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Benchmarking the BFGS Algorithm on the BBOB-2009 Function Testbed

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ABSTRACT

The BFGS quasi-Newton method is benchmarked on the noiseless BBOB-2009 testbed. A multistart strategy is applied with a maximum number of function evaluations of 10^5 times the search space dimension, resulting in the algorithm solving six functions.

Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—global optimization, unconstrained optimization; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, BFGS, Quasi-newton

1. INTRODUCTION

The Broyden-Fletcher-Goldfarb-Shanno (BFGS) method [1, 3, 4, 7] is a real-parameter unconstrained non-linear optimization method. The BFGS method belongs to the class of quasi-Newton methods which, by supposing that the objective function can be locally approximated by a quadratic function near its optimum, tries to approximate the Hessian matrix of this quadratic function. The BFGS method is tested here on a testbed of noiseless functions.

2. ALGORITHM PRESENTATION

Quasi-Newton methods address the problem of unconstrained black-box optimization by the determination of the stationary point of a function using a second-order approximation. The Hessian matrix is iteratively approximated by finding a search direction using the gradient of the current point and operating a line search to find the step size. For

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each iteration D + 1 function evaluations (where D denotes the problem dimension) or double if the step size changes, are done to evaluate the gradient and the function value at the current point. A simple stochastic independent restart procedure (as advised in [5]) was added.

3. EXPERIMENTAL PROCEDURE

The Matlab implementation of the BFGS method was used. It is accessible using the generic function fminunc (revision 1.1.6.3) that proposes, among others, the BFGS method for the update of the Hessian Matrix. The starting point is chosen uniformly in $[-5,5]^D$. The stopping criteria were chosen such that a restart occurs due to numerical errors. The multistart strategy was used with at most 100 restarts to reduce the duration of an experiment. For the same reason, a run is limited to at most $10^5 \times D$ function evaluations. The algorithm used is presented in Figure 1. No parameter tuning was done, the CrE [5] is computed to zero.

4. RESULTS AND DISCUSSION

Results from experiments according to [5] on the benchmark functions given in [2, 6] are presented in Figures 2 and 3 and in Table 1. As expected, poor results were obtained on multimodal functions f_3 , f_4 , f_{15-19} , f_{24} . The algorithm could not solve the rugged functions f_7 , f_{13} , f_{23} neither. It only solves f_6 , in dimensions lesser than 5. Though the functions f_{21} and f_{22} are multimodal and have weak global structure, the algorithm still solves them even for dimensions larger than 10. The algorithm solves f_1 and f_5 with a scaling of its performances close to linear. The scaling is close to quadratic for f_2 , f_8 and f_9 . The algorithm is coordinate-dependent as a result of the computation of the gradient using finite differences which can yield numerical errors on ill-conditioned functions as can be seen in the results on f_{10} compared to f_9 . The effects of ill-conditioning on the performances of the algorithm can also be seen in the results on f_{11} and f_{14} .

5. CPU TIMING EXPERIMENT

For the timing experiment, the proposed algorithm was run on f_8 and restarted until at least 30 seconds have passed (according to Figure 2 in [5]). The experiments were conducted with an Intel Core 2 6700 processor (2.66GHz) with Matlab R2008a on Linux 2.6.24.7. The results were 6.0, 4.7, 3.7, 3.0, 2.9, 2.9 and 2.8 ×10⁻⁴ seconds per function evaluations in dimension 2, 3, 5, 10, 20, 40 and 80 respectively.

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f1 in 5-D, N=15, mFE=18 $f1$ in 20-D, N=15, mFE=18 $#$ EBT 10% 90% BTeuron $#$ EBT 10% 90%	mFE=63 BTauga	<i>f2</i> in 5-D, N=15, mFE=948 <i>f2</i> in 20-D, N=15, mFE=12516
10 15 1.3e1 1.3e1 1.3e1 1.3e1 1.5e1 4.3e1 4.3e1 4.3e1	1 4.3e1	$\begin{array}{c} 10 \\ 15 \\ 15 \\ 4 \\ 5 \\ 2 \\ 2 \\ 3 \\ 2 \\ 2 \\ 3 \\ 2 \\ 2 \\ 3 \\ 2 \\ 2$
1 = 1 = 1 = 1.3	1 4.3e1	1 = 15 + 3.5 + 2 + 3.62 + 3.42 + 3.42 = 15 + 3.263 + 3.463 + 3.463 + 3.263 $1e - 1 = 15 + 5.5 + 2 + 5.162 + 5.92 + 5.5 + 2 = 15 + 1.0 + 4 + 1.0 + 4$
1e-3 15 1.3e1 1.3e1 1.3e1 1.3e1 1.3e1 15 4.3e1 4.3e1 4.3e1 1e-5 15 1.3e1 1.3e1 1.3e1 1.3e1 1.3e1 15 4.3e1 4.3e1 4.3e1	l 4.3el l 4.3el	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
1e-8 15 1.3e1 1.3e1 1.3e1 1.3e1 1.3e1 15 4.3e1 4.3e	L 4.3e1 E=129192	1e-8 15 6.9e2 6.4e2 7.3e2 6.9e2 15 1.1e4 1.1e
$\Delta f \ \# \ \text{ERT} \ 10\% \ 90\% \ \text{RT}_{\text{succ}} \ \# \ \text{ERT} \ 10\% \ 90\%$	RT _{succ}	Δf # ERT 10% 90% RT _{succ} # ERT 10% 90% RT _{succ}
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	5.6e4	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
1e-1		1e-1
1e-5		1e-5
f5 in 5-D, N=15, mFE=48 f5 in 20-D, N=15, mF	FE=189	$\begin{array}{c} f_{6} \text{ in 5-D, N=15, mFE=155790} \\ f_{6} \text{ in 5-D, N=15, mFE=2.00e6} \\ f_{6} \text{ in 20-D, N=15, mFE=2.00e6} \\ f_{6} \text{ in 20-D, N=15, mFE=2.00e6} \\ f_{6} \text{ in 5-D, N=15, mFE=2.00e6} \\ f_$
$\frac{\Delta f}{10} = \frac{\#}{15} \frac{\#}{1.9e1} \frac{10\%}{1.8e1} \frac{90\%}{2.0e1} \frac{\#}{1.9e1} \frac{\#}{15} \frac{\#}{9.6e1} \frac{10\%}{9.1e1} \frac{90\%}{1.0e2}$	9.6e1	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1.1e2 1.1e2	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
le-3 15 3.1e1 2.9e1 3.3e1 3.1e1 15 1.1e2 1.1e2 1.2e2 le-5 15 3.1e1 2.9e1 3.3e1 3.1e1 15 1.1e2 1.1e2 1.2e2	1.1e2 1.1e2	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
1e-8 15 3.1e1 2.9e1 3.3e1 3.1e1 15 1.1e2 1.1e2 1.2e2	1.1e2	1e-8 15 7.1e4 5.5e4 8.7e4 7.1e4 0 44e-9 31e-9 77e-9 8.9e5
$\Delta f = \frac{1}{4} \text{ ERT } 10\% 90\% \text{ RT}_{\text{succ}} = \frac{1}{4} \text{ ERT } 10\% 90\%$	$6 RT_{succ}$	Δf # ERT 10% 90% RT _{succ} # ERT 10% 90% RT _{succ}
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	-z 1.3e3	10 15 1.5e2 1.3e2 1.8e2 1.5e2 15 3.7e3 3.5e3 3.8e3 3.7e3 1 15 4.9e2 3.7e2 6.3e2 4.9e2 15 4.7e3 4.6e3 4.9e3 4.7e3
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		$\begin{array}{cccccccccccccccccccccccccccccccccccc$
1e-5	•	1e-5 15 6.0e2 4.9e2 7.4e2 6.0e2 15 5.2e3 5.0e3 5.3e3 5.2e3 1e-8 15 6.2e2 4.9e2 7.5e2 6.2e2 15 5.2e3 5.1e3 5.4e3 5.2e3
f9 in 5-D , N=15, mFE=936 f9 in 20-D , N=15, mFE	E=14028	f10 in 5-D, N=15, mFE=78708 f10 in 20-D, N=15, mFE=1.11e6
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	RT _{succ} 3.8e3	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	6.7e3 6.9e3	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$1e^{-3}$ 15 4.8e2 4.2e2 5.5e2 4.8e2 15 7.1e3 6.1e3 8.1e3	7.1e3	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	7.2e3	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$ \begin{array}{c} f_{11} \text{ in } 5\text{-}\text{D}, \text{ N}=15, \text{ mFE}=49422 \\ \Delta f & \# \text{ ERT } 10\% 90\% \text{ RT}_{\text{SUCC}} \end{array} \begin{array}{c} f_{11} \text{ in } 20\text{-}\text{D}, \text{ N}=15, \text{ mF} \\ \# \text{ ERT } 10\% 90\% \end{array} $	E=211239 RT _{succ}	$ \begin{array}{c} f_{12} \text{ in } 5\text{-D}, \text{ N} = 15, \text{ mFE} = 97074 \\ \# \text{ ERT } 10\% 90\% \text{ RT}_{\text{SUCC}} \end{array} \begin{array}{c} f_{12} \text{ in } 20\text{-D}, \text{ N} = 15, \text{ mFE} = 612423 \\ \# \text{ ERT } 10\% 90\% \text{ RT}_{\text{SUCC}} \end{array} $
10 15 1.4e2 1.5e2 1.4e2 15 1.0e3 8.5e2 1.2e3 1 15 2.0e2 1.8e2 2.3e2 2.0e2 15 2.2e3 1.8e3 2.7e3	1.0e3 2.2e3	10 15 1.2e2 9.4e1 1.5e2 1.2e2 15 1.6e3 1.1e3 2.2e3 1.6e3 1 15 2.7e2 2.3e2 3.1e2 2.7e2 15 3.2e3 2.4e3 4.0e3 3.2e3
$\begin{array}{c} 1 \\ 1e^{-1} \\ 15 \\ 8.4e^{2} \\ 6.4e^{2} \\ 1.1e^{3} \\ 8.4e^{2} \\ 1.1e^{3} \\ 8.4e^{2} \\ 1.1e^{3} \\ 8.4e^{2} \\ 1.1e^{3} \\ 1.1e^{3$	8.3e3	1 = 1 = 1 = 2.762 = 2.562 = 5.162 = 2.762 = 15 = 5.265 = 2.765 = 5.265 = 5.265 = 16 = 5.265 = 15 = 5.265 = 15 = 5.26
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	2.0e5 7.9e4	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	E=580545	le-8 5 1.4e5 8.5e4 2.9e5 3.7e4 1 7.9e6 3.7e6 >8e6 1.2e5 f14 in 5-D, N=15, mFE=37158 f14 in 20-D, N=15, mFE=257859
$\Delta f \ \# \ \text{ERT} \ 10\% \ 90\% \ \text{RT}_{\text{succ}} \ \# \ \text{ERT} \ 10\% \ 90\%$	RT _{succ}	$\frac{\Delta f}{4} \# \text{ ERT } 10\% 90\% \text{ RT}_{\text{succ}} \# \text{ ERT } 10\% 90\% \text{ RT}_{\text{succ}}$
10 13 1.3e2 1.2e2 1.4e2 1.3e2 13 1.1e3 1.1e3 1.1e3 1 15 1.9e2 1.9e2 1.9e2 1.9e2 15 2.0e3 2.0e3 2.0e3	2.0e3	10 13 2.2e1 1.7e1 2.2e1 13 2.0e2 1.8e2 2.3e2 2.0e2 1 15 7.0e1 5.7e1 8.4e1 7.0e1 15 4.4e2 4.0e2 4.7e2 4.4e2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	2.8e3 2.4e5	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1.8e5	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
f_{15} in 5-D, N=15, mFE=18486 f_{15} in 20-D, N=15, mF	E=128646 BT	f_{16} in 5-D, N=15, mFE=43056 f_{16} in 20-D, N=15, mFE=334509
$\begin{array}{c} 10 \\ 5 \\ 4.4e4 \\ 2.9e4 \\ 7.9e4 \\ 1.4e4 \\ 0 \\ 25e+1 \\ 18e+1 \\ 32e+1 \\ 16e+1 \\ 32e+1 \\ 18e+1 \\ 18$	7.1e4	$\begin{array}{c} 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 $
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	•	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
1e-8	E=410025	1e-8
Δf # ERT 10% 90% RT _{succ} # ERT 10% 90%	RT _{succ}	Δf # ERT 10% 90% RT _{succ} # ERT 10% 90% RT _{succ}
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	2.3e4 2.5e5	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	•	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		$\begin{array}{cccccccccccccccccccccccccccccccccccc$
f19 in 5-D, N=15, mFE=37734 f19 in 20-D, N=15, mF	E=255570	f20 in 5-D , N=15, mFE=28008 f20 in 20-D , N=15, mFE=420504
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	2.5e5	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	1.6e5	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		$\begin{array}{cccccccccccccccccccccccccccccccccccc$
1e-8	E=140112	1e-8 1 3.9e5 1.9e5 >4e5 2.7e4
Δf # ERT 10% 90% RT _{succ} # ERT 10% 90%	RT _{succ}	Δf # ERT 10% 90% RT _{succ} # ERT 10% 90% RT _{succ}
10 15 1.6e2 9.4e1 2.3e2 1.6e2 15 1.1e3 6.4e2 1.6e3 1 15 1.6e3 1.1e3 2.1e3 1.6e3 15 3.6e4 2.4e4 4.7e4	1.1e3 3.6e4	10 15 2.2e2 1.6e2 2.8e2 2.2e2 15 1.2e3 8.2e2 1.6e3 1.2e3 1 15 1.1e3 7.9e2 1.5e3 1.1e3 15 1.0e4 5.7e3 1.5e4 1.0e4
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$5.3e4 \\ 5.4e4$	le-1 15 2.0e3 1.5e3 2.4e3 2.0e3 7 1.9e5 1.2e5 3.2e5 7.2e4 le-3 15 2.1e3 1.6e3 2.5e3 2.1e3 7 1.9e5 1.3e5 3.2e5 7.3e4
1e-5 15 3.4e3 2.3e3 4.4e3 3.4e3 13 6.7e4 5.0e4 8.7e4 1e-8 15 3.7e3 2.5e3 4.9e3 3.7e3 2.86e5 4.3e5 2.2e6	5.5e4 9.1e4	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
f23 in 5-D, N=15, mFE=26718 f23 in 20-D, N=15, mF	E=115269	$\begin{array}{c} f_{24} \text{ in 5-D, N=15, mFE=16734} \\ f_{24} \text{ in 5-D, N=15, mFE=16734} \\ f_{24} \text{ in 20-D, N=15, mFE=122619} \\ \end{array}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	RT _{succ} 1.5e2	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	1.1e5 5.6e4	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
		1e-3
1e-8		1e-8

Table 1: Shown are, for a given target difference to the optimal function value Δf : the number of successful trials (#); the expected running time to surpass $f_{opt} + \Delta f$ (ERT, see Figure 2); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value (RT_{succ}). If $f_{opt} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf -value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 2 for the names of functions.

```
function [x, ilaunch] = MY_OPTIMIZER(FUN, D, ftarget, maxfunevals)
% minimizes FUN in D dimensions by independent restarts of fminunc (BFGS).
% ftarget and maxfunevals are additional external termination conditions.
% Search space is [-5, 5]^D
  % set options, make sure we always terminate
  options = optimset('fminunc');
  options = optimset(options, 'LargeScale', 'off'); % BFGS algorithm
options = optimset(options, 'MaxIter', inf, 'Tolfun', 1e-11, 'TolX', 0, ...
                       'OutputFcn', @callback, 'Display', 'off');
  maxfunevals = min(1e4*DIM, maxfunevals);
  % multistart such that ftarget is reached with reasonable prob.
  for ilaunch = 1:100; % relaunch optimizer up to 100 times
    options = optimset(options, 'MaxFunEvals', ...
                        maxfunevals - feval(FUN, 'evaluations'));
    x = fminunc(FUN, 10*rand(DIM,1)-5, options);
    if (feval(FUN, 'fbest') < ftarget || ...</pre>
        feval(FUN, 'evaluations') >= maxfunevals)
      break:
    end
  end
  function stop = callback(x, optimValues, state)
    stop = false;
    if optimValues.fval < ftarget
      stop = true;
    end
  end
end % function
```

6. CONCLUSION

The results of a quasi-Newton method algorithm with restarts were presented. The algorithm performs well on smooth functions, but its performances decrease on rugged and multimodal functions. Furthermore due to numerical error in the computation of the gradient in ill-conditioned problems, the algorithm is affected by coordinate systems transformations.

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Figure 2: Expected Running Time (ERT, •) to reach $f_{opt} + \Delta f$ and median number of function evaluations of successful trials (+), shown for $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ (the exponent is given in the legend of f_1 and f_{24}) versus dimension in log-log presentation. The ERT(Δf) equals to $\#FEs(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{opt} + \Delta f$ was surpassed during the trial. The $\#FEs(\Delta f)$ are the total number of function evaluations while $f_{opt} + \Delta f$ was not surpassed during the trial from all respective trials (successful and unsuccessful), and f_{opt} denotes the optimal function value. Crosses (×) indicate the total number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.



Figure 3: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus Δf (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D, to fall below $f_{opt} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of D, 10D, 100D... function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: separable functions; third row: misc. moderate functions; fourth row: ill-conditioned functions; fifth row: multi-modal functions with adequate structure; last row: multi-modal functions with weak structure. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.