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# Benchmarking the NEWUOA on the BBOB-2009 Function Testbed

Raymond Ros  
Univ. Paris-Sud, LRI  
UMR 8623 / INRIA Saclay, projet TAO  
F-91405 Orsay, France  
raymond.ros@lri.fr

## ABSTRACT

The NEWUOA which belongs to the class of Derivative-Free optimization algorithms is benchmarked on the BBOB-2009 noise-free testbed. A multistart strategy is applied with a maximum number of function evaluations of up to  $10^5$  times the search space dimension resulting in the algorithm solving 11 functions in 20-D. The results of the algorithm using the recommended number of interpolation points for the underlying model and the full model are shown and discussed.

## Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

## General Terms

Algorithms

## Keywords

Benchmarking, Black-box optimization, Derivative-free optimization

## 1. ALGORITHM PRESENTATION

The NEWUOA (New Unconstrained Optimization Algorithm) [4] is a Derivative-Free Optimization (DFO) algorithm using the trust region paradigm. NEWUOA computes a quadratic interpolation of the objective function in the current trust region and performs a truncated conjugate gradient minimization of the surrogate model in the trust region. It then updates either the current best point or the radius of the trust region, based on the a posteriori interpolation error. The time complexity of the algorithm is  $\mathcal{O}(m^2n)$  in the worst case but in practice closer to  $\mathcal{O}(mn)$ , where  $m$  is the number of interpolation points used for the determination of the quadratic model and  $n$  is the dimension

of the search space. The number of interpolation points is a parameter of the algorithm and needs to be chosen in the range  $[n+2, \frac{(n+1)(n+2)}{2}]$ . Other parameters of the algorithm are the initial and final radii of the trust region, respectively governing the initial ‘granularity’ and the precision of the search. A simple stochastic independent restart procedure (as advised in [2]) was added to improve the probability of the algorithm reaching a target function value.

## 2. EXPERIMENTAL PROCEDURE

The implementation used for our experiments is the one provided by Matthieu Guibert<sup>1</sup> which delivers Powell’s original Fortran source code of the algorithm. This Fortran code has been adapted to the BBOB experimental paradigm. In this paper, we will test two numbers of interpolation points:  $2n+1$  which is recommended in [4] and  $\frac{(n+1)(n+2)}{2}$  which is the full model. An intermediate model using a number of interpolation points that is the integer closer to  $\sqrt{(n+1/2)(n+1)(n+2)}$  was also tested with results that were in-between those of the two models we are considering. The initial radius  $\rho_{\text{beg}}$  of the search region has been set to 10, the range of the search space. Preliminary experiments show very few dependencies on this parameter, given it is not too small (ie. by many orders of magnitude) for the problem considered. A final radius  $\rho_{\text{end}} = 10^{-16}$  was chosen close to the limit being the machine precision to prevent numerical errors. The starting point  $x_0$  is chosen uniformly in  $[-5, 5]^n$ . The multistart strategy was used with at most 100 restarts to reduce the duration of an experiment. For the same reason, the maximum number of function evaluations is  $10^5 \times n$  for  $m = 2n+1$ ,  $10^4 \times n$  otherwise. An example of the algorithm used is presented in Figure 1. No parameter tuning was done, the CrE [2] is computed to zero.

## 3. RESULTS AND DISCUSSION

Results from experiments according to [2] on the benchmark functions given in [1, 3] are presented in Figures 2 and 3 and in Table 1 for  $m = 2n+1$ . The algorithm performs well on the convex quadratic functions  $f_1$ . It solves  $f_2$  and  $f_{11}$ . The algorithm performs well on functions with low or moderate conditioning.

On multimodal functions, the algorithm fails or only solves 2, 3 and/or 5-D, though it does well on the Gallagher functions. As we can see in Figures 4 and 5 and in Table 2 for the full model, these results cannot be improved by using more

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<sup>1</sup><http://www.inrialpes.fr/bipop/people/guilbert/newuoa/newuoa.html>

$f_1$ in 5-D, N=15, mFE=139					$f_1$ in 20-D, N=15, mFE=471					$f_2$ in 5-D, N=15, mFE=62427					$f_2$ in 20-D, N=15, mFE=315319						
$\Delta f$	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	
10	15	1.2e1	1.2e1	1.2e1	1.2e1	15	4.3e1	4.3e1	4.3e1	4.3e1	10	15	4.8e2	3.1e2	6.6e2	4.8e2	15	7.0e3	6.2e3	7.8e3	7.0e3
1	15	1.2e1	1.2e1	1.2e1	1.2e1	15	4.3e1	4.3e1	4.4e1	4.3e1	1	15	1.9e3	1.5e3	2.3e3	1.9e3	15	1.6e4	1.4e4	1.8e4	1.6e4
1e-1	15	1.2e1	1.2e1	1.2e1	1.2e1	15	4.3e1	4.3e1	4.4e1	4.3e1	1e-1	15	4.0e3	3.4e3	4.6e3	4.0e3	15	2.7e4	2.5e4	3.0e4	2.7e4
1e-3	15	1.2e1	1.2e1	1.2e1	1.2e1	15	4.4e1	4.3e1	4.4e1	4.4e1	1e-3	15	7.6e3	6.8e3	8.4e3	7.6e3	15	4.9e4	4.5e4	5.3e4	4.9e4
1e-5	15	1.2e1	1.2e1	1.2e1	1.2e1	15	4.4e1	4.3e1	4.4e1	4.4e1	1e-5	15	1.2e4	1.1e4	1.3e4	1.2e4	15	6.8e4	6.4e4	7.3e4	6.8e4
1e-8	15	1.2e1	1.2e1	1.2e1	1.2e1	15	4.4e1	4.3e1	4.4e1	4.4e1	1e-8	15	2.6e4	2.1e4	3.2e4	2.6e4	15	1.2e5	1.0e5	1.5e5	1.2e5
$f_3$ in 5-D, N=15, mFE=25753					$f_3$ in 20-D, N=15, mFE=133533					$f_4$ in 5-D, N=15, mFE=36591					$f_4$ in 20-D, N=15, mFE=255640						
10	15	4.3e3	2.9e3	5.9e3	4.3e3	0	<i>13e+1</i>	<i>11e+1</i>	<i>16e+1</i>	6.3e4	10	12	2.1e4	1.6e4	2.8e4	1.7e4	0	<i>17e+1</i>	<i>13e+1</i>	<i>22e+1</i>	8.9e4
1	1	3.7e5	1.8e5	>4e5	2.5e4						1	1	5.0e5	2.4e5	>5e5	3.5e4					
1e-1	0	<i>40e-1</i>	<i>30e-1</i>	<i>80e-1</i>	1.3e4						1e-1	0	<i>60e-1</i>	<i>20e-1</i>	<i>11e+0</i>	2.0e4					
1e-3											1e-3										
1e-5											1e-5										
1e-8											1e-8										
$f_5$ in 5-D, N=15, mFE=207					$f_5$ in 20-D, N=15, mFE=806					$f_6$ in 5-D, N=15, mFE=10778					$f_6$ in 20-D, N=15, mFE=25866						
10	15	1.3e1	1.2e1	1.3e1	1.3e1	15	5.0e1	4.8e1	5.2e1	5.0e1	10	15	2.0e2	1.6e2	2.4e2	2.0e2	15	1.3e3	1.1e3	1.5e3	1.3e3
1	15	1.5e1	1.4e1	1.5e1	1.5e1	15	5.9e1	5.5e1	6.3e1	5.9e1	1	15	5.0e2	4.0e2	6.2e2	5.0e2	15	2.3e3	2.1e3	2.6e3	2.3e3
1e-1	15	1.5e1	1.4e1	1.6e1	1.5e1	15	6.5e1	6.1e1	7.0e1	6.5e1	1e-1	15	1.0e3	7.9e2	1.2e3	1.0e3	15	3.4e3	3.0e3	3.9e3	3.4e3
1e-3	15	1.5e1	1.5e1	1.6e1	1.5e1	15	6.5e1	6.1e1	7.0e1	6.5e1	1e-3	15	1.9e3	1.6e3	2.2e3	1.9e3	15	5.8e3	4.9e3	6.6e3	5.8e3
1e-5	15	1.5e1	1.5e1	1.6e1	1.5e1	15	6.5e1	6.1e1	7.0e1	6.5e1	1e-5	15	2.8e3	2.4e3	3.3e3	2.8e3	15	8.4e3	7.1e3	9.7e3	8.4e3
1e-8	15	1.5e1	1.5e1	1.6e1	1.5e1	15	6.5e1	6.1e1	7.0e1	6.5e1	1e-8	15	4.3e3	3.8e3	4.9e3	4.3e3	15	1.1e4	9.9e3	1.3e4	1.1e4
$f_7$ in 5-D, N=15, mFE=78650					$f_7$ in 20-D, N=15, mFE=2.00e6					$f_8$ in 5-D, N=15, mFE=1485					$f_8$ in 20-D, N=15, mFE=10852						
10	15	2.3e2	1.3e2	3.4e2	2.3e2	0	<i>18e+0</i>	<i>15e+0</i>	<i>22e+0</i>	1.3e5	10	15	7.3e1	5.9e1	8.9e1	7.3e1	15	2.0e3	1.9e3	2.2e3	2.0e3
1	15	4.1e3	2.6e3	5.6e3	4.1e3						1	15	3.0e2	2.2e2	4.0e2	3.0e2	15	3.9e3	3.2e3	4.6e3	3.9e3
1e-1	6	7.1e4	4.7e4	1.3e5	2.7e4						1e-1	15	3.9e2	3.2e2	4.8e2	3.9e2	15	4.0e3	3.3e3	4.8e3	4.0e3
1e-3	0	<i>32e-2</i>	<i>21e-3</i>	<i>47e-2</i>	7.9e3						1e-3	15	4.5e2	3.8e2	5.4e2	4.5e2	15	4.2e3	3.5e3	4.9e3	4.2e3
1e-5											1e-5	15	4.9e2	4.1e2	5.7e2	4.9e2	15	4.4e3	3.7e3	5.1e3	4.4e3
1e-8											1e-8	15	5.2e2	4.4e2	6.0e2	5.2e2	15	4.5e3	3.8e3	5.3e3	4.5e3
$f_9$ in 5-D, N=15, mFE=1843					$f_9$ in 20-D, N=15, mFE=10808					$f_{10}$ in 5-D, N=15, mFE=76895					$f_{10}$ in 20-D, N=15, mFE=77382						
10	15	6.3e1	5.7e1	6.9e1	6.3e1	15	1.8e3	1.7e3	1.8e3	1.8e3	10	15	1.1e3	7.3e2	1.5e3	1.1e3	15	1.3e4	1.2e4	1.4e4	1.3e4
1	15	4.6e2	3.3e2	5.9e2	4.6e2	15	3.1e3	2.5e3	3.8e3	3.1e3	1	15	2.7e3	2.0e3	3.5e3	2.7e3	15	2.3e4	2.1e4	2.5e4	2.3e4
1e-1	15	5.3e2	4.1e2	6.5e2	5.3e2	15	3.3e3	2.7e3	4.0e3	3.3e3	1e-1	15	4.6e3	3.7e3	5.6e3	4.6e3	15	3.6e4	3.3e4	3.9e4	3.6e4
1e-3	15	5.8e2	4.7e2	7.1e2	5.8e2	15	3.5e3	2.8e3	4.2e3	3.5e3	1e-3	15	9.0e3	7.7e3	1.0e4	9.0e3	15	6.0e4	5.6e4	6.3e4	6.0e4
1e-5	15	6.3e2	5.0e2	7.4e2	6.3e2	15	3.6e3	2.9e3	4.3e3	3.6e3	1e-5	15	1.3e4	1.2e4	1.5e4	1.3e4	15	8.1e4	7.7e4	8.4e4	8.1e4
1e-8	15	6.5e2	5.0e2	7.8e2	6.5e2	15	3.8e3	3.2e3	4.5e3	3.8e3	1e-8	15	3.0e4	2.4e4	3.7e4	3.0e4	15	2.3e5	1.7e5	3.0e5	2.3e5
$f_{11}$ in 5-D, N=15, mFE=8585					$f_{11}$ in 20-D, N=15, mFE=131357					$f_{12}$ in 5-D, N=15, mFE=4396					$f_{12}$ in 20-D, N=15, mFE=28383						
10	15	5.0e2	4.4e2	5.6e2	5.0e2	15	1.5e4	1.4e4	1.5e4	1.5e4	10	15	3.7e2	2.6e2	5.1e2	3.7e2	15	3.1e3	2.2e3	4.1e3	3.1e3
1	15	9.5e2	8.3e2	1.1e3	9.5e2	15	2.9e4	2.8e4	3.0e4	2.9e4	1	15	6.9e2	5.0e2	8.9e2	6.9e2	15	5.9e3	4.4e3	7.3e3	5.9e3
1e-1	15	1.4e3	1.3e3	1.5e3	1.4e3	15	3.6e4	3.5e4	3.7e4	3.6e4	1e-1	15	9.2e2	6.9e2	1.2e3	9.2e2	15	8.3e3	6.9e3	9.7e3	8.3e3
1e-3	15	2.1e3	2.0e3	2.2e3	2.1e3	15	6.0e4	5.9e4	6.1e4	6.0e4	1e-3	15	1.2e3	9.2e2	1.5e3	1.2e3	15	1.0e4	9.1e3	1.2e4	1.0e4
1e-5	15	2.9e3	2.8e3	3.0e3	2.9e3	15	8.1e4	8.0e4	8.2e4	8.1e4	1e-5	15	1.5e3	1.1e3	1.8e3	1.5e3	15	1.2e4	1.1e4	1.4e4	1.2e4
1e-8	15	4.7e3	4.2e3	5.2e3	4.7e3	15	1.1e5	1.1e5	1.1e5	1.1e5	1e-8	15	1.8e3	1.4e3	2.2e3	1.8e3	15	1.5e4	1.3e4	1.6e4	1.5e4
$f_{13}$ in 5-D, N=15, mFE=42403					$f_{13}$ in 20-D, N=15, mFE=186688					$f_{14}$ in 5-D, N=15, mFE=500000					$f_{14}$ in 20-D, N=15, mFE=2.00e6						
10	15	4.2e2	2.8e2	5.6e2	4.2e2	15	6.5e2	2.9e2	1.0e3	6.5e2	10	15	1.6e1	1.5e1	1.8e1	1.6e1	15	1.1e2	9.8e1	1.3e2	1.1e2
1	15	1.8e3	1.3e3	2.4e3	1.8e3	15	6.2e3	3.8e3	8.6e3	6.2e3	1	15	4.1e1	3.7e1	4.4e1	4.1e1	15	2.4e2	2.1e2	2.6e2	2.4e2
1e-1	15	8.7e3	6.3e3	1.1e4	8.7e3	15	2.6e4	1.8e4	3.4e4	2.6e4	1e-1	15	5.8e1	5.4e1	6.2e1	5.8e1	15	3.0e2	2.8e2	3.3e2	3.0e2
1e-3	7	7.0e4	5.0e4	1.1e5	2.9e4	6	3.5e5	2.3e5	6.0e5	1.3e5	1e-3	15	1.7e2	1.6e2	1.8e2	1.7e2	15	9.3e2	8.9e2	9.7e2	9.3e2
1e-5	1	5.9e5	2.9e5	>6e5	4.0e4	0	<i>43e-4</i>	<i>23e-5</i>	<i>21e-3</i>	8.9e4	1e-5	15	1.4e3	1.3e3	1.5e3	1.4e3	15	1.5e4	1.4e4	1.5e4	1.5e4
1e-8	0	<i>17e-4</i>	<i>18e-6</i>	<i>51e-4</i>	2.0e4						1e-8	0	<i>12e-8</i>	<i>74e-9</i>	<i>17e-8</i>	2.0e5	0	<i>40e-9</i>	<i>31e-9</i>	<i>98e-9</i>	1.4e6
$f_{15}$ in 5-D, N=15, mFE=25959					$f_{15}$ in 20-D, N=15, mFE=134552					$f_{16}$ in 5-D, N=15, mFE=36861					$f_{16}$ in 20-D, N=15, mFE=233591						
10	15	2.9e3	2.2e3	3.7e3	2.9e3	0	<i>12e+1</i>	<i>11e+1</i>	<i>15e+1</i>	5.0e4	10	15	2.6e2	1.2e2	4.0e2	2.6e2	15	2.2e4	1.4e4	3.1e4	2.2e4
1	1	3.8e5	1.9e5	>4e5	2.5e4						1	14	1.8e4	1.3e4	2.3e4	1.5e4	0	<i>53e-1</i>	<i>40e-1</i>	<i>63e-1</i>	1.0e5
1e-1	0	<i>30e-1</i>	<i>20e-1</i>	<i>40e-1</i>	8.9e3						1e-1	0	<i>50e-2</i>	<i>23e-2</i>	<i>81e-2</i>	2.0e4					
1e-3											1e-3										
1e-5											1e-5										
1e-8											1e-8										
$f_{17}$ in 5-D, N=15, mFE=76530					$f_{17}$ in 20-D, N=15, mFE=2.00e6					$f_{18}$ in 5-D, N=15, mFE=500000					$f_{18}$ in 20-D, N=15, mFE=2.00e6						
10	15	1.2e1	9.9e0	1.4e1	1.2e1	15	1.0e3	1.3e2	2.0e3	1.0e3	10	15	3.3e3	1.2e3	5.4e3	3.3e3	3	7.4e6	4.4e6	2.2e7	1.6e6
1	14	8.6e3	5.3e3	1.3e4	6.5e3	0	<i>38e-1</i>	<i>30e-1</i>	<i>46e-1</i>	7.9e5	1	4	5.1e5	2.7e5	1.2e6	1.2e5	0	<i>11e+0</i>	<i>93e-1</i>	<i>16e+0</i>	7.1e5
1e-1	1	5.6e5	2.4e5	>6e5																	

f1 in 5-D, N=15, mFE=22					f1 in 20-D, N=15, mFE=236					f2 in 5-D, N=15, mFE=50000					f2 in 20-D, N=15, mFE=200000						
$\Delta f$	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	
10	15	2.1e1	2.0e1	2.2e1	2.1e1	15	2.3e2	2.3e2	2.3e2	2.3e2	10	15	5.8e2	4.2e2	6.9e2	5.8e2	12	1.7e5	1.6e5	1.9e5	1.3e5
1	15	2.2e1	2.2e1	2.2e1	2.2e1	15	2.3e2	2.3e2	2.3e2	2.3e2	1	15	1.6e3	1.3e3	2.0e3	1.6e3	2	1.5e6	1.4e6	1.5e6	1.7e5
1e-1	15	2.2e1	2.2e1	2.2e1	2.2e1	15	2.3e2	2.3e2	2.4e2	2.3e2	1e-1	15	3.2e3	2.6e3	3.8e3	3.2e3	0	<i>47e-1</i>	<i>58e-2</i>	<i>22e+0</i>	2.0e5
1e-3	15	2.2e1	2.2e1	2.2e1	2.2e1	15	2.3e2	2.3e2	2.3e2	2.3e2	1e-3	15	6.2e3	5.8e3	6.7e3	6.2e3					
1e-5	15	2.2e1	2.2e1	2.2e1	2.2e1	15	2.3e2	2.3e2	2.3e2	2.3e2	1e-5	15	9.5e3	9.3e3	1.0e4	9.5e3					
1e-8	15	2.2e1	2.2e1	2.2e1	2.2e1	15	2.3e2	2.3e2	2.3e2	2.3e2	1e-8	14	2.3e4	2.1e4	2.8e4	2.2e4					
f3 in 5-D, N=15, mFE=37504					f3 in 20-D, N=15, mFE=200000					f4 in 5-D, N=15, mFE=50000					f4 in 20-D, N=15, mFE=200000						
$\Delta f$	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	
10	15	3.0e3	2.4e3	3.6e3	3.0e3	0	<i>88e+0</i>	<i>71e+0</i>	<i>11e+1</i>	8.9e4	10	15	1.0e4	7.9e3	1.2e4	1.0e4	0	<i>13e+1</i>	<i>10e+1</i>	<i>16e+1</i>	1.0e5
1	2	2.7e5	2.6e5	2.8e5	3.4e4						1	0	<i>30e-1</i>	<i>20e-1</i>	<i>60e-1</i>	2.2e4					
1e-1	0	<i>20e-1</i>	<i>99e-2</i>	<i>30e-1</i>	1.8e4						1e-1										
1e-3											1e-3										
1e-5											1e-5										
1e-8											1e-8										
f5 in 5-D, N=15, mFE=32					f5 in 20-D, N=15, mFE=313					f6 in 5-D, N=15, mFE=8034					f6 in 20-D, N=15, mFE=11418						
$\Delta f$	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	
10	15	2.2e1	2.1e1	2.3e1	2.2e1	15	2.5e2	2.5e2	2.6e2	2.5e2	10	15	1.3e2	1.1e2	1.7e2	1.3e2	15	1.9e3	1.8e3	1.9e3	1.9e3
1	15	2.4e1	2.4e1	2.5e1	2.4e1	15	2.6e2	2.5e2	2.7e2	2.6e2	1	15	2.1e2	2.0e2	3.0e2	2.1e2	15	2.5e3	2.4e3	2.6e3	2.5e3
1e-1	15	2.4e1	2.3e1	2.5e1	2.4e1	15	2.7e2	2.6e2	2.8e2	2.7e2	1e-1	15	2.8e2	2.2e2	3.2e2	2.8e2	15	3.5e3	3.3e3	3.7e3	3.5e3
1e-3	15	2.4e1	2.3e1	2.5e1	2.4e1	15	2.7e2	2.6e2	2.7e2	2.7e2	1e-3	15	5.8e2	4.7e2	6.7e2	5.8e2	15	5.2e3	5.0e3	5.4e3	5.2e3
1e-5	15	2.4e1	2.3e1	2.5e1	2.4e1	15	2.7e2	2.6e2	2.8e2	2.7e2	1e-5	15	1.0e3	7.2e2	1.3e3	1.0e3	15	6.7e3	6.5e3	6.8e3	6.7e3
1e-8	15	2.4e1	2.4e1	2.5e1	2.4e1	15	2.7e2	2.6e2	2.7e2	2.7e2	1e-8	15	2.2e3	1.5e3	3.3e3	2.2e3	15	9.4e3	9.1e3	9.6e3	9.4e3
f7 in 5-D, N=15, mFE=43917					f7 in 20-D, N=15, mFE=200000					f8 in 5-D, N=15, mFE=1514					f8 in 20-D, N=15, mFE=15124						
$\Delta f$	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	
10	15	2.4e1	2.3e1	2.4e1	2.4e1	15	6.3e3	5.1e3	7.9e3	6.3e3	10	15	1.2e2	8.6e1	1.3e2	1.2e2	15	2.8e3	2.7e3	3.0e3	2.8e3
1	15	3.7e2	3.0e2	5.0e2	3.7e2	1	3.0e6	3.0e6	3.0e6	2.0e5	1	15	2.7e2	1.9e2	4.6e2	2.7e2	15	6.0e3	4.3e3	7.0e3	6.0e3
1e-1	15	4.7e3	2.9e3	6.2e3	4.7e3	0	<i>27e-1</i>	<i>15e-1</i>	<i>39e-1</i>	7.1e4	1e-1	15	3.4e2	2.5e2	5.1e2	3.4e2	15	6.7e3	5.2e3	8.4e3	6.7e3
1e-3	11	3.7e4	3.3e4	4.3e4	2.7e4						1e-3	15	3.9e2	2.7e2	4.6e2	3.9e2	15	7.2e3	6.1e3	8.5e3	7.2e3
1e-5	11	3.7e4	3.4e4	4.2e4	2.7e4						1e-5	15	4.1e2	3.1e2	4.9e2	4.1e2	15	7.4e3	6.1e3	8.9e3	7.4e3
1e-8	11	3.7e4	3.1e4	4.2e4	2.7e4						1e-8	15	4.2e2	3.3e2	5.8e2	4.2e2	15	7.5e3	6.1e3	8.6e3	7.5e3
f9 in 5-D, N=15, mFE=1278					f9 in 20-D, N=15, mFE=15428					f10 in 5-D, N=15, mFE=50000					f10 in 20-D, N=15, mFE=200000						
$\Delta f$	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	
10	15	9.1e1	7.2e1	1.0e2	9.1e1	15	3.2e3	2.9e3	3.5e3	3.2e3	10	15	1.2e3	1.0e3	1.5e3	1.2e3	10	2.5e5	2.4e5	2.7e5	1.6e5
1	15	3.4e2	2.8e2	4.0e2	3.4e2	15	6.9e3	6.1e3	9.0e3	6.9e3	1	15	3.1e3	2.8e3	3.3e3	3.1e3	0	<i>67e-1</i>	<i>38e-1</i>	<i>60e-1</i>	2.0e5
1e-1	15	4.1e2	3.4e2	5.3e2	4.1e2	15	7.6e3	7.0e3	9.3e3	7.6e3	1e-1	15	5.0e3	4.7e3	5.4e3	5.0e3					
1e-3	15	4.6e2	3.4e2	4.9e2	4.6e2	15	8.1e3	6.9e3	9.0e3	8.1e3	1e-3	15	8.2e3	7.8e3	8.8e3	8.2e3					
1e-5	15	4.8e2	3.7e2	5.5e2	4.8e2	15	8.3e3	7.4e3	9.3e3	8.3e3	1e-5	15	1.1e4	1.1e4	1.2e4	1.1e4					
1e-8	15	4.9e2	4.4e2	5.6e2	4.9e2	15	8.4e3	7.1e3	9.9e3	8.4e3	1e-8	12	3.5e4	2.7e4	3.9e4	2.6e4					
f11 in 5-D, N=15, mFE=50000					f11 in 20-D, N=15, mFE=200000					f12 in 5-D, N=15, mFE=6362					f12 in 20-D, N=15, mFE=200000						
$\Delta f$	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	
10	15	1.5e3	1.3e3	1.6e3	1.5e3	15	5.8e4	5.5e4	6.6e4	5.8e4	10	15	4.0e2	2.0e2	7.9e2	4.0e2	15	1.1e4	8.2e3	1.5e4	1.1e4
1	15	2.9e3	2.6e3	3.0e3	2.9e3	15	1.0e5	9.3e4	1.1e5	1.0e5	1	15	7.0e2	4.6e2	9.4e2	7.0e2	15	2.9e4	2.5e4	3.7e4	2.9e4
1e-1	15	4.2e3	3.8e3	4.6e3	4.2e3	15	1.3e5	1.3e5	1.4e5	1.3e5	1e-1	15	1.0e3	6.2e2	1.3e3	1.0e3	14	7.2e4	5.5e4	8.8e4	6.9e4
1e-3	15	6.3e3	6.0e3	6.7e3	6.3e3	8	3.5e5	3.5e5	3.6e5	1.8e5	1e-3	15	1.5e3	1.3e3	2.1e3	1.5e3	12	1.6e5	1.4e5	1.9e5	1.3e5
1e-5	15	8.6e3	7.9e3	8.9e3	8.6e3	0	<i>79e-5</i>	<i>32e-6</i>	<i>30e-4</i>	2.0e5	1e-5	15	2.1e3	1.7e3	2.7e3	2.1e3	6	4.4e5	4.2e5	4.7e5	1.8e5
1e-8	13	2.2e4	1.8e4	2.9e4	1.8e4	0	<i>30e-4</i>	<i>68e-6</i>	<i>31e-3</i>	1.4e5	1e-8	15	2.8e3	2.0e3	3.4e3	2.8e3	0	<i>42e-6</i>	<i>19e-9</i>	<i>19e-4</i>	1.4e5
f13 in 5-D, N=15, mFE=50000					f13 in 20-D, N=15, mFE=200000					f14 in 5-D, N=15, mFE=50000					f14 in 20-D, N=15, mFE=200000						
$\Delta f$	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	
10	15	2.4e2	1.3e2	3.3e2	2.4e2	15	1.2e3	1.1e3	1.2e3	1.2e3	10	15	2.7e1	2.5e1	2.9e1	2.7e1	15	4.4e2	4.1e2	4.6e2	4.4e2
1	15	1.3e3	7.7e2	1.8e3	1.3e3	15	1.2e4	9.1e3	1.6e4	1.2e4	1	15	4.3e1	4.0e1	4.6e1	4.3e1	15	7.2e2	6.9e2	7.5e2	7.2e2
1e-1	15	5.6e3	4.0e3	8.7e3	5.6e3	15	5.1e4	4.1e4	7.1e4	5.1e4	1e-1	15	6.5e1	6.3e1	6.7e1	6.5e1	15	1.1e3	1.1e3	1.1e3	1.1e3
1e-3	5	1.3e5	1.1e5	1.4e5	4.7e4	5	4.8e5	4.1e5	5.2e5	2.0e5	1e-3	15	1.4e2	1.3e2	1.5e2	1.4e2	15	2.4e3	2.3e3	2.5e3	2.4e3
1e-5	0	<i>19e-4</i>	<i>16e-5</i>	<i>11e-3</i>	2.5e4	1	2.8e6	2.7e6	3.0e6	2.0e5	1e-5	15	1.8e2	1.7e2	1.9e2	1.8e2	15	3.2e4	3.2e4	3.3e4	3.2e4
1e-8						0	<i>30e-4</i>	<i>68e-6</i>	<i>31e-3</i>	1.4e5	1e-8	0	<i>48e-9</i>	<i>38e-9</i>	<i>59e-9</i>	3.2e4	0	<i>12e-7</i>	<i>11e-7</i>	<i>12e-7</i>	2.0e5
f15 in 5-D, N=15, mFE=35310					f15 in 20-D, N=15, mFE=200000					f16 in 5-D, N=15, mFE=48848					f16 in 20-D, N=15, mFE=200000						
$\Delta f$	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	#	ERT	10%	90%	RT <sub>succ</sub>	
10	15	3.2e3	2.1e3	4.0e3	3.2e3	0	<i>77e+0</i>	<i>66e+0</i>	<i>11e+1</i>	1.4e5	10	15	3.2e2	1.8e2	3.8e2	3.2e2	15	6.4e3	2.7e3	8.4e3	6.4e3
1	1	5.1e5	5.0e5	5.2e5	3.4e4						1	15	7.1e3	5.6e3	9.1e3	7.1e3	1	2.9e6	2.8e6	3.0e6	2.0e5
1e-1	0	<i>50e-1</i>	<i>20e-1</i>	<i>70e-1</i>	1.1e4						1e-1	7	7.7e4	6.4e4	8.9e4	3.5e4	0	<i>25e-1</i>	<i>12e-1</i>		

Figure 1: Multistart NEWUOA, the number of interpolation points is two times the dimension plus one.

---

```
#include <stdlib.h>
#include <math.h>
#include <stdio.h>
#include "bbobStructures.h"

/* Call to the Fortran function */
extern void newuoa_(unsigned int* n, int* m, double* x0, double* rhobeg,
                  double* rhoend, int* verbose, int* maxfun,
                  double* W, double* ftarget);

/* The Multistart NEWUOA */
void newuoa(unsigned int dim, unsigned int maxfunevals, double ftarget)
{
    int m, iprint = 0, curmaxfun;
    double * x = malloc(sizeof(double) * dim);
    unsigned int iter = 0, i;
    double rhobeg = 10, rhoend = 1e-16;
    /* internal variable of NEWUOA */
    double * w = malloc(1000000 * sizeof(double));

    m = 2 * dim + 1;

    curmaxfun = maxfunevals - fgeneric_evaluations();
    while (curmaxfun > 0 && fgeneric_best() > ftarget && iter < 100)
    {
        /* Generate a starting point */
        for (i = 0; i < dim; i++)
            x[i] = 10. * ((double)rand() / RAND_MAX) - 5.;
        /* Call NEWUOA */
        newuoa_(&dim, &npt, x, &rhobeg, &rhoend, &iprint, &curmaxfun, w, &ftarget);
        /* Update */
        curmaxfun = maxfunevals - fgeneric_evaluations();
        iter++;
    }
    free(x);
    free(w);
}
```

---

points on the interpolation of the model. To the contrary, the performances only seem to scale only worse resulting in failures in larger dimensions, for instance on  $f_2$  or  $f_{12}$  with the exception of  $f_7$  which the full model NEWUOA solves in 5-D.

#### 4. CPU TIMING EXPERIMENT

The proposed algorithm was run on  $f_8$  and restarted until at least 30 seconds have passed. The experiments were conducted with an Intel Core 2 6700 processor (2.66GHz) on Linux 2.6.24.7. The results were 130, 73, 45, 18, 2.2, 26 for  $m = 2n + 1$  and 200, 86, 45, 7.9, 3.7,  $36 \times 10^{-3}$  seconds per function evaluations for the full model in dimension 2, 3, 5, 10, 20 and 40 respectively.

#### Acknowledgments

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#### 5. REFERENCES

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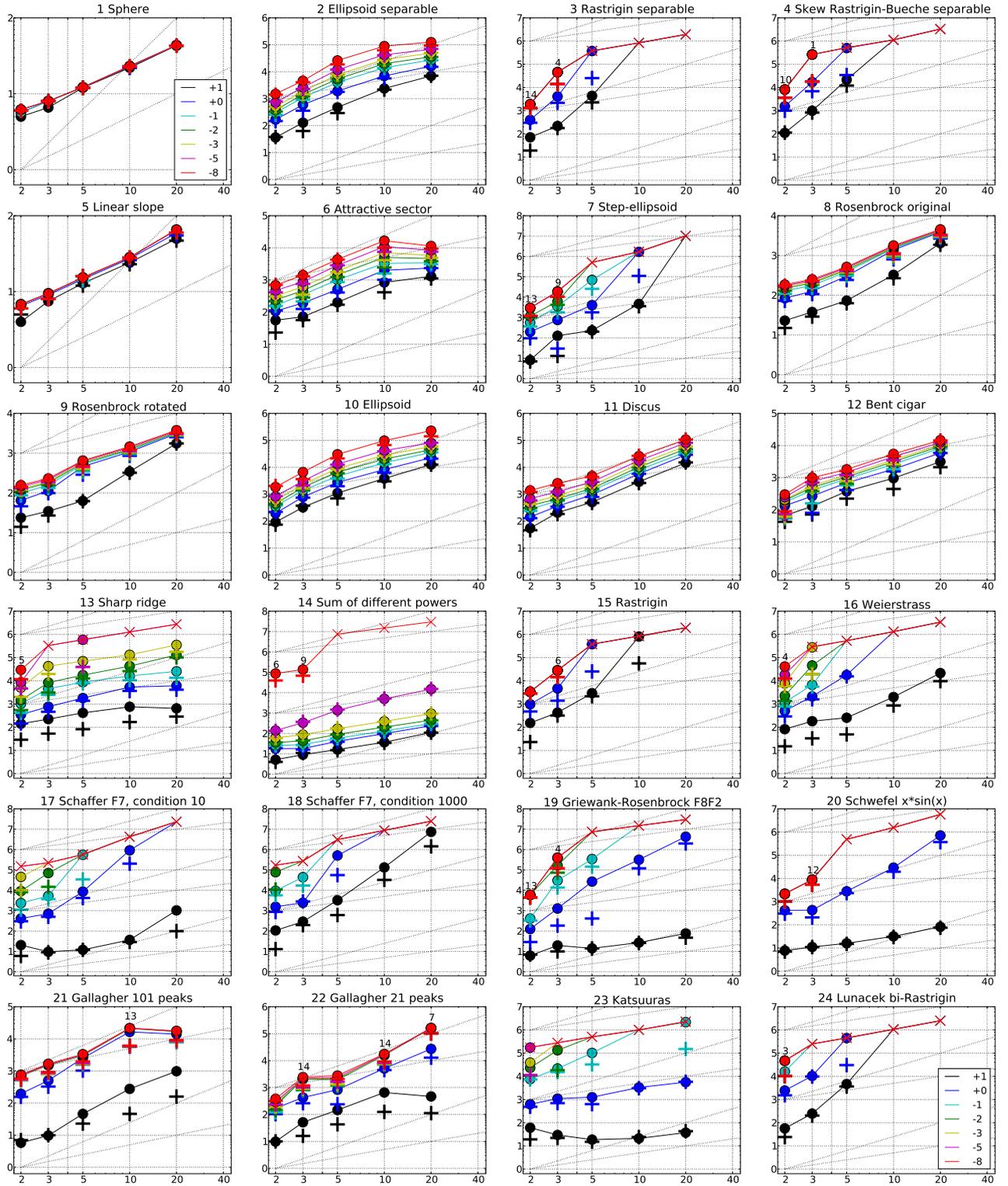
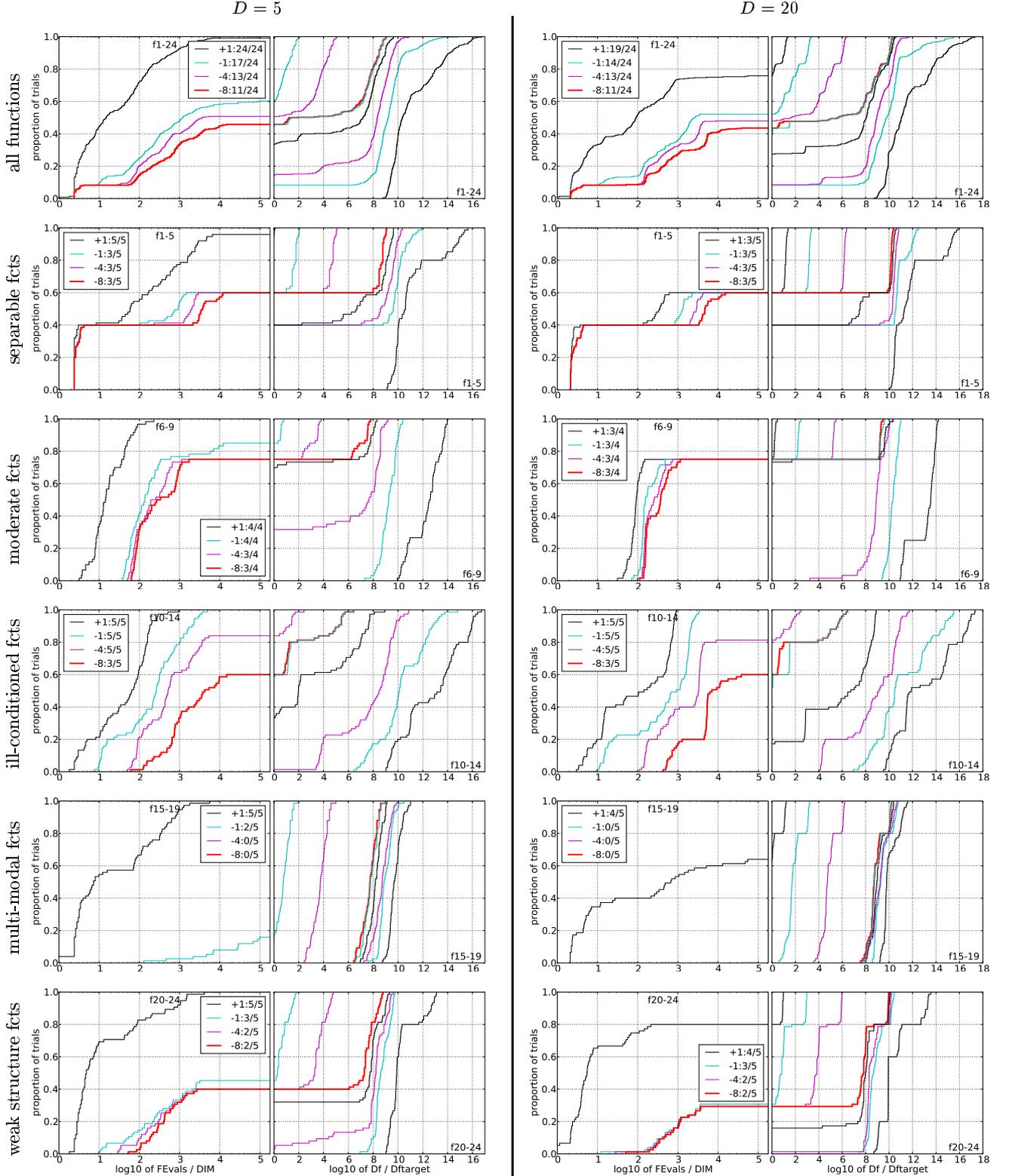


Figure 2: NEWUOA,  $2n + 1$  interpolation points. Expected Running Time (ERT,  $\bullet$ ) to reach  $f_{\text{opt}} + \Delta f$  and median number of function evaluations of successful trials (+), shown for  $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$  (the exponent is given in the legend of  $f_1$  and  $f_{24}$ ) versus dimension in log-log presentation. The  $\text{ERT}(\Delta f)$  equals to  $\#FES(\Delta f)$  divided by the number of successful trials, where a trial is successful if  $f_{\text{opt}} + \Delta f$  was surpassed during the trial. The  $\#FES(\Delta f)$  are the total number of function evaluations while  $f_{\text{opt}} + \Delta f$  was not surpassed during the trial from all respective trials (successful and unsuccessful), and  $f_{\text{opt}}$  denotes the optimal function value. Crosses ( $\times$ ) indicate the total number of function evaluations  $\#FES(-\infty)$ . Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.



**Figure 3: NEWUOA,  $2n + 1$  interpolation points.** Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus  $\Delta f$  (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension  $D$ , to fall below  $f_{\text{opt}} + \Delta f$  with  $\Delta f = 10^k$ , where  $k$  is the first value in the legend. Right subplots: ECDF of the best achieved  $\Delta f$  divided by  $10^k$  (upper left lines in continuation of the left subplot), and best achieved  $\Delta f$  divided by  $10^{-8}$  (lower left lines in continuation of the left subplot) for running times of  $D, 10D, 100D \dots$  function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: separable functions; third row: misc. moderate functions; fourth row: ill-conditioned functions; fifth row: multi-modal functions with adequate structure; last row: multi-modal functions with weak structure. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations,  $D$  and DIM denote search space dimension, and  $\Delta f$  and Df denote the difference to the optimal function value.

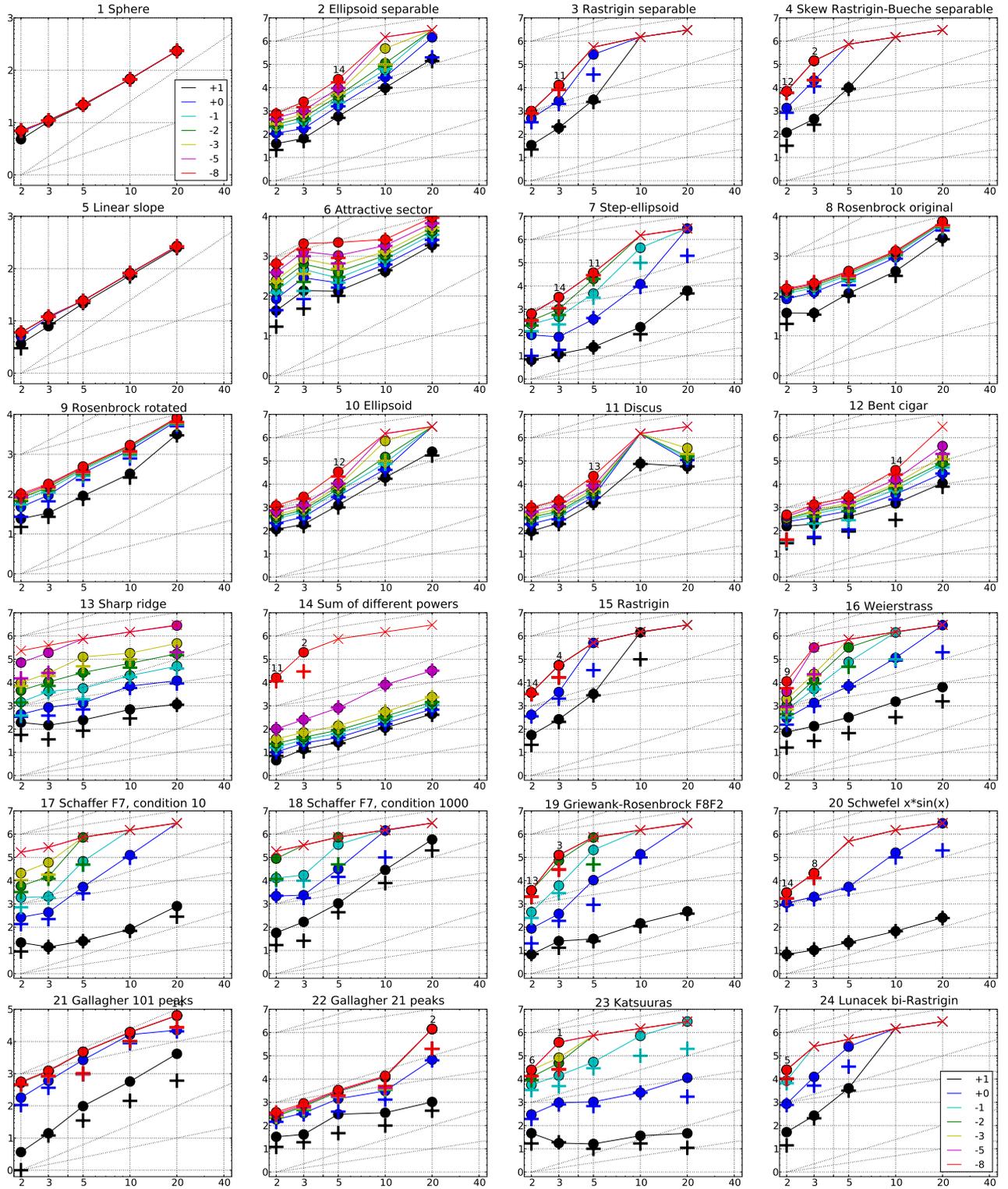
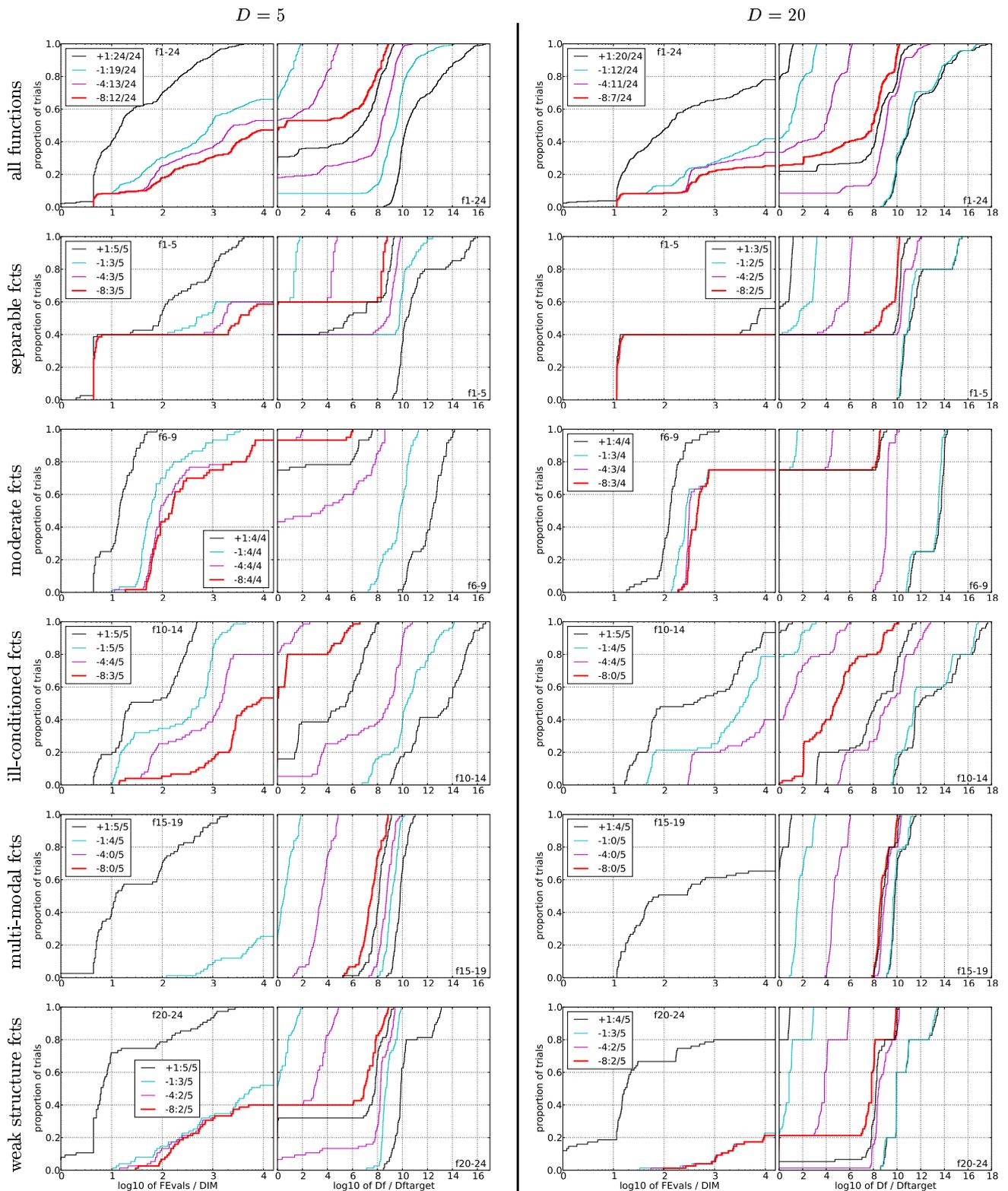


Figure 4: NEWUOA, full model. Expected Running Time (ERT,  $\bullet$ ) to reach  $f_{\text{opt}} + \Delta f$  and median number of function evaluations of successful trials (+), shown for  $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$  (the exponent is given in the legend of  $f_1$  and  $f_{24}$ ) versus dimension in log-log presentation. The ERT( $\Delta f$ ) equals to  $\#FEs(\Delta f)$  divided by the number of successful trials, where a trial is successful if  $f_{\text{opt}} + \Delta f$  was surpassed during the trial. The  $\#FEs(\Delta f)$  are the total number of function evaluations while  $f_{\text{opt}} + \Delta f$  was not surpassed during the trial from all respective trials (successful and unsuccessful), and  $f_{\text{opt}}$  denotes the optimal function value. Crosses ( $\times$ ) indicate the total number of function evaluations  $\#FEs(-\infty)$ . Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.



**Figure 5: NEWUOA, full model.** Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus  $\Delta f$  (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension  $D$ , to fall below  $f_{\text{opt}} + \Delta f$  with  $\Delta f = 10^k$ , where  $k$  is the first value in the legend. Right subplots: ECDF of the best achieved  $\Delta f$  divided by  $10^k$  (upper left lines in continuation of the left subplot), and best achieved  $\Delta f$  divided by  $10^{-8}$  for running times of  $D, 10D, 100D \dots$  function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: separable functions; third row: misc. moderate functions; fourth row: ill-conditioned functions; fifth row: multi-modal functions with adequate structure; last row: multi-modal functions with weak structure. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations,  $D$  and DIM denote search space dimension, and  $\Delta f$  and Df denote the difference to the optimal function value.