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► **To cite this version:**

Thomas Bréhard, Emmanuel Duflos, Philippe Vanheeghe, Pierre-Arnaud Coquelin. Optimal Policies Search for Sensor Management. FUSION 2008, Jun 2008, Cologne, Germany. pp.1 - 8. inria-00368875

**HAL Id: inria-00368875**

**<https://inria.hal.science/inria-00368875v1>**

Submitted on 19 Mar 2009

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# Optimal Policies Search for Sensor Management : Application to the ESA Radar

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**Abstract**—This paper introduces a new approach to solve sensor management problems. Classically sensor management problems can be well formalized as Partially-Observed Markov Decision Processes (POMDP). The original approach developed here consists in deriving the optimal parameterized policy based on stochastic gradient estimation. We assume in this work that it is possible to learn the optimal policy off-line (in simulation) using models of the environment and of the sensor(s). The learned policy can then be used to manage the sensor(s). In order to approximate the gradient in a stochastic context, we introduce a new method to approximate the gradient, based on Infinitesimal Approximation (IPA). The effectiveness of this general framework is illustrated by the managing of an Electronically Scanned Array Radar.

**Keywords:** Sensor(s) Management, Partially Observable Markov Decision Process, Stochastic Gradient Estimation, AESA Radar.

## Sensor(s) Management Special Session

### I. INTRODUCTION

Years after years the complexity and the performances of many sensors have increased leading to more and more complex sensor(s)-based systems which supply the decision centers with an increasing amount of data. The number, the types and the agility of sensors along with the increased quality of data far outstrip the ability of a human to manage them: it is often difficult to compare how much information can be gained by way of a given management scheme [1]. It results from this the necessity to derive unmanned sensing platforms that have the capacity to adapt to their environment [2]. This problem is often referred as the *Sensor(s) Management Problem*. In more simple situations, the operational context may lead to works on sensor(s) management like in the *radar - infrared sensor* case [3]. A general definition of this problem could then be : sensor management is the effective use of available sensing and database capabilities to meet the mission goals. Many applications deal with military applications, a classical one being to detect, to track and to identify smart targets (a smart target can change its way of moving or its way of sensing when it detects it is under analysis) with several

sensors. The questions are then the following at each time: how must we group the sensors, how long, in which direction, and with which functioning mode? The increasing complexity of the targets to be detected, tracked and identified, makes the management even more difficult and led to the development of researches on the definition of an optimal sensor management scheme in which the targets and the sensors are treated altogether in a complex dynamic system [4].

Sensor Management has become very popular this last years and many approaches can be found in the litterature. In [5] and [6] the authors use a the modelling of the detection process of an Electronically Scanned Array (ESA) Radar to propose management scheme during the detection step. In [7]–[9] an information-based approach is use to manage a set of sensors. From a theoretical point of view the sensor management can be modelled as a Partially Observable Markov Decision Process (POMDP) [10]–[12]. Whatever the underlying application, the sensor management problem consists in choosing at each time  $t$  an action  $A_t$  within the set  $\mathcal{A}$  of available actions. The choice of  $A_t$  is generally based on the density state vector  $X_t$  describing the environment of the system and variables of the system itself. It is generally assumed that the state or at least a part of this state is Markovian. Moreover in most of the applications, we only have access to a partial information of the state and  $X_t$  must be estimated from the measurements  $\{Y_s\}_{1 \leq s \leq t}$ . This estimation process is often derived within a Bayesian framework where we use state-dynamics and observation models such as:

$$X_{t+1} = F(X_t, A_t, N_t) \quad (1)$$

$$Y_t = H(X_t, W_t) \quad (2)$$

where  $N_t$ ,  $W_t$ ,  $F$  and  $H$  respectively stands for the state noise, the measurements noise, the state-dynamics and the measurement function.  $F$  and  $H$  are generally time varying functions. The control problem consists in finding the scheduling policy  $\pi$  i.e. select  $A_t$  given the past and the possible

futures. However, this control problem may have a theoretical solution, it is generally untractable in practice. However few works propose optimal solution in the frame of POMDPs like [12]. Beside, several works have been carried out to find sub-optimal policies like for instance myopic policies. Reinforcement Learning and Q-Learning have also been used to propose a solution ([13], [14]).

We propose in this paper to look for a policy within a class of parametrized policy  $\pi_\theta$  and to learn it which means learn the optimal value of  $\theta$ . Funding our work on the approach described in [15] we assume that it is possible to learn this policy *in simulation* using models of the overall system. Once the optimal parameter has been found it is used to manage the sensor(s). The frame of this work being the detection and localization of targets, we show in the last part of this paper how it may be applied the the management of an ESA radar.

The section II described the modelling of a sensor management problem using a POMDP approach. In the section III we derive the algorithm to learn the parameter of the policy. In section IV we show how this method may be used for the tasking of an ESA radar. Finally section V exhibits firsts simulations results.

## II. MODELLING

### A. POMDP Modelling

Let us consider three measurable continuous spaces denoted by  $\mathcal{X}$ ,  $\mathcal{A}$  and  $\mathcal{Y}$ ,  $\mathcal{X}$  is called the *state space*,  $\mathcal{Y}$  the observation space and  $\mathcal{A}$  the *action space*. We call  $\mathcal{M}(\mathcal{X})$  the set of all the measures defined on  $\mathcal{X}$ . A Partially-Observable Decision Process is defined by a *state process*  $(X_t)_{t \geq 0} \in \mathcal{X}$ , an *observation process*  $(Y_t)_{t \geq 1} \in \mathcal{Y}$  and a set of actions  $(A_t)_{t \geq 1} \in \mathcal{A}$ . In these definitions  $t$  stands usually for the *time*. The state process is an homogeneous Markov chain with initial probability measure  $\mu(dx_0) \in \mathcal{M}(\mathcal{X})$  and with Markov transition kernel  $K(dx_{t+1}|x_t)$  ([16]):

$$\forall t \geq 0, X_{t+1} \sim K(\cdot|X_t) \quad (3)$$

$$X_0 \sim \mu \quad (4)$$

$(Y_t)_{t \geq 1}$  is called the observation is linked with the state process by the conditional probability measure:

$$\mathcal{P}(Y_t \in dy_t | X_t = x_t) = g(x_t, y_t) dy_t \quad (5)$$

where  $g : \mathcal{X} \times \mathcal{Y} \rightarrow [0, 1]$  is the marginal density function of  $Y_t$  given  $X_t$ . In a general way, the state process evolves continuously with respect to time  $t$  whereas the observations are made at sampled time  $t_n$ . A new observation is used to derive a new action. We will therefore consider in the following the processes  $(X_t)_{t \geq 0}$ ,  $(Y_n)_{n \in \mathbb{N}}$ ,  $(A_n)_{n \in \mathbb{N}}$  where  $n$  stands for the index of the observation. We also assume that there exists two generative functions  $F_\mu : U \rightarrow \mathcal{X}$  and  $F : \mathcal{X} \times U \rightarrow \mathcal{X}$ , where  $(U, \sigma(U), \nu)$  is a probability space, such that for any measurable *test function*  $f$  defined over  $\mathcal{X}$  we have:

$$\int_{\mathcal{X}} f(x_t) K(dx_t | x_{t-1}) = \int f(F(x_{t-1}, u)) \nu(du) \quad (6)$$

and

$$\int_{\mathcal{X}} f(x_0) \mu(dx_0) = \int f(F_\mu(u)) \nu(du). \quad (7)$$

In many practical situations,  $U = [0, 1]^{n_U}$ , and  $u$  is a  $n_U$ -uple of pseudo random numbers generated by a computer. For sake of simplicity, we adopt the notations  $K(dx_0 | x_{-1}) \triangleq \mu(dx_0)$  and  $F(x_{-1}, u) \triangleq F_\mu(u)$ . Under this framework, the Markov Chain  $(X_t)_{t \geq 0}$  is fully specified by the following dynamical equation:

$$X_{t+1} = F(X_t, U_t), U_t \stackrel{i.i.d.}{\sim} \nu \quad (8)$$

The observation process  $(Y_n)_{n \in \mathbb{N}}$ , defined on the measurable space  $(\mathcal{Y}, \sigma(Y))$ , is actually linked with the state process by the following conditional probability measure

$$\mathbb{P}(Y_n \in dy_n | X_{t_n} = x_{t_n}, A_n) = g(y_n, x_{t_n}, A_n) \lambda(dy_t) \quad (9)$$

where  $A_n \in \mathcal{A}$  is defined on the measurable space  $(\mathcal{A}, \sigma(A))$  and  $\lambda$  is a fixed probability measure on  $(\mathcal{Y}, \sigma(Y))$ . As we assume that observations are conditionally independent given the state process we can write  $\forall 1 \leq i, j \leq t, i \neq j$ :

$$\begin{aligned} & \mathbb{P}(Y_i \in dy_i, Y_j \in dy_j | X_{0:t}, A_i, A_j) = \\ & \mathbb{P}(Y_i \in dy_i | X_{0:t}, A_i) \mathbb{P}(Y_j \in dy_j | X_{0:t}, A_j) \end{aligned} \quad (10)$$

where we have adopted the usual notation  $z_{i:j} = (z_k)_{i \leq k \leq j}$ .

### B. Filtering distribution in a Partially-Observable Markov Decision Process

Given a sequence of action  $A_{1:n}$  and a sample trajectory of the observation process  $y_{1:n}$  and indices  $\{n_1, n_2, t_1, t_2\}$  such that  $1 \leq n_1 \leq n_2 \leq n$  and  $0 \leq t_1 \leq t_{n_1} \leq t_{n_2} \leq t_2 \leq t_n$ , we define, using the , the posterior probability distribution  $M_{t_1:t_2|n_1:n_2}(dx_{t_1:t_2})$  by ([17]):

$$\mathbb{P}(X_{t_1:t_2} \in dx_{t_1:t_2} | Y_{n_1:n_2} = y_{n_1:n_2}, A_{n_1:n_2}) \quad (11)$$

Using the Feynman-Kac framework, the probability 11 can be written:

$$\frac{\prod_{t=t_1}^{t_2} K(dx_t | x_{t-1}) \prod_{j=n_1}^{n_2} G_{t_j}(x_{t_j})}{\int_{\mathcal{X}^{t_2-t_1}} \prod_{t=t_1}^{t_2} K(dx_t | x_{t-1}) \prod_{j=n_1}^{n_2} G_{t_j}(x_{t_j})}, \quad (12)$$

where for simplicity's sake,  $G_{t_n}(x_{t_n}) \triangleq g(y_n, x_{t_n}, A_n)$  and  $G_0(x_0) \triangleq 0$ . One of the main interest here is to estimate the state at time  $t$  from noisy observations  $y_{1:n_t}$  with  $n_t$  the index of the last observation just before time  $t$ . From a bayesian point of view this information is completely contained in the so-called *filtering distribution*  $M_{t:t|1:n_t}$ . In the following, the filtering distribution will simply be denoted as  $M_t$ .

### C. Numerical methods for estimating the filtering distribution

Given a measurable test function  $f : \mathcal{X} \rightarrow \mathbb{R}$ , we want to evaluate

$$M_t(f) = \mathbb{E}[f(X_t) | Y_{1:n_t} = y_{1:n_t}, A_{1:n_t}] \quad (13)$$

which is equal, using the Feynman Kac framework, to:

$$\frac{\mathbb{E}[f(X_t) \prod_{j=1}^{n_t} G_{t_j}(X_{t_j})]}{\mathbb{E}[\prod_{j=1}^{n_t} G_{t_j}(X_{t_j})]} \quad (14)$$

In general, it is impossible to find  $M_t(f)$  exactly except for simple cases such as linear/gaussian (using Kalman filter) or for finite state space Hidden Markov Models. In the general dynamics, continuous space case considered here, possible numerical methods for computing  $M_t(f)$  include the Extended Kalman filter, quantization methods, Markov Chain Monte Carlo methods and Sequential Monte Carlo methods (SMC), also called particle filtering. The basic SMC method, called Bootstrap Filter, approximates  $M_t(f)$  by an empirical distribution  $M_t^N(f) = \frac{1}{N} \sum_{i=1}^N f(x_i^N)$  made of  $N$  so-called *particles* ([18]). It consists in a sequence of transition/selection steps: at time  $t$ , given observation  $y_t$  ([15]):

- **Transition step:** (also called **importance sampling** or **mutation**) a successor particles population  $\tilde{x}_t^{1:N}$  is generated according to the state dynamics from the previous population  $x_{t-1}^{1:N}$ . The (importance sampling) weights  $w_t^{1:N} = \frac{g(\tilde{x}_t^{1:N}, y_t)}{\sum_{j=1}^N g(\tilde{x}_t^j, y_t)}$  are evaluated.
- **Selection step:** Resample (with replacement)  $N$  particles  $x_t^{1:N}$  from the set  $\tilde{x}_t^{1:N}$  according to the weights  $w_t^{1:N}$ . We write  $x_t^{1:N} = \tilde{x}_t^{k_t^{1:N}}$  where  $k_t^{1:N}$  are the selection indices.

Resampling is used to avoid the problem of degeneracy of the algorithm, i.e. that most of the weights decreases to zero. It consists in selecting new particle positions such as to preserve a consistency property :

$$\sum_{i=1}^N w_t^i \phi(\tilde{x}_t^i) = \mathbf{E}\left[\frac{1}{N} \sum_{i=1}^N \phi(x_t^i)\right] \quad (15)$$

The simplest version introduced in [19] consists in choosing the selection indices  $k_t^{1:N}$  by an independent sampling from the set  $1 : N$  according to a multinomial distribution with parameters  $w_t^{1:N}$ , i.e.  $\mathbb{P}(k_t^i = j) = w_t^j$ , for all  $1 \leq i \leq N$ . The idea is to replicate the particles in proportion to their weights. The reader can find some convergence results of  $M_t^N(f)$  to  $M_t(f)$  (e.g. Law of Large Numbers or Central Limit Theorems) in [17], but for our purpose we note that under weak conditions on the test function and on the HMM dynamics, we have the asymptotic consistency property in probability, i.e.  $\lim_{N \rightarrow \infty} M_t^N(f) \stackrel{\mathbb{P}}{=} M_t(f)$ .

## III. POLICY LEARNING ALGORITHM

### A. Optimal Parameterized Policy for Partially-Observable Markov Decision Process

Let  $R_t$  be a real value reward function

$$R_t \triangleq R(X_t, M_t(f)) . \quad (16)$$

The goal is to find a policy

$$\pi : \mathcal{A}^n \times \mathcal{Y}^n \rightarrow \mathcal{A} \quad (17)$$

that maximizes the criterion performance :

$$J_\pi = \int_0^T \mathbb{E}[R_t] dt \quad (18)$$

where  $T$  is the duration of the scenario. Designing in practice policies that depend on the whole trajectory of the past observations/actions is unrealistic. It has been proved that the class of stationary policies that depend on the filtering distribution conditionally to past observations/actions  $M_t$  contains the optimal policy. In general the filtering distribution is an infinite dimensional object, and it cannot be represented in a computer and so is the policy. We therefore propose to look for the optimal policy in a class of parameterized policies  $(\pi_\alpha)_{\alpha \in \Gamma}$  that depend on a statistic of the filtering distribution :

$$A_{n+1} = \pi_\alpha(M_{t_n}(f)) \quad (19)$$

where  $f$  is any test function. As the policy  $\pi$  is parameterized by  $\alpha$ , the performance criterion now depends only on  $\alpha$ . Thus we can maximize it by achieving a stochastic gradient ascent with respect to  $\alpha$  :

$$\alpha_{k+1} = \alpha_k + \eta_k \nabla J_{\alpha_k}, \quad k \geq 0 \quad (20)$$

where  $\nabla J_{\alpha_k}$  denotes the gradient of  $J_{\alpha_k}$  w.r.t  $\alpha_k$ . By convention  $\nabla J_{\alpha_k}$  is column vector whose  $i$ -th component is the partial derivative with respect to  $\alpha_i$ .  $(\eta_k)_{k \geq 0}$  is a non-increasing positive sequence tending to zero. We present in the two following subsection a possible approach to estimate  $\nabla J_{\alpha_k}$  based on Infinitesimal Perturbation Analysis (IPA).

### B. Infinitesimal Perturbation Analysis for gradient estimation

We assume that we can write the following equality at each  $k$ :

$$\nabla J_\alpha = \int_0^T \nabla_\alpha \mathbb{E}[R_t] dt \quad (21)$$

**Proposition 1:** We have the following decomposition of the gradient

$$\begin{aligned} \nabla_\alpha \mathbb{E}[R_t] &= \mathbb{E}[M_t(f) S_t \nabla_{M_t(f)} R_t] \\ &- \mathbb{E}[M_t(f) M_t(S_t) \nabla_{M_t(f)} R_t] \\ &+ \mathbb{E}[R_t S_t] \end{aligned} \quad (22)$$

where

$$S_t = \sum_{j=1}^{p_t} \frac{\nabla_\alpha G_{t_j}(X_{t_j})}{G_{t_j}(X_{t_j})} \quad (23)$$

*Proof:* First let us rewrite  $\nabla_\alpha \mathbb{E}[R_t]$  as following:

$$\nabla_\alpha \mathbb{E}[R_t] = \nabla_\alpha \int_{\mathcal{X}^t \times \mathcal{Y}^{n_t}} R_t U_t V_t \prod_{j=1}^{n_t} \lambda(dy_j) \quad (24)$$

where:

$$\begin{cases} U_t &= \prod_{i=0}^t K(dx_i|x_{i-1}), \\ V_t &= \prod_{j=1}^{m_t} G_{t_j}(x_{t_j}) \end{cases} \quad (25)$$

Remarking that only  $R_t$  and  $V_t$  depends on  $\alpha$  so that we obtain

$$\begin{cases} \nabla_\alpha V_t &= S_t V_t, \\ \nabla_\alpha R_t &= \nabla_\alpha M_t(f) \nabla_{M_t(f)} R_t \end{cases} \quad (26)$$

where  $S_t$  is given by eq.(23). Incorporating (24) in (26), we obtain

$$\nabla_\alpha \mathbb{E}[R_t] = \mathbb{E}[\nabla_\alpha M_t(f) \nabla_{M_t(f)} R_t] + \mathbb{E}[R_t S_t]. \quad (27)$$

Now using one more time (26), we have

$$\begin{aligned} \nabla_\alpha M_t(f) &= \nabla_\alpha \mathbb{E}\left[f(X_t) \frac{V_t}{\mathbb{E}[V_t]}\right] \\ &= \mathbb{E}\left[f(X_t) \frac{\nabla_\alpha V_t}{\mathbb{E}[V_t]}\right] - \mathbb{E}\left[f(X_t) \frac{V_t \mathbb{E}[\nabla_\alpha V_t]}{\mathbb{E}[V_t]^2}\right] \\ &= \mathbb{E}\left[f(X_t) S_t \frac{V_t}{\mathbb{E}[V_t]}\right] - M_t S_t \mathbb{E}\left[\frac{V_t}{\mathbb{E}[V_t]}\right] \\ &= M_t(f S_t) - M_t(f) M_t(S_t) \end{aligned} \quad (28)$$

so that we obtain (22) by incorporating (28) in (27).  $\blacksquare$

We can deduce directly Algorithm 1 from (22). It is important to note that we must deal with two time-scales. This first and the shorter one allows to simulate the continuous state  $X_t$ . On the contrary the observation and action process are updated only each time we get a new observation. These specific time is denoted  $t_n$  in the algorithm. That is the reason while there is an alternative to update the variables  $S_t$  and  $\tilde{w}_{t-1}^{(i)}$ . A new action  $A_n$  is also calculated each  $t_n$  as already explained above. One can also be surprised to calculate  $R(X_t, M_t(f))$  using the sampled value of  $X_t$ . To well understand this algorithm we must remind that **the learning is made off-line** using a simulated process. It is therefore possible to use the *real* value of  $X_t$  in this case.

#### IV. APPLICATION TO THE ESA RADAR

The ESA is an agile beam radar which means that it is able to point its beam in any direction of the environment almost instantaneously without inertia. However, the targets in the environment are detected w.r.t a probability of detection which depends on the direction of the beam and the time of observation in this direction. In the following, we precise first the nature of an action, then the influence of the action onto the probability of detection and finally the nature of the observations.

*Definition of the action:* The main property of an ESA is that it can point its beam without mechanically adjusting the antenna. An ESA radar provides measurements in a direction  $\theta$ . We note  $\delta$ , the time of observation in this direction. In this work the  $n$ -th action is :

$$A_n = [\theta_n \quad \delta_n]^T \quad (29)$$

---

#### Algorithm 1 Policy Gradient in POMDP via IPA

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Initialize  $\alpha_0 \in \Gamma$

**for**  $k = 1$  **to**  $\infty$  **do**

**for**  $t = 1$  **to**  $T$  **do**

Sample  $u_t \sim \nu$

Set  $x_t = F(x_{t-1}, u_t)$ ,

If  $t = t_n$ , sample  $y_n \sim g(\cdot, x_t, a_n) \lambda(\cdot)$

Set  $s_t = \begin{cases} s_{t-1} + \frac{\partial g(x_t, y_n, a_n)}{g(x_t, y_n, a_n)} & \text{if } t = t_n \\ s_{t-1} & \text{else} \end{cases}$

Set  $\forall i \in \{1, \dots, I\}$

$\tilde{x}_t^{(i)} = F(x_{t-1}^{(i)}, a_{t-1}, u_t^{(i)})$  where  $u^{(i)} \stackrel{iid}{\sim} \nu$

$\tilde{s}_t^{(i)} = \begin{cases} s_{t-1}^{(i)} + \frac{\partial g(x_t^{(i)}, y_n, a_n)}{g(x_t^{(i)}, y_n, a_n)} & \text{if } t = t_n \\ s_{t-1}^{(i)} & \text{else} \end{cases}$

$\tilde{w}_t^{(i)} = \begin{cases} \frac{g(x_t^{(i)}, y_n, a_n) \tilde{w}_{t-1}^{(i)}}{\sum_j g(x_t^{(j)}, y_n, a_n) \tilde{w}_{t-1}^{(j)}} & \text{if } t = t_n \\ \tilde{w}_{t-1}^{(i)} & \text{else} \end{cases}$

Set  $(x_t^{(i)}, s_t^{(i)})_{i \in \{1, \dots, I\}} = (\tilde{x}_t^{(i)}, \tilde{s}_t^{(i)})_{i \in \{k_1, \dots, k_I\}}$ ,  $k_{1:I}$  are selection indices associated to  $(\tilde{w}_t^{(i)})_{i \in \{1, \dots, I\}}$ ,

$m_t(f) = \frac{1}{I} \sum_i f(x_t^{(i)})$ ,  $m_t(s_t) = \frac{1}{I} \sum_i s_t^{(i)}$ ,

$m_t(f s_t) = \frac{1}{I} \sum_i f(x_t^{(i)}) s_t^{(i)}$ ,

$a_{n+1} = \pi_{\alpha_k}(m_t)$  if  $t = t_n$

$r_t = R(x_t, m_t(f))$

$\nabla r_t = (m_t(f s_t) - m_t(f) m_t(s_t)) \frac{\partial R}{\partial m_t(f)}(x_t, m_t(f)) + r_t s_t$

$\nabla J_{\alpha_k} = \nabla J_{\alpha_k} + \nabla r_t$

**end for**

$\alpha_{k+1} = \alpha_k + \eta_k \nabla J_{\alpha_k}$

**end for**

---

with

$$\begin{cases} \theta_n \in [-\frac{\pi}{2}, \frac{\pi}{2}], \\ \delta_n \in \mathbb{R}^+ \end{cases}, \quad \forall n \geq 0. \quad (30)$$

This is a simple possible action. One could increase the number of components of an action by adding the emitted frequency for instance. The action does not influence directly the observation produced by the ESA but the probability of detection of a target.

*The probability of detection  $P_d$ :* It refers to the probability to detect a target and therefore to the probability to obtain an estimation of the state of a target  $p$  at time  $t_n$  denoted  $X_{t_n, p}$  with action  $A_n$ . In this work,  $X_{t_n, p}$  is composed of the localisation and velocity components of the target  $p$  at time  $t_n$  in the x-y plane:

$$X_{t_n, p} = [rx_{t_n, p} \quad ry_{t_n, p} \quad vx_{t_n, p} \quad ry_{t_n, p}]^T \quad (31)$$

where the subscript  $T$  stands for *matrix transpose*. The terms  $rx_{t_n, p}$  and  $ry_{t_n, p}$  refers here to the position and  $vx_{t_n, p}$  and  $vy_{t_n, p}$  the velocity of target  $p$  at time  $t_n$ . We also denote  $D_{n, p}$  the random variable which takes values 1 if the radar produces a detection (and therefore an estimation) for target  $p$  and 0 else :

$$D_n = [D_{n,1} \quad \dots \quad D_{n,P}]^T. \quad (32)$$

As said previously, this probability also depends on the time of observation  $\delta_n$ . Aerial targets being considering here, the reflectivity of a target can be modelled using a Swerling I model [20]. We then have the following relation between the probability of detection and the probability of false alarm  $P_{fa}$  (i.e. the probability that the radar produce a detection knowing that there is no target) ([5], [21]):

$$P_d(x_{t_n,p}, A_n) = P_{fa}^{\frac{1}{1+\rho(x_{t_n,p}, A_n)}} \quad (33)$$

where  $\rho(x_{t_n,p}, A_n)$  is the target signal-to-noise ratio. In the case of an ESA radar, it is equal to :

$$\rho(x_{t_n,p}, A_n) = \alpha \delta_n \frac{\cos^2 \theta_n}{r_{t_n,p}^4} e^{-\frac{(\beta_{t_n,p} - \theta_n)^2}{2B^2}} \quad (34)$$

where  $r_{t_n,p}$  is the target range and  $\beta_{t_n,p}$  the azimuth associated to target  $p$  at instant time  $t_n$ .  $\alpha$  is a coefficient which includes all the parameters of the sensor and  $B$  is the beamwidth of the radar. It is reminded in Appendix A how the equations 33 and 34 may be derived. If we make the assumption that all the detections are independant, we can write :

$$\mathbb{P}(D_n = d_n | X_{t_n} = x_{t_n}, A_n) = \prod_p \mathbb{P}(D_{n,p} = d_{n,p} | X_{t_n,p} = x_{t_n,p}, A_n) \quad (35)$$

where

$$\mathbb{P}(D_{n,p} = d_{n,p} | X_{t_n,p} = x_{t_n,p}, A_n) = P_d(x_{t_n,p}, A_n) \delta_{d_{n,p}=1} + (1 - P_d(x_{t_n,p}, A_n)) \delta_{d_{n,p}=0} \quad (36)$$

*Observation equation:* At instant time  $t_n$ , the radar produces a raw observation  $Y_n$  composed of  $P$  measurements :

$$Y_n = [Y_{n,1} \ \dots \ Y_{n,P}]^T \quad (37)$$

where  $Y_{n,p}$  is the observation related to target of state value  $x_{t_n,p}$  obtained with action  $A_n$  (we do not consider here the problem of measurement-target association). Moreover, we assume that the number of targets  $P$  is known. Each of these measurements has the following formulation :

$$Y_{n,p} = [r_{n,p} \ \beta_{n,p} \ \dot{r}_{n,p}]^T \quad (38)$$

where  $r_{n,p}$ ,  $\beta_{n,p}$ ,  $\dot{r}_{n,p}$  are range, azimuth and range rate. The equation observation can be written

$$\mathbb{P}(Y_n \in dy_n | X_{t_n} = x_{t_n}, A_n) = \quad (39)$$

$$\prod_p \mathbb{P}(Y_{n,p} \in dy_{n,p} | X_{t_n,p} = x_{t_n,p}, A_n) \quad (40)$$

where

$$\mathbb{P}(Y_{n,p} \in dy_{n,p} | X_{t_n,p} = x_{t_n,p}, A_n) = \quad (41)$$

$$= g(y_{n,p}, x_{t_n,p}, A_n) \lambda(dy_{n,p}) \quad (42)$$

$$g(y_{n,p}, x_{t_n,p}, A_n) = \left( \frac{\mathcal{N}(h_t(x_{t_n,p}), \Sigma_y) P_d(x_{t_n,p}, A_n)}{1 - P_d(x_{t_n,p}, A_n)} \right)^T \quad (43)$$

and

$$\lambda(dy_{n,p}) = \lambda_{cont}(dy_{n,p}) + \lambda_{disc}(dy_{n,p}) \quad (44)$$

The relation between the state and the raw observations is given by :

$$Y_{n,p} = h_{t_n}(X_{t_n,p}) + W_{n,p} \quad (45)$$

with  $h_{t_n}(x_{t_n,p})$  equals to:

$$\left( \begin{array}{c} \sqrt{(rx_{t_n,p} - rx_{t_n}^{obs})^2 + (ry_{t_n,p} - ry_{t_n}^{obs})^2} \\ \text{atan} \left\{ \frac{ry_{t_n,p} - ry_{t_n}^{obs}}{rx_{t_n,p} - rx_{t_n}^{obs}} \right\} \\ \frac{(rx_{t_n,p} - rx_{t_n}^{obs})(vx_{t_n,p} - vx_{t_n}^{obs}) + (ry_{t_n,p} - ry_{t_n}^{obs})(vy_{t_n,p} - vy_{t_n}^{obs})}{\sqrt{(rx_{t_n,p} - rx_{t_n}^{obs})^2 + (ry_{t_n,p} - ry_{t_n}^{obs})^2}} \end{array} \right) \quad (46)$$

and  $W_{n,p}$  a gaussian noise the covariance matrix of which is given by :

$$\Sigma_y = \text{diag}(\sigma_r^2, \sigma_\beta^2, \sigma_{\dot{r}}^2) \quad (47)$$

*State equation:* First let us introduce the definition of the unknown state  $X_t$  at time  $t$  and its evolution through time.  $X_{t,p}$  is the state of the target  $p$ . It has been defined above. Let  $P$  be the known number of targets in the space under analysis at time  $t$ .  $X_t$  has the following form :

$$X_t = [X_{t,1} \ \dots \ X_{t,P}]^T \quad (48)$$

Based on [22] works, we classically assume that all the targets follow a nearly constant velocity model. We use a discretized version of this model ([23]) :

$$X_{t,p} = F(X_{t-1,p}, U_t) \text{ where } U_t \sim \mathcal{N}(0, \sigma^2 Q) \quad (49)$$

where

$$F = \begin{bmatrix} 1 & 0 & \beta & 0 \\ 0 & 1 & 0 & \beta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } Q = \begin{bmatrix} \frac{\beta^3}{3} & 0 & \frac{\beta^2}{2} & 0 \\ 0 & \frac{\beta^3}{3} & 0 & \frac{\beta^2}{2} \\ \frac{\beta^2}{2} & 0 & \beta & 0 \\ 0 & \frac{\beta^2}{2} & 0 & \beta \end{bmatrix} \quad (50)$$

## V. SIMULATIONS

## VI. CONCLUSION

### APPENDIX A

We show in this Appendix how the probability of detection is derived. First, the radar transmits a pulse expressed as follows

$$s(t) = \alpha(t) \cos(w_c t) \quad (51)$$

$$= \text{Re}\{\alpha(t)e^{jw_c t}\} \quad (52)$$

where  $\alpha(t)$  is the envelope also called the transmitted pulse and  $w_c$  the carrier frequency. This pulse is modified by the process of reflection. A target is modelled as a set of elementary reflectors, each reflecting: time delayed, Doppler shift,

Phase shift and attenuated version of the transmitted signal. We usually assume that the reflection process is linear and frequency independent within the bandwidth of the transmitted pulse. The return signal has the following formulation:

$$s_r(t) = G \sum_i \alpha(t - \tau_i) g_i e^{j(w_c(t - \tau_i + \frac{2\dot{r}_i}{c}t) + \theta_i)} + n(t) \quad (53)$$

where

- $g_i$  is the radar cross section associated to reflector  $i$ ,
- $\theta_i$  is the phase shift associated to reflector  $i$ ,
- $\dot{r}_i$  is the radial velocity between the antenna and the object (Doppler frequency shift),
- $G$ : others losses heavily range dependent due to spatial spreading of energy,
- $n(t)$  is a thermal noise of the receiver such that  $\text{Re}\{n(t)\}, \text{Im}\{n(t)\} \sim \mathcal{N}(0, \sigma_n^2)$ .

We make the following approximations:

$$\begin{cases} \dot{r}_i \approx \dot{r} \\ \alpha(t - \tau_i) \approx \alpha(t - \tau) \end{cases} \quad (54)$$

where  $\dot{r}$  is the mean radial velocity of the target  $\tau$  is the mean time delay of the target. Using these approximations, the return signal can be rewritten as follows:

$$s_r(t) = \alpha(t - \tau) G e^{jw_D t} b + n(t) \quad (55)$$

where

$$\begin{cases} w_D = w_c(1 + \frac{2\dot{r}}{c}) \\ b = \sum_i g_i e^{j(-w_c \tau_i + \theta_i)} \end{cases} \quad (56)$$

The fluctuations of  $b$  are known and modelled using Swerling 1 model [20]. There are different models available (Swerling 1, 2, 3,...) corresponding to different types of targets. Swerling 1 given below is convenient for aircrafts. We can then write :

$$\text{Re}\{b\}, \text{Im}\{b\} \sim \mathcal{N}(0, \sigma_{RCS}^2) \quad (57)$$

This modelling of  $b$  assumes that the phase shifts  $\theta_i$  are independent and uniformly distributed and the magnitudes  $g_i$  are identically distributed. If the number of reflector is large, the central limit theorem gives that  $b$  is a complex-valued Gaussian random variable centered at the origin. Now, a matching filter is applied to our return signal

$$s_m(t) = \int_{-\infty}^{+\infty} s_r(t) h(s) ds \quad (58)$$

where  $h(t)$  is a shifted, scaled and reversed copy of  $s_r(t)$

$$h(s) = \alpha(\delta - t) e^{-jw_D(\delta - t)} \quad (59)$$

We choose  $t = \delta + \tau$  which yields the best signal to noise ratio where  $\delta$  is the length of the transmitted pulse. The probability of detection is based on quantity  $|s_m(\delta + \tau)|^2$ . We can show that

$$s_m(\delta + \tau) = G e^{jw_D \tau} b + \int_{-\infty}^{+\infty} n(\delta + \tau - s) h(s) ds \quad (60)$$

One can remark that  $s_m(\delta + \tau)$  is the sum of two complex-value Gaussian variables. We look at the following statistic

$$\Lambda = \frac{|s_m(\delta + \tau)|^2}{2\sigma_n^2} \quad (61)$$

and we introduce the following notation

$$\sigma_s^2 = G^2 \sigma_{RCS}^2 \quad (62)$$

Now we construct the test

$$\begin{cases} \mathcal{H}_1 : \text{data generated by signal + noise} \\ \mathcal{H}_0 : \text{data generated by noise} \end{cases} \quad (63)$$

$$\begin{cases} \mathcal{H}_1 : p_\Lambda(x) = \frac{1}{\frac{\sigma_s^2}{\sigma_n^2} + 1} e^{-\frac{x}{\frac{\sigma_s^2}{\sigma_n^2} + 1}} \\ \mathcal{H}_0 : p_\Lambda(x) = e^{-x} \end{cases} \quad (64)$$

Then, we derive the probability of detection and false alarm.

$$\begin{cases} P_d = \int_\gamma^{+\infty} p_\Lambda(x | \mathcal{H}_1 \text{ is true}) = e^{-\frac{\gamma}{\frac{\sigma_s^2}{\sigma_n^2} + 1}} \\ P_{fa} = \int_\gamma^{+\infty} p_\Lambda(x | \mathcal{H}_0 \text{ is true}) = e^{-\gamma} \end{cases} \quad (65)$$

Consequently

$$P_d = P_{fa}^{\frac{1}{\frac{\sigma_s^2}{\sigma_n^2} + 1}} \quad (66)$$

The ratio  $\frac{\sigma_s^2}{\sigma_n^2}$  is called the Signal-to-Noise Ratio noted  $\rho$ . This SNR is related to the parameters of the system and the target. The classical radar equation is given by the following formula ([21]):

$$\rho = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 r^4} \quad (67)$$

where  $P_t$  is the energy of the transmitted pulse,  $G_t$  is the gain of the transmitted antenna,  $G_r$  is the gain of the received antenna,  $\sigma$  is the radar cross section (for an aircraft between 0.1 and 1  $m^2$ ),  $r$  is the target range,  $\gamma$  is the system noise temperature and  $L$  is a general loss term. However, the above formula does not take into account for the sake of simplicity the losses due to atmospheric attenuation and to the imperfection of the radar. Thus, extra terms must be added :

$$\rho = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 k b L \gamma r^4} \quad (68)$$

where  $b$  is the receiver noise bandwidth (generally consider equal to the signal bandwidth so that  $b = \frac{1}{\delta t}$ ),  $k$  is Boltzmann's constant,  $\gamma$  is the temperature of the system and  $L$  some losses. Moreover, the gain reduces with the deviation of the beam from the antenna normal in an array antenna.

$$G_t = G_0 \cos^\alpha(\theta_t) \quad (69)$$

$$G_r = G_0 \cos^\alpha(\theta_r) \quad (70)$$

where  $G_0$  is the gain of the antenna. In [24],  $\alpha = 2$ , in [21],  $\alpha = 2.7$ . According [25], there is also a beam loss because the

radar beam is not pointing directly so that the radar equation is:

$$\rho = \frac{P_t G_0^2 \lambda^2 \sigma \delta_t \cos^2(\theta_t)}{(4\pi)^3 k L \gamma r^4} e^{-\frac{(\theta_t - \beta_t)^2}{2B^2}} \quad (71)$$

where is  $B$  is the beamwidth.

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