A note on cycle covering

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[Extended Abstract]

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ABSTRACT
This study considers the design of a survivable WDM network based on covering the initial network with sub-networks, which are protected independently from each other.

Categories and Subject Descriptors
C.2.1 [**Computer-communication networks**]: Network Architecture and Design; C.2.5 [**Computer-communication network**]: Local and Wide-Area Networks; G.2.2 [**Discrete Mathematics**]: Graph Theory

General Terms
Design, Reliability, Theory

Keywords
Cycle covering, WDM, graph, survivability

The planning of an optical layer can be divided into two sub-problems: computing the routing of the wavelength demands on the physical layer (routing problem) and allocating the resources to the routed demands (resource allocation problem) [1, 5, 8]. The survivability against equipment or link failure consists in computing new routes for the demands affected by a failure; thus the optical layer must be over-dimensioned. Two survivability schemes can be implemented : protection or restoration. Protection can be done by using a pre-assigned capacity between nodes in order to replace the failed or degraded transport entities. On the other hand, restoration can be realized by using any capacity available between nodes in order to find a transport entity that can replace the failed one. Dividing the network into independent sub-networks provides an intermediate solution for survivability. Indeed it allows resource sharing within the limits of a given sub-network, and uses of fast automatic protection in case of failure [9].

We model the optical telecommunications network by a graph G, where vertices and edges represent optical switches and fiber-optics links respectively. In fact, G is a symmetric directed multigraph; but for reasons stated later, we consider the underlying undirected graph obtained by replacing each pair of symmetric arcs by an edge.

Usually the physical graph has no regularity properties, and it has just enough connectivity in order to ensure a good routing for the demands even in the case of failures. However, many designers of optical networks builds it with loops interconnected between themselves. Thus, the first case to consider is when the physical graph is a loop (or ring), which means that the graph G is a cycle (ring). So, in the following, we will suppose that **the graph G is an undirected cycle of length** n, denoted by C_n.

The family of requests (or demands), called **instance of communications**, is modelled by a graph, called logical (or virtual) graph and denoted by I. The vertices represent the nodes of the physical graph and the edges correspond to the requests between these nodes. In general, it is a digraph but in telecommunication networks used for applications like telephone the requests are symmetric (i.e. for each communication request from node A to node B there is a similar request from node B to node A). Thus, the logical graph will be undirected. Furthermore, we will suppose that the routing of symmetric requests is done by a symmetric routing. There is no constraint forcing that, but it is the way requests are actually managed. So the symmetry of the routing explains why we consider undirected graphs for the physical one. Finally the instance of communication called *total exchange* (or All-to-All), where each node wants to communicate with all the others simultaneously, is important. In such a case, the logical graph I will be the complete graph. In the following sections, we will suppose that **the logical graph I is the complete graph on n vertices, K_n.**

Routing an instance consists in finding a set of paths such that each (symmetric) request (edge of I) is associated with

a path in the physical graph G between the pairs of nodes communicating in the request. Here we do not consider the allocation of wavelengths to the request (that is done later in the last phase of the network design). The securization problem considered here can be modelled by finding a covering of the edges of the logical graph I by subgraphs I_k. In general, we wish the I_k to have a simple structure and a small number of vertices. Therefore the interesting case is when I_k is a small cycle. Indeed, on the cycle we use half of the capacity for the demands, and in case of failure we reroute the traffic through the failed link via the remaining part of the cycle using the other half of the capacity. It will be interesting to get very small cycles as subnetworks as they are easier to manage and less costly to reroute. Also, we will associate a wavelength to each cycle (in fact two: one for the normal traffic and one for the spare one).

The problem of finding a covering of the edges of a graph (in particular K_n) by small cycles has been studied in the literature. For example the minimum number of 3-cycles required to cover the edges of K_n is $\left\lceil \frac{n}{3} \left\lceil \frac{n-1}{2} \right\rceil \right\rceil$ (See [6, 7]). The covering by C_k, $k > 3$, has been considered in [2], where in particular, the minimum number of 4-cycles required to cover K_n is determined.

But here there is another constraint due to the fact that the requests have to be routed on the physical network G, which can be modelled as follows (one can think that to each subnetwork, here cycle, we associate a wavelength): for each subgraph I_k of the covering there should exist **edge disjoint routing in** G i.e. the paths associated in G to any pair of requests of I_k should be edge disjoint. We call this property **the disjoint routing constraint** (DRC). As an illustration, let G be $C_4 = (1, 2, 3, 4, 1)$ and I be K_4. One covering is given by the two C_4's $(1, 2, 3, 4, 1)$ and $(1, 3, 4, 2, 1)$ but there does not exist an edge disjoint routing for the cycle $(1, 3, 4, 2, 1)$, as it is impossible to associate the requests $(1, 3)$ and $(2, 4)$ to two edge disjoint paths in G. On the otherhand, the covering given by the C_4 $(1, 2, 3, 4, 1)$ and the two C_3's $(1, 2, 4, 1)$ and $(1, 3, 4, 1)$ satisfies the edge disjoint routing property.

The general problem can be summarized as follows: *Find a covering of the edges of a logical graph I by subgraphs I_k, such that, for each I_k, there exists in the physical graph G a disjoint routing of the edges of I_k and such that the cost of the network is minimized.*

The cost is a very complex function depending on the size of the ADM (Add and Drop Multiplexer) in each node, the number of wavelengths (associated to the subnetworks) in transit in each optical node and a cost of regeneration and amplification of the signal. When the physical graph is a ring that corresponds **to minimize the number of subgraphs I_k in the covering** (as there is a unique physical path associated to a request). Furthermore it reduces the complexity (in terms of number of sub-networks) of the whole network.

Compared to previous studies based on a similar approach, we provide an analytical model and optimal solutions for the ring as the physical topology. A similar problem is considered in [3] and [4]. They use the same ring survivability conditions but their aim is to minimize the sum of the num-

ber of vertices of the rings.

Here, we consider covering of the edges of K_n by cycles satisfying the *disjoint routing constraint*. Such a covering will be called a *DRC-covering* of K_n, and $\rho(n)$ will denote the minimum number of cycles needed in a DRC-covering of K_n. We obtain the following results (proof omitted) :

THEOREM 1. *When* $n = 2p + 1$, $\rho(n) = \frac{p(p+1)}{2}$. *Furthermore, the DRC-covering of K_{2p+1} consists of p C_3 and $\frac{p(p-1)}{2}$ C_4.*

THEOREM 2. *When* $n = 2p$, $p \geq 3$, $\rho(n) = \left\lceil \frac{p^2+1}{2} \right\rceil$. *Furthermore, when $n = 4q$, the DRC-covering of K_{4q} consists of 4 C_3 and $2q^2 - 3$ C_4, and when $n = 4q+2$ the DRC-covering of K_{4q+2} consists of 2 C_3 and $2q^2 + 2q - 1$ C_4.*

As an extension of this problem, we are now investigating cases with other communication instances such as λK_n (or more general logical graphs). We also consider other network topologies, for example, trees of rings, grids or tori. Moreover it will be interesting to consider the real cost function in these extensions.

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