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## **An algorithm for Coulomb's frictional contact**

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## 1 Introduction

- Goal: simulate dynamics of a mechanical system
  - with **unilateral contact**
  - and **Coulomb's friction** at contact points.
- Example: granular materials, robotics



Granular materials

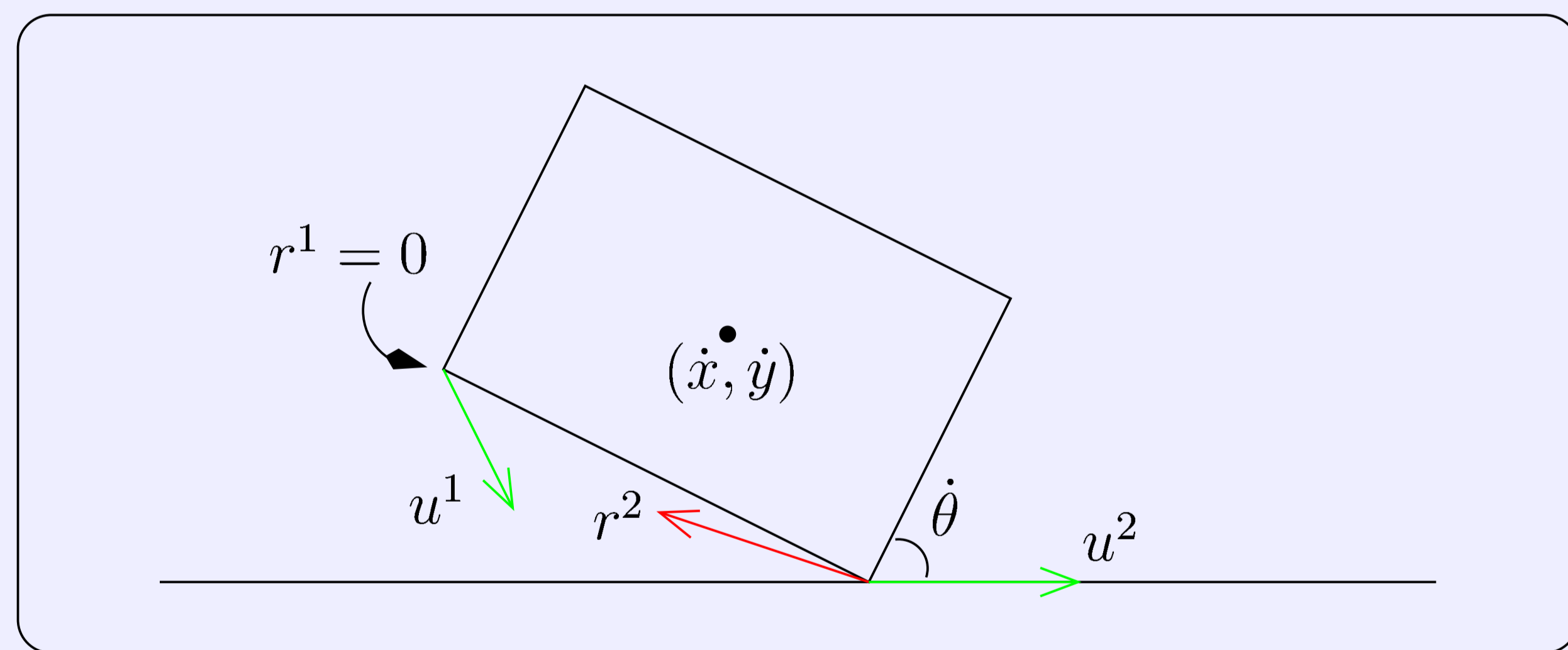


Robotics

- Mechanical system must have finitely many degrees of freedom
- No other nonlinearity than Coulomb's law

## 2 Unknowns

- Time is discretized
- We want to compute at each time step:
  - parameters  $(x, y, \theta)$
  - **generalized velocity** :  $v = (\dot{x}, \dot{y}, \dot{\theta})$
  - **velocity** at contact points :  $u = (u^1, u^2)$
  - **reaction** at contact points :  $r = (r^1, r^2)$



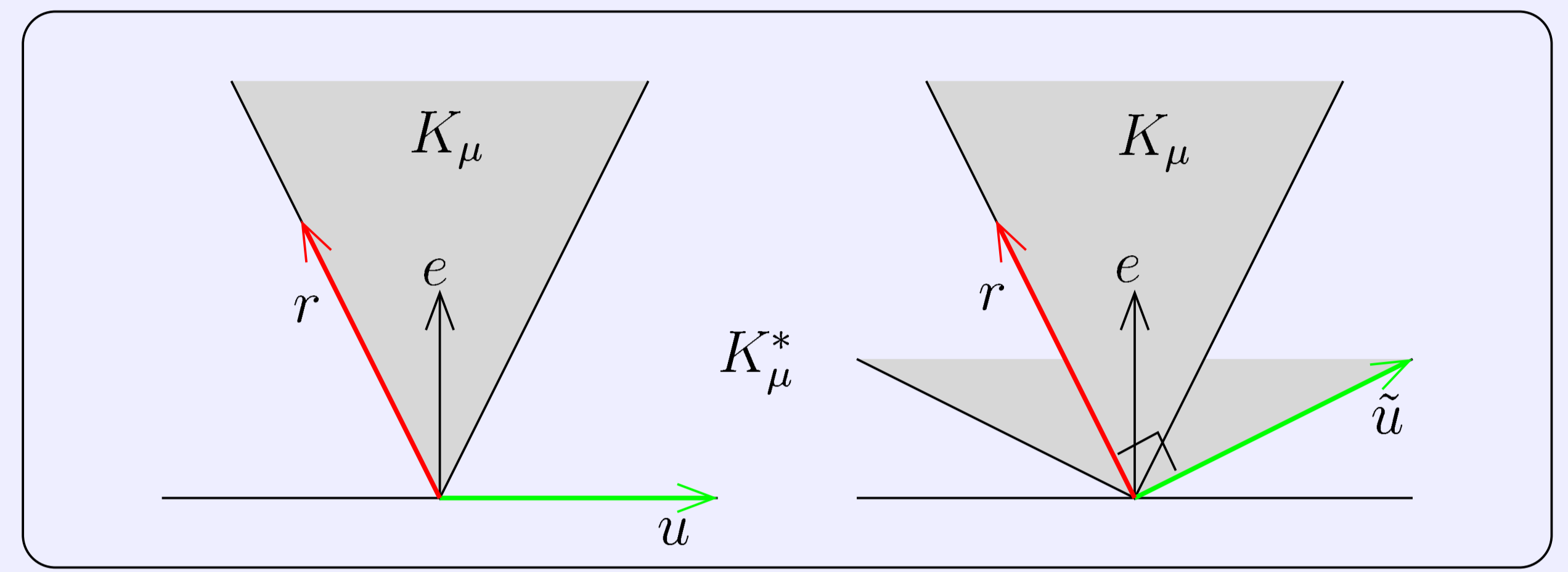
## 3 Coulomb's law

- Let  $K_\mu = \{\|r_T\| \leq \mu r_N\} \subset \mathbb{R}^3$  (second order cone)
- Coulomb's law : usually formulated as 3-case **disjunction**:
  - **take off** :  $r = 0$  and  $u_N \geq 0$
  - **sticking** :  $r \in \text{int}(K_\mu)$  and  $u = 0$
  - **friction** :  $r \in \partial K_\mu \setminus 0$ ,  $u_N = 0$ ,  $u_T$  opposed to  $r_T$ .

## 4 Coulomb's law revisited

- Formulation as disjunction is not convenient
- More compact (and practical) formulation: **complementarity**

$$\begin{cases} \tilde{u} = u + \mu \|u_T\| e \\ K_\mu \ni r \perp \tilde{u} \in K_\mu^* \end{cases} \quad (1)$$



## 5 Formulation

- Altogether, we want to solve:

$$\begin{cases} Hr = Mv + f & \text{[Newton's law]} \quad (a) \\ u = H^T v + Es & \text{[kinematics]} \quad (b) \\ L \ni r \perp u \in L^* & \text{[Coulomb's law]} \quad (c) \\ s^i = \|u_T^i\| \quad \forall i & (d) \end{cases} \quad (2)$$

- $H, M \in \mathbb{S}_n^{++}$  (mass matrix),  $f, E$  are constant
- $L$  is a product of several cones  $K_\mu$  (one for each contact)

- Key observation : (a-c) are the **optimality conditions** of :

$$\begin{cases} \min J(v) := \frac{1}{2} v^T M v + f^T v & \text{(quadratic, strictly convex)} \\ H^T v + Es \in L^* & \text{(conic constraints)} \end{cases}$$

- The (equivalent) dual problem can also be used

## 6 Algorithm

- Consider  $s$  as a parameter
- Solve (2.a-2.c) as a **second order cone program** (SOCP)
- Adapt  $s$  iteratively in **damped Newton algorithm** to satisfy (2.d)
- Need to differentiate solution of SOCP with respect to  $s$

## 7 Results

- Theoretical : simple proof of **existence of a solution**
- Numerical : **stability, very fast convergence**
  - Ex. : 3D, 200 degrees of freedom, 150 contacts,  $\mu \leq 2$
  - only 3 iterations (ie 3 SOCP subproblems) are enough !

Iteration	1	2	3
Infeasibility in (2.d)	$1.1 * 10^3$	$4.8 * 10^{-3}$	$1.7 * 10^{-10}$

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