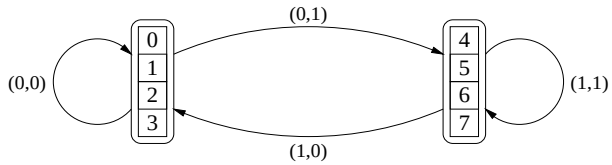

Fig. 5. $POPS(4,2)$ modeled as $\varsigma(4, K_2^+)$.

2. *There is an arc from a vertex* $x = (x_1, \cdots, x_k)$ *to all vertices* y *such that* $y = (x_2, \cdots, x_k, z)$, $z \in \Sigma$, $z \neq x_k$.

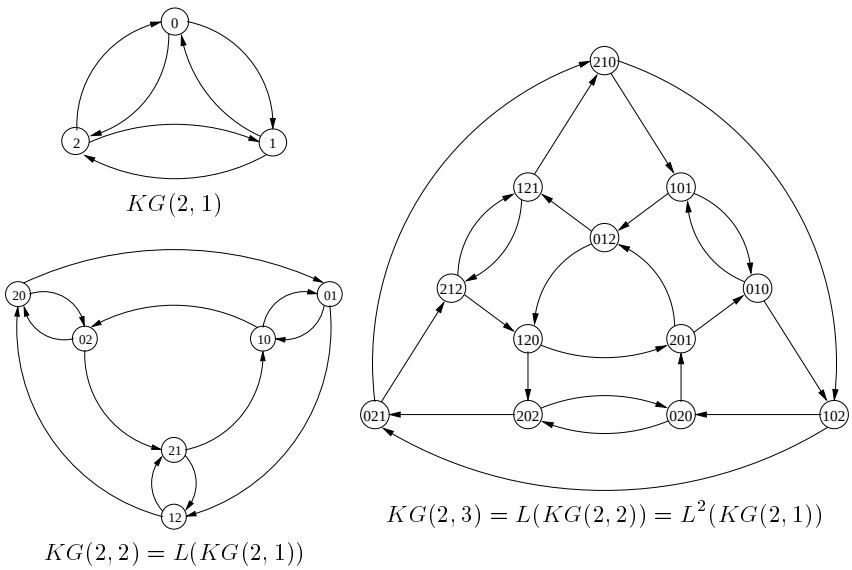

Fig. 6. Three line digraph iterations of the Kautz graph $KG(d,k)$.

Another definition of direct Kautz graph, in terms of *line digraph iteration*, was proposed in [13]. They show that $KG(d,1)$ is the complete digraph K_{d+1} and that $KG(d,k) = L^{k-1}(K_{d+1})$, where $L(G)$ is the line digraph of a digraph G. Figure 6 shows three of these iterations.

The Kautz graph $KG(d,k)$ has $N = d^{k-1}(d+1)$ nodes, constant degree d and diameter k ($k \leq \log_d N$). It is both Eulerian and Hamiltonian and optimal with respect to the number of nodes if $d > 2$ [18]. As an example, $KG(5,4)$ has $N = 3750$ nodes, degree 5 and diameter 4.

Notice that routing on the Kautz graph is very simple, since a shortest path routing algorithm (every path is of length at most k) is induced by the label of

the nodes. It can be extended to generate a path of length at most $k+2$ which survives $d-1$ link or node faults [17].

2.6 Imase and Itoh

Kautz graphs are very interesting in terms of number of nodes for fixed degrees and diameters. However, its definition does not yield graphs of any size. Graphs of Imase and Itoh have thus been introduced in [15] as a Kautz graph generalization in order to obtain graphs of every size.

Imase and Itoh's graphs are defined using a congruence relation, as follows.

Definition 3. *[15] The* **graph of Imase and Itoh** $II(d,n)$*, of degree* d *with* n *nodes, is the directed graph in which:*

1. *Nodes are integers modulo* n*.*
2. *There is an arc from the node* u *to all nodes* v *such that* $v \equiv (-du-\alpha) \bmod n$*,* $1 \leq \alpha \leq d$*.*

Figure 10, left, shows an example of Imase and Itoh's graph with $II(3,12)$. It is drawn such that the left nodes and the right nodes represents the same nodes. As $II(3,12)$ is a directed graph, the left nodes are connected to the outgoing arcs and the right nodes to the incoming arcs.

It has been shown in [15] that the diameter of $II(d,n)$ is $\lceil \log_d n \rceil$ and in [16] that the graph $II(d,n)$ is the Kautz graph $KG(d,k)$ when $n = d^{k-1}(d+1)$ (for example, in Fig. 10, left, $II(3,12)$ is $KG(3,2)$).

2.7 Stack-Kautz

We now dispose of a good model for multi-OPS networks (the stack-graphs) and also of a graph having good properties to build a multi-hop network (the Kautz graph). Hence, we can recall the multi-hop multi-OPS architecture based on the *stack-Kautz*.

In order to define the optical interconnection network called stack-Kautz, introduced in [11], we use the Kautz graph with loops $KG^+(d,k)$, where every node has a loop and hence degree $d+1$.

Thus, we can define the stack-Kautz graph as follows.

Definition 4. *[11] The* **stack-Kautz graph** $SK(s,d,k)$ *is the stack-graph of stacking-factor* s*, degree* $d+1$ *and diameter* k*,* $\varsigma(s, KG^+(d,k))$ *(see Fig. 7).*

The *stack-Kautz* network has the topology of the stack-Kautz graph $SK(s,d,k)$ and $N = sd^{k-1}(d+1)$ nodes. Each node is a processor labeled by a pair (x,y) where x is the label of the stack in $KG(d,k)$ and y is an integer $0 \leq y < s$, i.e., x is the label of a processor group and y is the label of a processor in this group. Since the stack-Kautz network inherits most of the properties of the Kautz graph, like shortest path routing, fault tolerance and others, in a previous work, we showed that it is a good candidate for the topology of an OPS-based lightwave network [11].

We note that the definition of stack-Kautz network can be trivially extended to the stack-Imase-Itoh network.

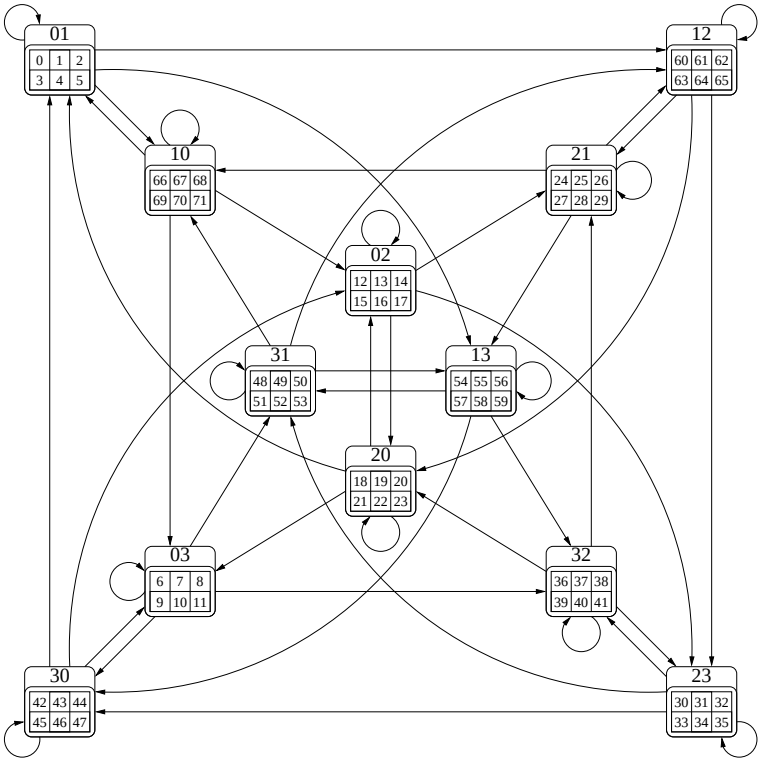

Fig. 7. Stack-Kautz network $SK(6,3,2)$.

3 Implementing some building blocks with OTIS

In this section, we explain how the OTIS architecture can be used both to connect a group of processors to the inputs of OPS couplers, and to connect the outputs of OPS couplers to a group of processors, and we show that the optical interconnections of an Imase and Itoh's based network can be simply realized using the OTIS architecture.

3.1 Creating groups of processors

We can realize the optical interconnections between the t processors of a group and the inputs of g OPS couplers (each processor being connected to the g OPS couplers), in a simple way using one $OTIS(t,g)$ architecture plus g optical multiplexers (input part of an OPS coupler). Figure 8 shows how to connect the transmitters of a group of 6 processors to 4 optical multiplexers, using $OTIS(6,4)$.

Analogously, we can also realize the optical interconnections between the outputs of g OPS couplers and the t processors of a group, using one $OTIS(g,t)$ ar-

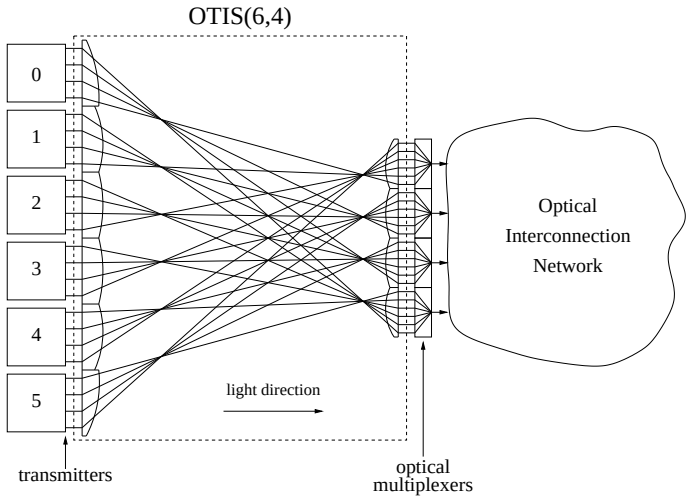

Fig. 8. Interconnections between a group of 6 processors and the input of 4 OPS couplers.

chitecture plus g beam-splitters. Figure 9 shows how to connect 3 beam-splitters to the receivers of a group of 5 processors, using $OTIS(3,5)$.

To build specific topologies, the output optical multiplexers are to be connected to its corresponding input beam-splitters, through the topology of the Optical Interconnection Network.

3.2 Optical implementation of the graph of Imase and Itoh

Proposition 1. *The optical interconnections of the graph of Imase and Itoh $II(d,n)$ of degree d with n nodes can be perfectly realized with the OTIS architecture $OTIS(d,n)$.*

Proof. The OTIS architecture $OTIS(d,n)$ has dn inputs (d groups of size n) and dn outputs (n groups of size d). The inputs $e=(i,j)$, $0 \le i < d$, $0 \le j < n$, are connected to the outputs $s=(n-j-1, d-i-1)$.

The graph of Imase and Itoh $II(d,n)$ of degree d with n nodes connect a node u to the nodes $v \equiv (-du-\alpha) \mod n$, $1 \le \alpha \le d$.

We associate d inputs of $OTIS(d,n)$ to each node of $II(d,n)$, such that the input $e=(i,j)$ is associated to the node $u = \lfloor \frac{ni+j}{d} \rfloor$. In other words, a node u is associated to inputs $e_{du+\alpha-1} = \left(\lfloor \frac{du+\alpha-1}{n} \rfloor, du+\alpha-1-\lfloor \frac{du+\alpha-1}{n} \rfloor n \right)$, $1 \le \alpha \le d$.

We also associate d outputs of $OTIS(d,n)$ to each node v of $II(d,n)$, an output $s=(n-j-1, d-i-1)$ being associated to node $v=n-j-1$, and a node v being associated to outputs $s=(v, d-\alpha)$, $1 \le \alpha \le d$.

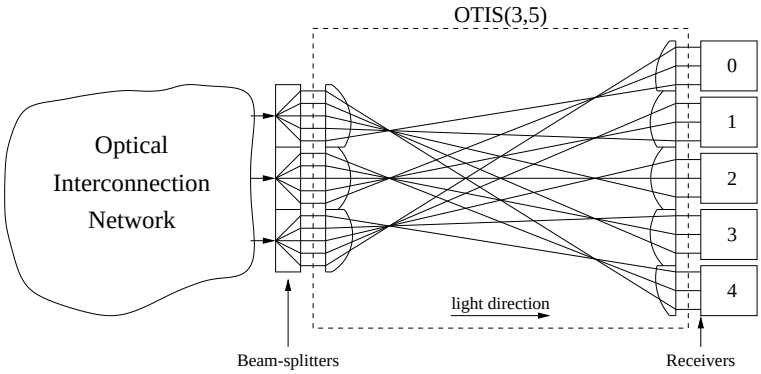

Fig. 9. Interconnections between the outputs of 3 OPS couplers and a group of 5 processors.

Due to the OTIS architecture, a node u is then connected to nodes

$$\begin{aligned} v_\alpha &= n - \left(du + \alpha - 1 - \left\lfloor \tfrac{du+\alpha-1}{n} \right\rfloor n\right) - 1,\ 1 \le \alpha \le d, \\ v_\alpha &= n\left(1 + \left\lfloor \tfrac{du+\alpha-1}{n} \right\rfloor\right) - du - \alpha,\ 1 \le \alpha \le d, \\ v_\alpha &\equiv (-du - \alpha) \mod n,\ 1 \le \alpha \le d. \end{aligned}$$

The neighborhood of u, obtained by using the OTIS architecture is exactly the neighborhood of u in the graph of Imase and Itoh. Finally, by associating inputs of the OTIS architecture $OTIS(d, n)$ to nodes of the graph of Imase and Itoh $II(d, n)$ as we did, we have perfectly realized the interconnections of $II(d, n)$ using the OTIS architecture $OTIS(d, n)$.

Figure 10 shows how the optical interconnections of the graph of Imase and Itoh are realized with the OTIS architecture.

Corollary 1. *The Kautz graph $KG(d, k)$ is the graph of Imase and Itoh $II(d, d^{k-1}(d+1))$. Hence, we can realize the optical interconnections of $KG(d, k)$ using $OTIS(d, d^{k-1}(d+1))$.*

4 Implementing multi-OPS networks with OTIS

We saw in the previous section that the OTIS architecture can be used to realize the optical interconnections of the graph of Imase and Itoh and consequently of the Kautz graph. It was also shown how to connect the processors of a group with its corresponding OPS couplers. In this section, we show how to build the POPS and the stack-Kautz networks using these building blocks

4.1 POPS with OTIS

$POPS(t, g)$ has g groups of t processors and g^2 OPS couplers. We have explain how to realize the optical interconnections between the inputs and the outputs

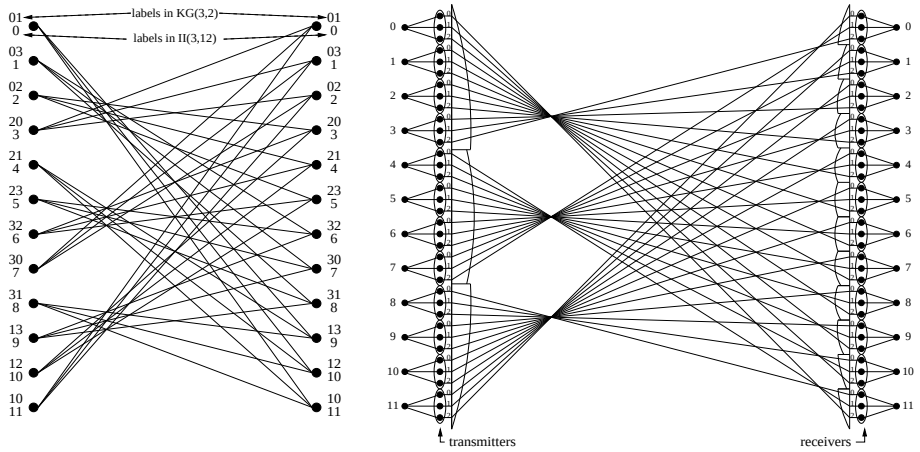

Fig. 10. $II(3,12)$ with $OTIS(3,12)$.

of t processors of a group through g OPS couplers using two $OTIS(t,g)$ architectures. We also have modeled the $POPS(t,g)$ network as a stack-K_g^+ of stacking factor t. As $OTIS(g,g)$ realize the optical interconnections of K_g^+, we can realize all the optical interconnections of the $POPS(t,g)$ network using two $OTIS(t,g)$ and one $OTIS(g,g)$, as shown in Fig. 11.

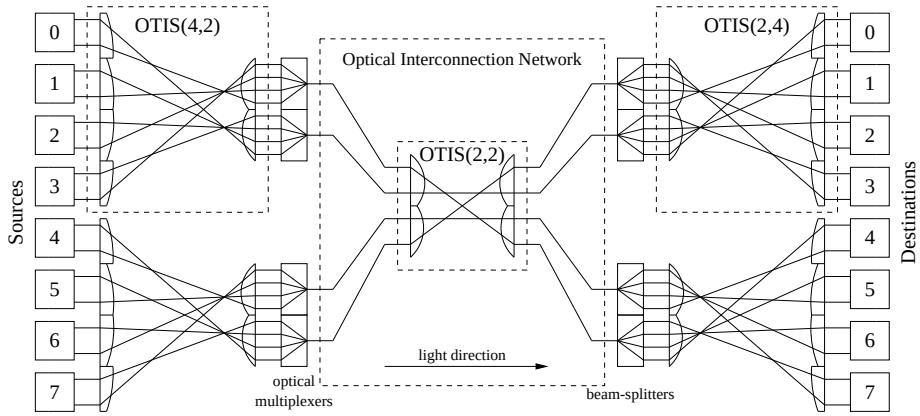

Fig. 11. Optical interconnections of $POPS(4,2)$ using the OTIS architecture.

4.2 Stack-Kautz with OTIS

The stack-Kautz network $SK(s,d,k)$ has $d^{k-1}(d+1)$ groups of s processors and $d^{k-1}(d+1)^2$ OPS couplers of degree s. Each processor has degree $d+1$.

The groups: We have to explain how to connect a group of processors to its corresponding OPS couplers. Using $d^{k-1}(d+1)$ $OTIS(s,d+1)$ plus $d^{k-1}(d+1)$

$OTIS(d+1, s)$ architectures, we can connect all the groups of s processors to its corresponding $d+1$ optical multiplexers and $d+1$ beam-splitters.

The Optical Interconnection Network: Now, we have to realize the optical interconnection network which connects the optical multiplexers to its corresponding beam-splitters. By Corollary 1, that the optical interconnections of a Kautz network $KG(d, k)$ can be realized using one $OTIS(d, d^{k-1}(d+1))$. The stack-Kautz network $SK(s, d, k)$ is based on the Kautz graph $KG(d, k)$. Hence, we can use one $OTIS(d, d^{k-1}(d+1))$ to realize the optical interconnections between the optical multiplexers and its corresponding beam-splitters. We remark that the loops are not taken into account in the optical interconnection network and we consider that they are connected using an appropriate technique (e.g., optical fiber).

An example: Figure 12 shows how those interconnections are realized for $SK(6, 3, 2)$ (modeled in Fig. 7) using 12 $OTIS(6, 4)$, 12 $OTIS(4, 6)$, 48 optical multiplexers, 48 beam-splitters and one $OTIS(3, 12)$. $SK(6, 3, 2)$ has 72 processors (12 groups of 6 processors) of degree 4, connected in a network of diameter 2.

5 Conclusion

In this paper, we showed the optical design of several graph-theoretical network topologies using available optical technology. Our work advances the interplay between graph theory and optics technologies. Indeed, the key of our work was the relationship it established between the OTIS architecture and the graph of Imase and Itoh. Also important was the proposed optical design of the concept of a group of processors, since such a concept seems to be central in multi-OPS communication networks. It is of interest to study the optical design of other multi-OPS networks using the tools developed in this paper.

Finally, a corollary of our results is that the OTIS architecture can be viewed as the graph of Imase and Itoh. Therefore, properties of existing OTIS-based networks can be studied using the properties of such a graph.

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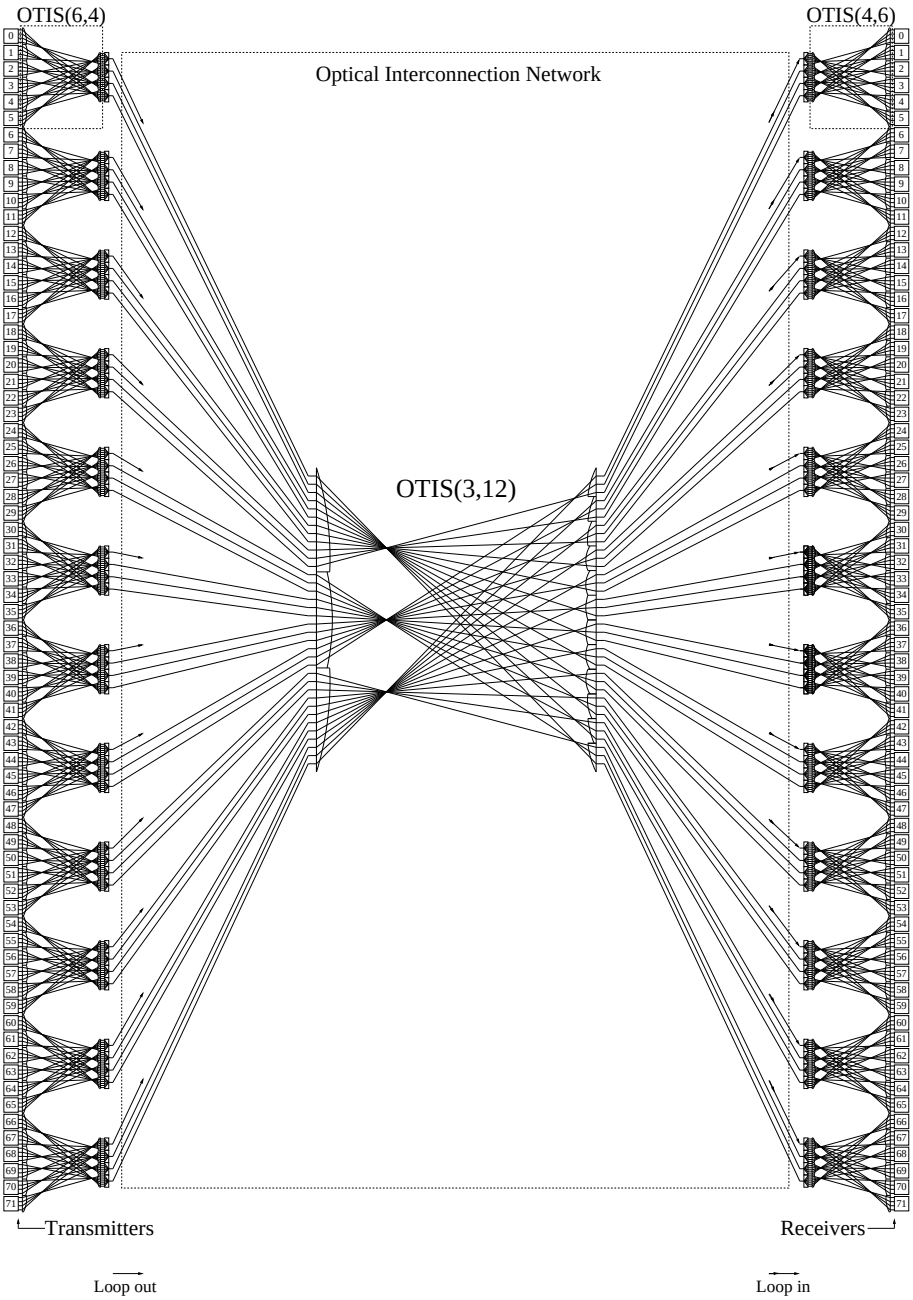

Fig. 12. Optical interconnections of $SK(6,3,2)$ using the OTIS architecture.