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Prioritizing linear equality and inequality systems: application to local motion planning for redundant robots

Oussama Kanoun, Florent Lamiraux, Pierre-Brice Wieber, Fumio Kanehiro, Eiichi Yoshida and Jean-Paul Laumond

***Abstract*— We present a novel method for prioritizing both linear equality and inequality systems and provide one algorithm for its resolution. This algorithm can be summarized as a sequence of optimal resolutions for each linear system following their priority order. We propose an optimality criterion that is adapted to linear inequality systems and characterize the resulting optimal sets at every priority level. We have successfully applied our method to plan local motions for the humanoid robot HPR-2. We will demonstrate the validity of the method using an original scenario where linear inequality constraints are solved at lower priority than equality constraints.**

I. INTRODUCTION

A. Statement of the problem and contribution

Let us recall the context of prioritized kinematic control of robots. For a robotic arm, a humanoid robot or any articulated structure, a motion of the structure's joints is calculated to achieve a goal task. The task is often a target position and/or an orientation in the workspace for a body in the structure. Call q the joints configuration of the robot and $T(q) = 0$ the goal value of a task whose current value is $T(q) = c$. By computing the jacobian $J = \frac{\partial T}{\partial q}(q)$, one can calculate velocities $\dot{q}$ to tend towards achieving $T(q) = 0$. $\dot{q}$ is solution of the following linear equality system [1]:

$$J\dot{q} = -\lambda c$$

where λ is a positive real. This linear system can be under-constrained for structures with a high number of degrees of freedom. As we naturally want to specify extra tasks to take advantage of this redundancy, comes a need to organize the tasks from most to least critical. The reason is that we want to avoid trade off between tasks of unequal importance and secure the most critical ones. Several works have been carried in this scope yielding efficient algorithms for task prioritization [2], [3], [4]. These algorithms have also been widely used in the robotics community [5], [6].

Some tasks are not naturally expressed as linear equality systems. There are for instance limits on the controls (e.g velocity and acceleration bounds in robot joints). Another example is the avoidance of collision with obstacles in the environment. Collision avoidance is a task naturally expressed as $T(q) \leq 0$, where $T(q)$ is a function defining the boundaries between colliding configurations, $\{q$ such that $T(q) > 0\}$, and non-colliding ones $\{q$ such that $T(q) < 0\}$. The critical nature of these unilateral constraints have inspired works such as [7], [8], [9], [10]. In these works, the inequality constraints are taken into account prior to solving any other task.

The contribution of our work is to overcome this restriction as our method will allow us to prioritize both linear equations and linear inequalities in any order. The algorithm we provide in this paper is general in the sense that it can be applied to any problem involving the resolution of a set of linear equality and inequality systems with priorities.

For the control of redundant robots, the inequalities at lower priority allow us to solve new kinds of scenarii. Consider for example a humanoid robot which has to grasp an object seen with embedded cameras. It is best if its reaching hand does not come between the cameras and the object too soon. This is because we would like to keep checking the visual target to maximize the chance of a successful grasp. In this scenario, the robot has to accomplish a primary reaching task and a secondary region-avoidance task. The available algorithms do not handle this problem including tasks expressed by inequalities with lower priority. Our algorithm, however, will provide a solution to this scenario.

B. Definition of linear systems

Let A and C be matrices in $\Re^{m\times n}$ and b and d vectors in $\Re^m$ with $(m, n) \in \mathbb{N}^2$. We'll consider in the following either a system of linear equalities

$$Ax = b \tag{1}$$

or a system of linear inequalities

$$Cx \leq d \tag{2}$$

or both. When $m = 1$, (1) is reduced to one linear equation and (2) to one linear inequality.

A system of linear equalities may have no solution or may define an affine subspace of $\Re^n$. For instance, in case $n = 3$ this affine subspace is either a point, a line, a plane or the whole space $\Re^3$.

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A linear inequality, when it has solutions, defines a halfspace. The set of solutions of a system of linear inequalities is the intersection of the halfspaces generated by its inequalities. This set is a volume of $\Re^n$ bounded by a convex polytope which may be closed or infinite (for example a halfspace is infinite). See figure 1 for an illustration of a system of linear inequalities in $\Re^2$.

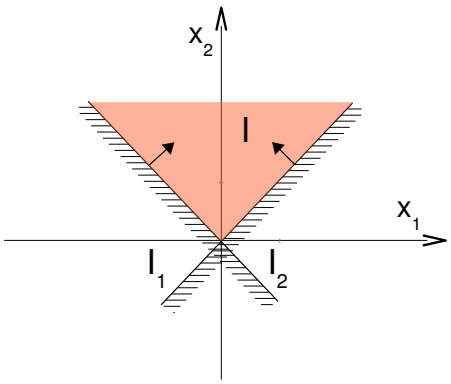


Fig. 1. Two linear inequalities determining a convex polygon in $\Re^2$

C. Approach

Affecting priorities to linear systems means that we leave some of the systems unsolved to respect the ones with higher priorities. Letting L_1 and L_2 be two linear systems without common solutions, prioritizing L_1 over L_2 means that we retain a solution which satisfies L_1 to the expense of L_2. Nonetheless, to take into account L_2, one may select a solution of L_1 which minimizes the euclidean distance to L_2's solutions set. Euclidean distance is one example of optimality criterion adapted to systems of linear equalities. As a matter of fact, a point realizing this shortest distance belongs to the orthogonal projection of L_2's solutions on L_1's and can be obtained analytically. Furthermore, the entire set of points realizing the shortest distance may also be determined analytically [3]. To solve a third system of linear equalities L_3, the resolution is done within L_2's optimal set.

For our problem, we adopt the same approach consisting in solving every linear system in the optimal set defined by higher priorities. When we introduce systems of linear inequalities, however, we introduce solution sets which are volumes of $\Re^n$ bounded by convex polytopes. In this particular case, euclidean distance is not a good optimality criterion (Figure 2).

In this work, we choose an optimality criterion adapted to both types of linear systems and study the nature of generated sets of solutions (Section II). For this choice, we prove that prioritizing linear systems results in optimal sets described with linear systems and we deduce a resolution algorithm that is relatively easy to implement (Section III). We illustrate the effectiveness of our method by describing an application to local motion planning for the humanoid robot HRP-2 (Section IV).

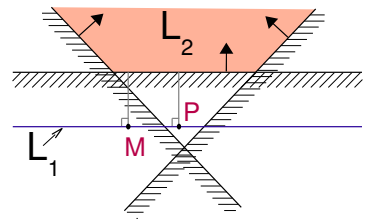


Fig. 2. The set of solutions to a primary system of linear equalities L_1 and a secondary system of linear inequalities L_2 are without common solutions. M and P are solutions of L_1 minimizing the euclidean distance to L_2's set, however, P satisfies two inequalities out of three while M satisfies none.

II. OPTIMAL RESOLUTION OF LINEAR SYSTEMS

In this section we construct optimization problems to solve each type of linear system. For each problem, we demonstrate the nature of the optimal set. A will denote a matrix in $\Re^{m\times n}$ and b a vector in $\Re^m$ unless indicated otherwise.

A. System of linear equalities

When trying to satisfy a system (1) of linear equalities while constrained to a non-empty convex set $\Omega \subset \Re^n$, we'll consider the set S_e of optimal solutions to the following minimization problem:

$$\min_{x\in\Omega} \frac{1}{2}\|w\|^2 \tag{3}$$

with

$$w = Ax - b. \tag{4}$$

Since the minimized function is coercive, the set S_e is non-empty. We also have the property:

$$x_1, x_2 \in S_e \Leftrightarrow x_1, x_2 \in \Omega \text{ and } Ax_1 = Ax_2, \tag{5}$$

from which we can conclude that the set S_e is convex.

Proof: Let's consider an optimal solution x^* to the minimization problem (3)-(4). The gradient of the minimized function at this point is

$$A^T(Ax^* - b).$$

The Karush-Kuhn-Tucker optimality conditions give us that the scalar product between this gradient and any vector v not pointing outside Ω from x^* is non-negative,

$$w^{*T}Av \geq 0$$

with

$$w^* = Ax^* - b.$$

Let's consider now two such optimal solutions, x_1^* and x_2^*. Since the set Ω is convex, the direction $x_2^* - x_1^*$ points towards its inside from x_1^*, so we have

$$w_1^{*T}A(x_2^* - x_1^*) \geq 0$$

which is equivalent to

$$w_1^{*T} w_2^* - \|w_1^*\|^2 \geq 0.$$

The same can be written from x_2^*,

$$w_2^{*T} w_1^* - \|w_2^*\|^2 \geq 0,$$

so that we obtain

$$\|w_2^* - w_1^*\|^2 = \|w_2^*\|^2 + \|w_1^*\|^2 - 2w_2^{*T} w_1^* \leq 0,$$

but this squared norm can't be negative, so it must be zero and $w_2^* = w_1^*$, what concludes the proof. ■

In the unconstrained case, when $\Omega = \Re^n$, the solutions of (3)-(4) are such that $A^T A x^* = A^T b$. This minimization problem corresponds therefore to a constrained pseudo-inverse solution of the system of linear equalities (1).

B. System of linear inequalities

When trying to satisfy a system (2) of linear inequalities while constrained to a non-empty convex set $\Omega \subset \Re^n$, we'll consider the set S_i of optimal solutions to the following minimization problem:

$$\min_{x \in \Omega, w \in \Re^m} \frac{1}{2}\|w\|^2 \tag{6}$$

with

$$w \geq Cx - d, \tag{7}$$

where w plays now the role of a vector in $\Re^m$ of *slack variables*. Once again, since the minimized function is coercive, the set S_i is non-empty. Considering each inequality $c^j x \leq b^j$ of the system (2) separately, we also have the property:

$$x_1, x_2 \in S_i \Leftrightarrow x_1, x_2 \in \Omega \text{ and } \forall j \left\{ \begin{array}{l} c^j x_1 \leq d^j \Leftrightarrow c^j x_2 \leq d^j, \\ c^j x_1 > d^j \Rightarrow c^j x_1 = c^j x_2, \end{array} \right. \tag{8}$$

which means that all the optimal solutions satisfy a same set of inequalities and violate the others by a same amount, and from which we can conclude that the set S_i is convex.

Proof: Let's consider an optimal solution x^*, w^* to the minimization problem (6)-(7). The Karush-Kuhn-Tucker optimality conditions give that for every vector v not pointing outside Ω from x^*,

$$w^{*T} C v \geq 0$$

and

$$w^* = \max\{0, Cx^* - d\}. \tag{9}$$

This last condition indicates that if an inequality in the system (2) is satisfied, the corresponding element of w^* is zero, and when an inequality is violated, the corresponding element of w^* is equal to the value of the violation.

Let's consider now two such optimal solutions, x_1^*, w_1^* and x_2^*, w_2^*. Since the set Ω is convex, the direction $x_2^* - x_1^*$ points towards its inside from x_1^*, so we have

$$w_1^{*T} C(x_2^* - x_1^*) \geq 0$$

which is equivalent to

$$w_1^{*T}(Cx_2^* - d) - w_1^{*T}(Cx_1^* - d) \geq 0.$$

The optimality condition (9) gives

$$w_1^{*T} w_2^* \geq w_1^{*T}(Cx_2^* - d)$$

and

$$w_1^{*T} w_1^* = w_1^{*T}(Cx_1^* - d),$$

so we obtain

$$w_1^{*T} w_2^* - \|w_1^*\|^2 \geq 0.$$

The same can be written from x_2^*,

$$w_2^{*T} w_1^* - \|w_2^*\|^2 \geq 0,$$

so that we obtain

$$\|w_2^* - w_1^*\|^2 = \|w_2^*\|^2 + \|w_1^*\|^2 - 2w_2^{*T} w_1^* \leq 0,$$

but this squared norm can't be negative, so it must be zero and $w_2^* = w_1^*$, what concludes the proof. ■

C. Mixed system of linear equalities and inequalities

We can observe that systems of linear equalities and systems of linear inequalities are dealt with optimization problems (3)-(4) and (6)-(7) which have similar lay-outs and similar properties (5) and (8). The generalization of these results to mixed systems of linear equalities and inequalities is therefore trivial and we'll consider in the following the minimization problem (in a more compact form)

$$\min_{x \in \Omega, w \in \Re^m} \frac{1}{2}\|Ax - b\|^2 + \frac{1}{2}\|w\|^2 \tag{10}$$

with

$$Cx - w \leq d. \tag{11}$$

The set of solutions to this minimization problem shares both properties (5) and (8).

III. PRIORITIZING LINEAR SYSTEMS

A. Formulation

Let's consider now the problem of trying to satisfy a set of systems of linear equalities and inequalities with a strict order of priority between these systems. At each level of priority $k \in \{1, \ldots p\}$, both a system of linear equalities (1) and a system of linear inequalities (2) are considered, with matrices and vectors A_k, b_k, C_k, d_k indexed by their priority level k. At each level of priority, we try to satisfy these systems while strictly enforcing the solutions found for the levels of higher priority. We propose to do so by solving at each level of priority a minimization problem such as (10)-(11). With levels of priority decreasing with k, that gives:

$$S_0 = \Re^n, \tag{12}$$

$$S_{k+1} = \operatorname*{Arg\,min}_{x \in S_k, w \in \Re^m} \frac{1}{2}\|A_k x - b_k\|^2 + \frac{1}{2}\|w\|^2 \tag{13}$$

$$\text{with } C_k x - w \leq d_k. \tag{14}$$

B. Properties

A first direct implication of properties (5) and (8) is that throughout the process (12)-(14),

$$S_{k+1} \subseteq S_k.$$

This means that the set of solutions found at a level of priority k is always strictly enforced at lower levels of priority, what is the main objective of all this prioritization scheme.

A second direct implication of these properties (5) and (8) is that if S_k is a non-empty convex polytope, S_{k+1} is also a non-empty convex polytope, the shape of which is given in properties (5) and (8). Figure 3 illustrates how these sets evolve in different cases. Classically, these convex polytopes can always be represented by systems of linear equalities and inequalities:

$$\forall k,\ \exists \bar{A}_k, \bar{b}_k, \bar{C}_k, \bar{d}_k \text{ such that } x \in S_k \Leftrightarrow \left\{ \begin{array}{l} \bar{A}_k x = \bar{b}_k \\ \bar{C}_k x \leq \bar{d}_k \end{array} \right.$$

With this representation, the step (13)-(14) in the prioritization process appears to be a simple Quadratic Program with linear constraints that can be solved efficiently.

Note that when only systems of linear equalities are considered, with the additionnal final requirement of choosing x^* with a minimal norm, the prioritization process (12)-(14) boils down to a reformulation of the well-known *task-priority* problem [3].

C. Algorithm

The proposed Algorithm consists in processing the priority levels from highest to lowest and solving at every level the corresponding Quadratic Program. The representation of the sets S_k by systems of linear equalities and inequalities is efficiently updated then by direct application of the properties (5) and (8).

It is naturally possible to optimize additional criteria over the final set of solutions. For instance, one might be interested in the solution with minimal norm, or in the solution that maximizes the distance to the boundaries of the optimal set, etc...

Note that a similar algorithm has already been described in [11], but in the setting of Constraint Programming on discrete variables: the structure and the logic are similar, but the inner workings are very different, especially the theoretical analysis of Section II.

IV. APPLICATION

We have applied the proposed algorithm to plan local motions for the humanoid robot HRP-2 [12]. We show in the following examples the ability of our algorithm to treat any order of priority with both equality and inequality tasks. The motions mentioned hereby may be viewed in the accompanying video.

Algorithm 1 Solve prioritized linear systems

Initialize the system of equalities $\bar{A}_0$, $\bar{b}_0$ to empty.
Initialize the system of inequalities $\bar{C}_0$, $\bar{d}_0$ to empty.

for $k = 0$ to $p - 1$ **do**

  Solve the Quadratic Program (13)-(14) to obtain S_{k+1}.

  $\bar{A}_{k+1} \leftarrow \begin{bmatrix} \bar{A}_k \\ A_k \end{bmatrix}$, $\bar{b}_{k+1} \leftarrow \begin{bmatrix} \bar{b}_k \\ A_k x_k^* \end{bmatrix}$.

  $\bar{C}_{k+1} \leftarrow \bar{C}_k$, $\bar{d}_{k+1} \leftarrow \bar{d}_k$.

  for all c_k^j in C_k **do**
    if $c_k^j x_k^* \leq d_k^j$ **then**

      $\bar{C}_{k+1} \leftarrow \begin{bmatrix} \bar{C}_{k+1} \\ c_k^j \end{bmatrix}$, $\bar{d}_{k+1} \leftarrow \begin{bmatrix} \bar{d}_{k+1} \\ d_k^j \end{bmatrix}$.

    else

      $\bar{A}_{k+1} \leftarrow \begin{bmatrix} \bar{A}_{k+1} \\ c_k^j \end{bmatrix}$, $\bar{b}_{k+1} \leftarrow \begin{bmatrix} \bar{b}_{k+1} \\ c_k^j x_k^* \end{bmatrix}$.

    end if
  end for
end for

A. Example 1: inequality tasks at higher priority

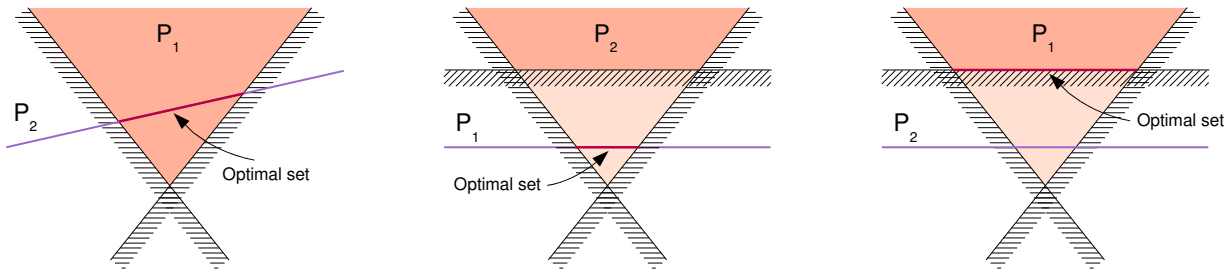
In this example, we illustrate the utility of prioritizing equality tasks after specification of inequality constraints. The goal of the motion is to reach a ball underneath an object (blue polyhedron in figure IV) while looking at it. Here is the stack of tasks sorted in decreasing priority:

1) Stability + Collision avoidance
2) Reach for the ball
3) Look at the ball + minimal joint velocity

The stability task ensures the quasi-static stability of the motion by fixating the center of mass projection and the feet on the ground. The collision avoidance task was built following Kanehiro's method[13] for smooth avoidance between non strictly convex polyhedra. This task is used for both obstacle avoidance and self-collision avoidance and it expresses as a linear inequality system. For the reaching we specified a three-dimensional position task on the center of the left hand. The gaze task was defined as the alignment of the principle axis of the head on the vector linking the center of the head to the ball. We added a final task to minimize the joint velocities, also called a damping task (see [14]).

In the resulting motion, the looking task could be maintained until the robot's head came close to the border of the table. When simultaneous looking and reaching became infeasible, the specified priorities made the robot continue the reaching while its gaze direction drifted off the target. Task 2) was satisfied at the end of this motion (frame 4(d)).

We tried to achieve the same goal while making the looking and the reaching tasks share the same priority. This



(a) Priority does not matter, the prioritized linear systems are compatible

(b) Equality has priority over inequality

(c) Inequality has priority over equality

Fig. 3. Optimal sets in different scenarii

time the robot was trapped in an intermediate configuration of the previous motion (frame 4(c)) where the looking task was maintained while the head over the table and the reaching hand stuck away from the ball.

One other method, presented by Park et al [8] and applied to move a robotic arm, should also permit to solve this scenario. For the next example, there is no available resolution method to our knowledge.

B. Example2: inequality tasks at lower priority

In this example we illustrate the ability of our algorithm to account for inequality tasks at low priority. The goal of the motion is to reach for a ball while avoiding, when possible, a region around the ball (represented in figure 5(a) by a green box). Here is the priority order for this motion:

1) Stability + auto-collision avoidance
2) Reach for the ball
3) Look at the ball
4) Avoid the green box + minimal joint velocity

The idea behind placing the tall box on the ball is to guide the hand out of the vision field to avoid the occlusion of the ball. In a more rigorous but less simple implementation of this scenario, one would consider the vision cone linking the robot's head to the ball. For the illustration of the method, however, the green box suffices.

For the box avoidance task, four points in the body of the hand were checked for collision. The reaching task placed at priority 2) is different from the first example as it is one-dimensional only. This is done by allowing the hand to move on the orthogonal plane to the vector separating it from the target.

In the resulting motion, the hand was forced by task 4) to stay behind a face of the green box (frame 5(b)) until it became incompatible with task 2). Then, the hand progressively entered the volume of the box (frame 5(c)) and achieved its goal (frame 5(d)). As we expected, the inequality task 4) was maintained as long as possible and ended unsatisfied to the benefit of equality task 2).

This motion was computed using an average processor in about four times its actual duration. The performance of the algorithm should be improved in the future with a more optimized implementation.

V. CONCLUSION

We presented the novel problem of prioritizing both linear equality and inequality systems and provided one algorithm for its resolution. This algorithm can be summarized as a sequence of optimal resolutions for each linear system following their priority order. We proposed an optimality criterion that is adapted to linear inequality systems and characterized the resulting optimal sets at every priority level. We successfully applied our method to plan local motions for the humanoid robot HPR-2. One planning scenario presented the originality of requiring inequality tasks be solved at lower priority than equality tasks. Further applications of our algorithm, such as to biped robots' locomotion, are to be considered in forthcoming works.

VI. ACKNOWLEDGMENTS

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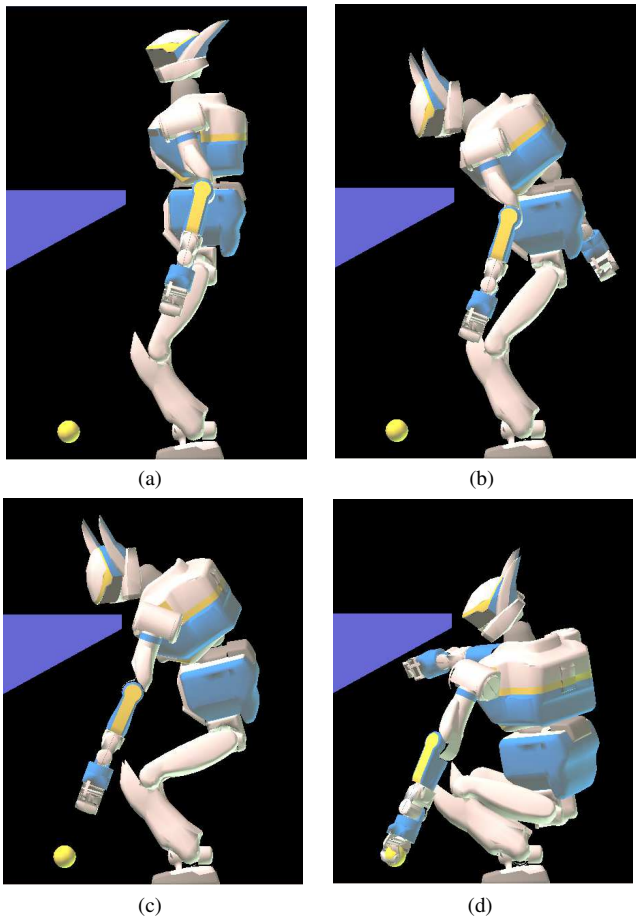

Fig. 4. Example 1

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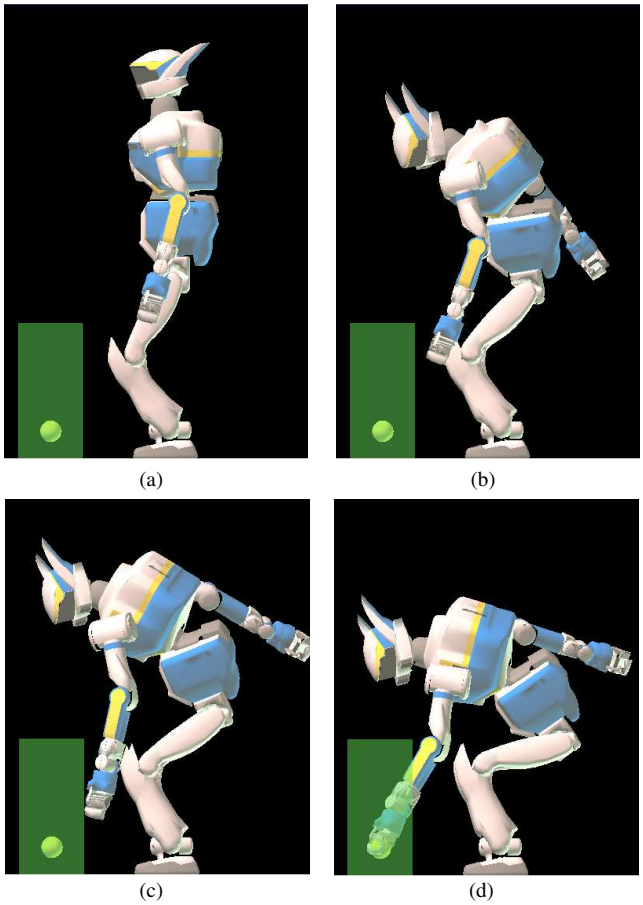

Fig. 5. Example 2