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# Simultaneous Learning and Alignment: Multi-Instance and Multi-Pose Learning<sup>\*</sup>

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**Abstract.** In object recognition in general and in face detection in particular, data alignment is necessary to achieve good classification results with certain statistical learning approaches such as Viola-Jones. Data can be aligned in one of two ways: (1) by separating the data into coherent groups and training separate classifiers for each; (2) by adjusting training samples so they lie in correspondence. If done manually, both procedures are labor intensive and can significantly add to the cost of labeling. In this paper we present a unified boosting framework for simultaneous learning and alignment. We present a novel boosting algorithm for *Multiple Pose Learning* (MPL), where the goal is to simultaneously split data into groups and train classifiers for each. We also review *Multiple Instance Learning* (MIL), and in particular MIL-BOOST, and describe how to use it to simultaneously train a classifier and bring data into correspondence. We show results on variations of LFW and MNIST, demonstrating the potential of these approaches.

## 1 Introduction

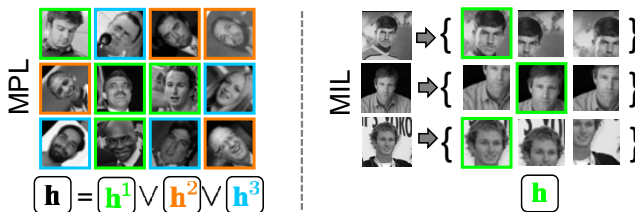
In object recognition in general and in face detection in particular, data alignment is often necessary to achieve good classification results. *E.g.*, consider the Viola-Jones face detector [1]. The detector works best when trained with images that come from a single coherent group (*e.g.* frontal faces) and lie in approximate correspondence (*e.g.* eyes aligned). Generalizing Viola-Jones to multiple poses and offset training data remains an active area of research.

In this work we are interested in strategies for simultaneous learning and alignment. Data can be aligned in one of two ways: (1) by separating the data into coherent groups and training separate classifiers for each; (2) by adjusting training samples so they lie in correspondence (see Fig. 1). The first strategy is appropriate for out of plane rotation of faces while the second strategy is appropriate for dealing with translational offset. If done manually, both alignment strategies are labor intensive and can significantly add to the cost of labeling.

In this paper we present a unified boosting framework for simultaneous learning and alignment derived using Friedman’s gradient boosting methodology [2]. We present a novel boosting algorithm for simultaneously splitting data into groups and training classifiers for each group. We refer to this problem as *Multiple Pose Learning* (MPL) and our algorithm as MPL-BOOST. We also review

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**Fig. 1.** We present two strategies for simultaneous learning and alignment. Data can be aligned by: (1) separating the data into coherent groups and training separate classifiers for each (MPL); (2) adjusting training samples so they lie in correspondence (MIL).

*Multiple Instance Learning* (MIL) [3], and describe how to use it to simultaneously train a classifier and bring data into correspondence. In particular, we re-derive and generalize MIL-BOOST, first presented in [4]. Throughout we show the deep connection between these two approaches.

We begin with a review of related work below. In Section 2 we give a detailed and accessible derivation of MPL-BOOST and MIL-BOOST. We show results on variations of the LFW face dataset [5] as well as MNIST digits [6] in Section 3, demonstrating the potential of these approaches. We conclude in Section 4.

### 1.1 Related Work

A number of papers have focused on grouping data either manually or automatically. *E.g.*, one approach for face detection is to train multiple detectors, each on a manually defined range of orientations [7, 8]. A pre-processing step can be used to predict orientation [7]. However, manual grouping of data can be labor intensive and suboptimal. Alternatively, data can be clustered prior to training [9], but the resulting clustering is likely suboptimal. Most related to this are methods that attempt to group the data during training [10–12], *e.g.* by learning a tree classifier that naturally splits the data [11, 12]. Although promising, these approaches often have heuristic splitting procedures and many parameters.

A number of papers have addressed the second form of alignment. The most direct approach is to try to bring a set of images into joint alignment via congealing [13–15] or thin plate splines [16]. This line of work has yielded promising results; however, alignment is independent of learning. Many weakly supervised recognition methods [17–19] perform alignment implicitly during training, but these alignment strategies are not easily transferable to boosted detection systems. Finally, MIL has been successfully applied to a number of vision problems [20, 21, 4]. MIL offers one way of simultaneously training a discriminative classifier and bringing data into correspondence. As mentioned, we re-derive and generalize MIL-BOOST [4]; details given in Sec. 2.4.

## 2 Multi-Pose & Multi-Instance Boosting

We begin with an overview of gradient boosting with the log likelihood in Section 2.1, followed by a discussion of smooth approximations of the max function in 2.2. This will serve as the foundation of MPL-BOOST and MIL-BOOST, presented in 2.3 and 2.4, respectively. Finally, we propose various extensions in 2.5.

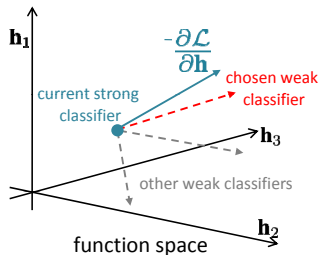


Fig. 2. Illustration of gradient boosting, see text.

## 2.1 Gradient Boosting Overview

We begin with an overview of the standard setting for supervised learning. Training data consists of  $n$  samples  $x_i \in \mathcal{X}$  and  $n$  corresponding labels  $y_i \in \mathcal{Y}$ , where typically  $\mathcal{X} = \mathbb{R}^d$  and in binary classification  $\mathcal{Y} = \{-1, 1\}$ . The goal is to learn a classification function  $h : \mathcal{X} \rightarrow \mathcal{Y}$  that generalizes well to unseen data.

Boosting has been shown to be a very successful approach for binary classification. In boosting, the goal is to train a classifier of the form:

$$\mathbf{h}(x) = \sum_{t=1}^T \alpha_t h_t(x), \quad (1)$$

where each  $h_t : \mathcal{X} \rightarrow \mathcal{Y}$  is a *weak learner* whose performance may only be slightly above chance, and the  $\alpha_t$  weigh the weak learners' relative importance. Boosting combines multiple weak learners into a single *strong* classifier with low error. Training proceeds sequentially. In each phase incorrectly classified examples receive more weight; details vary according to boosting algorithm.

Friedman [2] proposes an elegant method for deriving boosting algorithms for a wide range of loss functions. The general idea is to optimize a loss function  $\mathcal{L}(\mathbf{h})$  by performing gradient descent on  $\mathbf{h}$  (known as gradient descent in function space). Intuitively, the idea is to always select the  $h_t$  that most reduces the loss on the training data. Finding the best  $h_t$  proceeds in two stages: (1) compute the *optimal* weak classifier response and (2) from the available candidates select the weak classifier that best approximates the optimal response.

More formally, we can consider  $\mathbf{h}$  as an  $n$  vector whose  $i^{th}$  component  $\mathbf{h}_i$  has the value  $\mathbf{h}(x_i)$ . The loss  $\mathcal{L}$  is a function over  $\mathbf{h}$ , and the goal is to minimize  $\mathcal{L}(\mathbf{h})$  w.r.t. to  $\mathbf{h}$  via gradient descent. In other words to find the optimal weak classifier response in each phase  $t$  we compute  $-\frac{\partial \mathcal{L}}{\partial \mathbf{h}}$ , which is a vector with components  $w_i \equiv -\frac{\partial \mathcal{L}}{\partial \mathbf{h}_i}$ . Ideally we would select  $h_t$  such that  $w_i = h_t(x_i) \forall i$ ; however, in practice we are limited in the choice of  $h_t$ . Therefore, Friedman proposes to find the  $h_t$  which is as close as possible to the gradient in function space:

$$h_t = \operatorname{argmax}_h \sum_{i=1}^n w_i h(x_i) \quad (2)$$

The general process is illustrated in Fig. 2. Once we compute  $h_t$ , the step size  $\alpha_t$  can be found via a line search (again minimizing  $\mathcal{L}(\mathbf{h})$ ). Putting everything together, we obtain the boosting procedure in Fig. 3. Remaining details follow.

**Input:** Dataset  $\{x_1, \dots, x_n\}, \{y_1, \dots, y_n\}, y_i \in \{-1, 1\}$

- 1: **for**  $t = 1$  to  $T$  **do**
- 2:   Compute weights  $w_i = -\frac{\partial \mathcal{L}}{\partial \mathbf{h}_i}$
- 3:   Train weak classifier  $h_t$  using weights  $|w_i|$   
 $h_t = \operatorname{argmin}_h \sum_i \mathbf{1}(h(x_i) \neq y_i) |w_i|$
- 4:   Find  $\alpha_t$  via line search to minimize  $\mathcal{L}(\mathbf{h})$   
 $\alpha_t = \operatorname{argmin}_\alpha \mathcal{L}(\mathbf{h} + \alpha h_t)$
- 5:   Update strong classifier  $\mathbf{h} \leftarrow \mathbf{h} + \alpha_t h_t$ .
- 6: **end for**

**Fig. 3.** Boosting via gradient descent in function space.

**Log Likelihood:** In the remainder of this paper we use the negative log likelihood as the loss function. If we define  $p_i \equiv p(y_i = 1|x_i)$ , we can write:

$$\mathcal{L}(\mathbf{h}) = -\sum_{i=1}^n \left( \mathbf{1}(y_i = 1) \log p_i + \mathbf{1}(y_i = -1) \log(1 - p_i) \right). \quad (3)$$

We begin by deriving a standard boosting algorithm using this loss function and gradient descent boosting. This derivation will serve as the foundation for the derivations for MPL-BOOST and MIL-BOOST in Sections 2.3 and 2.4. Note that a related derivation is done in [2]; furthermore, the popular LogitBoost algorithm [22] uses similar criteria, though the actual optimization procedure differs.

First, we define  $p_i$  in terms of  $\mathbf{h}_i = \mathbf{h}(x_i)$ . We follow [2, 22] and define:

$$p_i = \sigma(2\mathbf{h}_i), \quad (4)$$

where  $\sigma(v) = \frac{1}{1+\exp(-v)}$  is the sigmoid. Note that  $\sigma(v) \in [0, 1]$  and  $\frac{\partial \sigma}{\partial v} = \sigma(v)(1 - \sigma(v))$ . Finally, taking the derivatives:

$$\frac{\partial \mathcal{L}}{\partial p_i} = \begin{cases} \frac{-1}{p_i} & \text{if } y_i = 1 \\ \frac{1}{1-p_i} & \text{if } y_i = -1 \end{cases} \quad (5)$$

$$\frac{\partial p_i}{\partial \mathbf{h}_i} = 2p_i(1 - p_i), \quad (6)$$

and using the chain rule, we get  $w_i = -\frac{\partial \mathcal{L}}{\partial \mathbf{h}_i} = -\frac{\partial \mathcal{L}}{\partial p_i} \frac{\partial p_i}{\partial \mathbf{h}_i}$ :

$$w_i = \begin{cases} 2(1 - p_i) & \text{if } y_i = 1 \\ -2p_i & \text{if } y_i = -1 \end{cases} \quad (7)$$

Intuitively, negatives with large  $p_i$  and positives with small  $p_i$  receive high weight.

**Training  $h_t$ :** In the original gradient boosting paper weak classifiers are chosen by optimizing Eqn. (2), while in most boosting algorithms weak classifiers are chosen by optimizing weighted error:

$$h_t = \operatorname{argmin}_h \sum_i \mathbf{1}(h(x_i) \neq y_i) w_i, \quad (8)$$

where  $y_i \in \{-1, 1\}$ . The criterion in Eqn. (8) is more common and many existing learning algorithms approximately minimize this cost function. Here we show that for binary classifiers the first cost function can be converted to the second. Let  $y'_i = \text{sign}(w_i)$  (typically  $y'_i = y_i$ ), and  $w'_i = |w_i| / \sum_i (|w_i|)$ . Then:

$$\begin{aligned} h_{t+1} &= \arg \min_h \sum_i -h(x_i)w_i \\ &= \arg \min_h \sum_i (2 \cdot \mathbf{1}(h(x_i) \neq y'_i) - 1)|w_i| \\ &= \arg \min_h \sum_i 2 \cdot \mathbf{1}(h(x_i) \neq y'_i)|w_i| \\ &= \arg \min_h \sum_i \mathbf{1}(h(x_i) \neq y'_i)w'_i \end{aligned}$$

This transformation allows us to use existing learning algorithms, *e.g.* decision stumps, that minimize weighted error or some approximation to it to train  $h_t$ .

**Initial distribution:** It is often useful to train with an initial (prior) distribution over the data, *e.g.* if more negative than positive training examples are available. Let  $\omega_i$  be the prior on the  $i^{\text{th}}$  example. We can modify  $\mathcal{L}$  as follows:

$$\mathcal{L}(\mathbf{h}) = - \sum_{i=1}^n \omega_i \left( \mathbf{1}(y_i = 1) \log p_i + \mathbf{1}(y_i = -1) \log(1 - p_i) \right). \quad (9)$$

Observe that doubling the weight of  $x_i$  is the same as having two copies of  $x_i$  in the training data. The derivative of  $\mathcal{L}$  becomes  $\frac{\partial \mathcal{L}}{\partial p_i} = \frac{-\omega_i}{p_i}$  if  $y_i = 1$  and  $\frac{\omega_i}{1-p_i}$  if  $y_i = -1$ . Modifying the corresponding equations for  $w_i$  is trivial.

**Real vs. Discrete  $h_t$ :** Thus far we have assumed that the weak classifiers are binary, *i.e.*  $h_t(x) \in \{-1, 1\}$ . If, however,  $h_t$  outputs a real valued confidence or score, we can use this score directly. The procedure outlined in Fig. 3 remains unchanged. The alternative is to just use  $\text{sign}(h_t(x))$ . Using the terminology of [22], we refer to these variations as *real* and *discrete*, respectively.

## 2.2 Softmax Functions

We now present an overview of differentiable approximations of the *max*, which we refer to as the *softmax*. The general idea is to approximate the max over  $\{v_1, \dots, v_m\}$  by a differentiable function  $g_\ell(v_\ell)$ , such that:

$$g_\ell(v_\ell) \approx \max_\ell(v_\ell) = v_* \quad (10)$$

$$\frac{\partial g_\ell(v_\ell)}{\partial v_i} \approx \frac{\mathbf{1}(v_i = v_*)}{\sum_\ell \mathbf{1}(v_\ell = v_*)} \quad (11)$$

Intuitively, if  $v_i$  is the unique max, then changing  $v_i$  changes the max by the same amount, otherwise changing  $v_i$  does not affect the max.

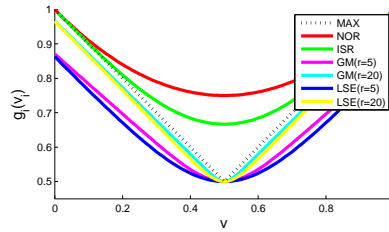


Fig. 4. Various models  $g_\ell(v_\ell)$  applied to  $v_\ell = (v, 1 - v)$ .

A number of approximations for  $g$  have been proposed. We summarize the choices used here in Table 1: a variant of log-sum-exponential (LSE) [23, 24], generalized mean (GM), noisy-or (NOR) [4], and the ISR model [20, 4]. In Fig. 4 we show the different models applied to  $(v, 1 - v)$  for  $v \in [0, 1]$ .

LSE and GM each have a parameter  $r$  that controls their sharpness and accuracy;  $g_\ell(v_\ell) \rightarrow v_*$  as  $r \rightarrow \infty$  (note that large  $r$  can lead to numerical instability). For LSE one can show that  $v_* - \log(m)/r \leq g_\ell(v_\ell) \leq v_*$  [23] and for GM that  $(1/m)^{1/r} v_* \leq g_\ell(v_\ell) \leq v_*$ , where  $m = |v_\ell|$ . NOR and ISR are only defined over  $[0, 1]$ . Both have probabilistic interpretations and work well in practice; however, these models are best suited for small  $m$  as  $g_\ell(v_\ell) \rightarrow 1$  as  $m \rightarrow \infty$ . Finally, all models are exact for  $m = 1$ , and if  $\forall \ell v_\ell \in [0, 1]$ , then  $0 \leq g_\ell(v_\ell) \leq 1$  for all models.

	$g_\ell(v_\ell)$	$\partial g_\ell(v_\ell)/\partial v_i$	domain
LSE	$\frac{1}{r} \ln \left( \frac{1}{m} \sum_\ell \exp(rv_\ell) \right)$	$\frac{\exp(rv_i)}{\sum_\ell \exp(rv_\ell)}$	$[-\infty, \infty]$
GM	$\left( \frac{1}{m} \sum_\ell v_\ell^r \right)^{\frac{1}{r}}$	$g_\ell(v_\ell) \frac{v_i^{r-1}}{\sum_\ell v_\ell^r}$	$[0, \infty]$
NOR	$1 - \prod_\ell (1 - v_\ell)$	$\frac{1 - g_\ell(v_\ell)}{1 - v_i}$	$[0, 1]$
ISR	$\frac{\sum_\ell v'_\ell}{1 + \sum_\ell v'_\ell}, v'_\ell = \frac{v_\ell}{1 - v_\ell}$	$\left( \frac{1 - g_\ell(v_\ell)}{1 - v_i} \right)^2$	$[0, 1]$

Table 1. Four softmax approximations  $g_\ell(v_\ell) \approx \max_\ell(v_\ell)$ .

### 2.3 Multiple Pose Learning

We now present a boosting approach for simultaneously splitting data into groups and training a classifier for each group. Similar to standard supervised learning, we are given  $n$  samples  $x_i \in \mathcal{X}$  and  $n$  corresponding labels  $y_i \in \{-1, +1\}$ . We assume, however, that there are  $K$  latent variables  $y_i^k \in \{-1, +1\}$  associated with each sample. Each latent variable defines membership to one of the  $K$  groups. A sample is considered positive if it belongs to at least one of these groups, which can be expressed as follows:

$$y_i = \max_k(y_i^k) \quad (12)$$

Our goal is to simultaneously split the positive data into  $K$  groups and train  $K$  classifiers  $\mathbf{h}^1, \dots, \mathbf{h}^K$ , one per group, so that  $\max_k(\mathbf{h}^k(x_i)) = y_i$ . We call this problem *Multiple Pose Learning*. Note that the latent variables  $y_i^k$  are not known; if they were, we could train each  $\mathbf{h}^k$  using standard learning approaches.

As before, we would like to optimize the negative log likelihood  $\mathcal{L}$ . However, we need to modify our definition of the probability of  $x_i$ . We define the probability  $p_i^k \equiv p^k(y_i = 1|x_i)$  according to a single classifier  $\mathbf{h}^k$  as  $p_i^k = \sigma(2\mathbf{h}_i^k)$ , where  $\mathbf{h}_i^k = \mathbf{h}^k(x_i)$ . This is similar to Eqn. (4). Given all of the classifiers, we define the probability  $p_i \equiv p(y_i = 1|x_i)$  as the *maximum* over the probabilities  $p_i^k$ . Using one of the approximations  $g$  of the max from Section 2.2, we write:

$$p_i = g_k(p_i^k) = g_k(\sigma(2\mathbf{h}_i^k)) \quad (13)$$

This can be viewed as the probabilistic approximation of Eqn. (12). Note that  $p_i$  and consequently  $\mathcal{L}$  depend on each  $\mathbf{h}_i^k$ ; to make this dependence explicit we write  $\mathcal{L}(\mathbf{h}^1, \dots, \mathbf{h}^K)$ . We optimize  $\mathcal{L}(\mathbf{h}^1, \dots, \mathbf{h}^K)$  by coordinate descent, cycling through  $k$ , where in each phase we add a weak classifier to  $\mathbf{h}^k$  while keeping all other weak classifiers fixed. The algorithm is summarized in Figure 5.

**Input:** Dataset  $\{x_1, \dots, x_n\}, \{y_1, \dots, y_n\}, y_i \in \{-1, 1\}, K$

- 1: **for**  $t = 1$  to  $T$  **do**
- 2:   **for**  $k = 1$  to  $K$  **do**
- 3:     Compute weights  $w_i^k = -\frac{\partial \mathcal{L}}{\partial \mathbf{h}_i^k}$
- 4:     Train weak classifier  $h_i^k$  using weights  $|w_i^k|$   
 $h_i^k = \operatorname{argmin}_h \sum_i \mathbf{1}(h(x_i) \neq y_i) |w_i^k|$
- 5:     Find  $\alpha_t$  via line search to minimize  $\mathcal{L}(\cdot, \mathbf{h}^k, \cdot)$   
 $\alpha_t = \operatorname{argmin}_\alpha \mathcal{L}(\cdot, \mathbf{h}^k + \alpha h_i^k, \cdot)$
- 6:     Update strong classifier  $\mathbf{h}^k \leftarrow \mathbf{h}^k + \alpha_t^k h_i^k$ .
- 7:   **end for**
- 8: **end for**

**Fig. 5.** MPL-BOOST

All that remains is to derive  $w_i^k = -\frac{\partial \mathcal{L}}{\partial \mathbf{h}_i^k}$ . Using the chain rule we get:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{h}_i^k} = \frac{\partial \mathcal{L}}{\partial p_i} \frac{\partial p_i}{\partial p_i^k} \frac{\partial p_i^k}{\partial \mathbf{h}_i^k}. \quad (14)$$

All other terms cancel. Plugging in  $\frac{\partial \mathcal{L}}{\partial p_i}$  and  $\frac{\partial p_i^k}{\partial \mathbf{h}_i^k}$ , derived in Eqns. (5-6), we get:

$$w_i^k = \begin{cases} \frac{2p_i^k(1-p_i^k)}{p_i} \frac{\partial p_i}{\partial p_i^k} & \text{if } y_i = 1 \\ \frac{-2p_i^k(1-p_i^k)}{1-p_i} \frac{\partial p_i}{\partial p_i^k} & \text{if } y_i = -1 \end{cases} \quad (15)$$

The form of  $\frac{\partial p_i}{\partial p_i^k}$  depends on the choice of softmax model  $g$ , see Table 1. We list the equations for  $w_i^k$  for different choices of the softmax in Table 2.



	$y_i = -1$	$y_i = 1$
LSE	$\frac{-2p_i^k(1-p_i^k)}{1-p_i} \frac{\exp(rp_i^k)}{\sum_{\ell} \exp(rp_i^{\ell})}$	$\frac{2p_i^k(1-p_i^k)}{p_i} \frac{\exp(rp_i^k)}{\sum_{\ell} \exp(rp_i^{\ell})}$
GM	$\frac{-2p_i}{1-p_i} \left( \frac{(p_i^k)^r - (p_i^k)^{r+1}}{\sum_{\ell} (p_i^{\ell})^r} \right)$	$2 \left( \frac{(p_i^k)^r - (p_i^k)^{r+1}}{\sum_{\ell} (p_i^{\ell})^r} \right)$
NOR	$-2p_i^k$	$\frac{2p_i^k(1-p_i)}{p_i}$
ISR	$\frac{-2\chi_i^k p_i}{\sum_{\ell} \chi_i^{\ell}}, \chi_i^k = \frac{p_i^k}{1-p_i^k}$	$\frac{2\chi_i^k(1-p_i)}{\sum_{\ell} \chi_i^{\ell}}, \chi_i^k = \frac{p_i^k}{1-p_i^k}$

**Table 2.** MPL equations for  $w_i^k$  for different choices of  $g$ .

## 2.4 Multiple Instance Learning

We now turn to Multiple Instance Learning (MIL), and re-derive and generalize MIL-BOOST [4]. MIL was introduced in [3] for scenarios where a training sample cannot be represented as a single feature vector, but rather as a set of  $m$  vectors, often called a ‘bag’,  $X_i = \{x_{i1}, \dots, x_{im}\}$ . It is assumed that every instance has some true label  $y_{ij} \in \{-1, +1\}$ , but its value is not known during training (it is latent, much like  $y_i^k$  in MPL). A bag is labeled positive if at least one of its instances is positive, which can be expressed as follows:

$$y_i = \max_j(y_{ij}) \quad (16)$$

Here our goal is to learn a classifier  $\mathbf{h}$  using only bag labels  $y_i$ , such that  $\max_j(\mathbf{h}(x_{ij})) = y_i$ . Similar to MPL, if the latent variables  $y_{ij}$  were known this classifier could be trained with standard supervised learning.

As in the previous cases, we would like to optimize the negative log likelihood  $\mathcal{L}$ . To do so we must define  $p_i \equiv p(y_i = 1|X_i)$ , the probability of a bag  $X_i$ . We begin by defining the probability  $p_{ij} \equiv p(y_{ij} = 1|x_{ij})$  of an instance  $x_{ij}$  in the same manner as before:  $p_{ij} = \sigma(2\mathbf{h}_{ij})$ , where  $\mathbf{h}_{ij} = \mathbf{h}(x_{ij})$ . Given the instance probabilities, we define the bag probability  $p_i$  as the *maximum* over the instance probabilities  $p_{ij}$ . Using the softmax  $g$  in place of the max we write:

$$p_i = g_j(p_{ij}) = g_j(\sigma(2\mathbf{h}_{ij})) \quad (17)$$

This can be viewed as the probabilistic approximation of Eqn. (16). Note the similarity between Eqns. (12) and (16), and also between Eqns. (13) and (17). Although the form of the input data and learned classifiers for MIL and MPL are different, the form of the weights  $w_{ij}$  is nearly identical to the form of the weights  $w_i^k$ . In fact, if we replace the superscript  $k$  with the subscript  $j$  in Eqns. (14) and (15), we get the equivalent MIL equations (not shown).

Algorithm details are given in Figure 6. The optimization procedure for MIL is similar to the optimization procedure for regular boosting described in Section 2.1; compare Figures 3 and 6.

**Input:** Dataset  $\{X_1, \dots, X_n\}, \{y_1, \dots, y_n\}, y_i \in \{-1, 1\}$

- 1: **for**  $t = 1$  to  $T$  **do**
- 2:   Compute weights  $w_{ij} = -\frac{\partial \mathcal{L}}{\partial \mathbf{h}_{ij}}$
- 3:   Train weak classifier  $h_t$  using weights  $|w_{ij}|$   
        $h_t = \operatorname{argmin}_{\mathbf{h}} \sum_{ij} \mathbf{1}(h(x_{ij}) \neq y_i) |w_{ij}|$
- 4:   Find  $\alpha_t$  via line search to minimize  $\mathcal{L}(\mathbf{h})$   
        $\alpha_t = \operatorname{argmin}_{\alpha} \mathcal{L}(\mathbf{h} + \alpha h_t)$
- 5:   Update strong classifier  $\mathbf{h} \leftarrow \mathbf{h} + \alpha_t h_t$ .
- 6: **end for**

**Fig. 6.** MIL-BOOST

## 2.5 Extensions

For completeness, we conclude by discussing a number of noteworthy extensions and variations to MPL and MIL, which we leave to future work.

**Combining MPL and MIL** Thus far we have considered the two solutions to the alignment problem independently. It is conceivable that in certain situation it would be beneficial to apply both alignment approaches simultaneously. Here we briefly describe such an approach.

We are given a training dataset consisting of bags  $X_i = \{x_{i1}, \dots, x_{im}\}$  and bag labels  $y_i \in \{-1, +1\}$ . We assume that there exist  $K$  latent variables  $y_{ij}^k$  for each sample  $x_{ij}$  which define group memberships as in MPL. A bag is labeled positive if at least one of its instances belongs to at least one of the  $K$  groups:

$$y_i = \max_k \max_j (y_{ij}^k) = \max_{jk} (y_{ij}^k) \quad (18)$$

The formulation of this seemingly more complex problem is actually similar to both MIL and MPL. We can define the probability of  $x_{ij}$  according to  $\mathbf{h}^k$  in the same manner as before:  $p_{ij}^k = \sigma(2\mathbf{h}_{ij}^k)$ , where  $\mathbf{h}_{ij}^k = \mathbf{h}^k(x_{ij})$ . Then, we can define the bag probability  $p_i$  as the *maximum* over the instances *and* the classifiers:

$$p_i = g_{jk}(p_{ij}^k) = g_j(\sigma(2\mathbf{h}_{ij}^k)) \quad (19)$$

Note the similarity of Eqn. (19) to Eqns. (13) and (17). The form of the weights  $w_{ij}^k = -\frac{\partial \mathcal{L}}{\partial \mathbf{h}_{ij}^k}$  is again very similar to both MIL and MPL. Although we omit further details, derivation of the resulting boosting algorithm is straightforward.

**Alternate MPL/MIL Formulation** When deriving MPL, we defined  $p^k(y_i = 1|x_i)$  as  $p_i^k = \sigma(2\mathbf{h}_i^k)$ , and the overall probability as  $p_i = g_k(p_i^k)$ . Instead of defining  $p_i$  in terms of the  $p_i^k$ , we could define  $\mathbf{h}_i = g_k(\mathbf{h}_i^k)$  and  $p_i$  in terms of  $\mathbf{h}_i$ :

$$p_i = \sigma(2\mathbf{h}_i) = \sigma(2g_k(\mathbf{h}_i^k)) \quad (20)$$

Compare with Eqn. (13). Since  $\mathbf{h}_i^k \in \mathbb{R}$ , only the LSE softmax model is appropriate (see Table 1). Taking derivatives as before, we obtain:

$$w_i^k = \frac{\partial \mathcal{L}}{\partial p_i} \frac{\partial p_i}{\partial \mathbf{h}_i} \frac{\partial \mathbf{h}_i}{\partial \mathbf{h}_i^k} = w_i \frac{\partial \mathbf{h}_i}{\partial \mathbf{h}_i^k} = w_i \frac{\exp(r \mathbf{h}_i^k)}{\sum_{\ell} \exp(r \mathbf{h}_i^{\ell})}, \quad (21)$$

where  $w_i$  are the weights given in Eqn. (7). These weights can be used directly in our MPL boosting algorithm, see Fig. 5. Note that we could apply the same exact re-derivation for MIL, we omit details for space.

**Negatives in MPL/MIL** In MIL we defined  $p_i = p(y_i = 1 | X_i)$  in terms of  $p_{ij}$ , allowing us to write an expression for  $\mathcal{L}$ . We couldn't write  $\mathcal{L}$  directly in terms of  $p_{ij}$  as we did not know the instance labels  $y_{ij}$ . However, for negatives, we know that  $y_{ij} = -1$  for every  $j$ . We can thus write an alternate expression for  $\mathcal{L}$ :

$$\mathcal{L}(\mathbf{h}) = - \sum_{i=1}^n \left( \mathbf{1}(y_i = 1) \log p_i + \mathbf{1}(y_i = -1) \sum_{j=1}^m \log(1 - p_{ij}) \right). \quad (22)$$

For positives,  $w_{ij}$  remains unchanged. For negatives,  $w_{ij} = -2p_{ij}$ , as in Eqn. 7. All negative instances are treated identically regardless of the bag  $X_i$  they came from, just as in the NOR model. In MPL, we can use the same variant of  $\mathcal{L}$ , obtaining  $w_i^k = -2p_i^k$  for negatives. This seems appropriate, as each classifier  $\mathbf{h}^k$  has to correctly classify every negative.



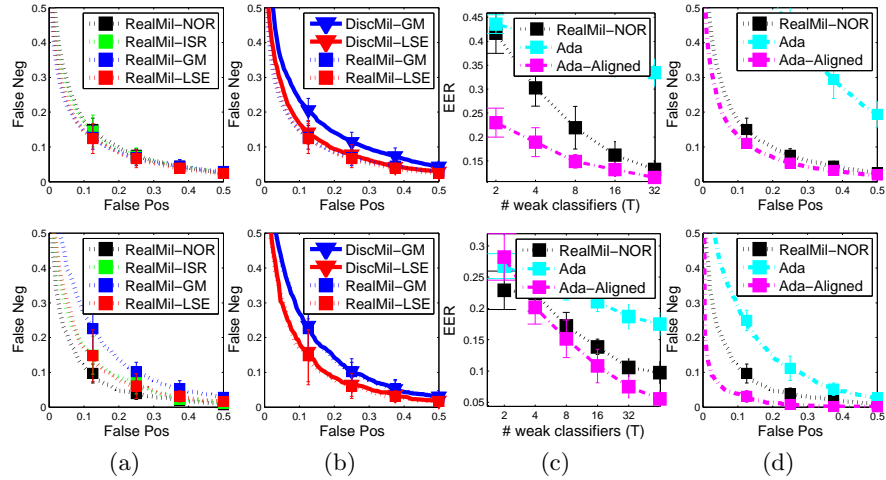
**Fig. 7.** Examples of positive (green) and negative (red) bags for the MIL experiments: *Left:* MNIST, *Middle:* LFW. For positive bags we highlight the true (unknown) object location. *Right:* Actual detections of a MIL-BOOST classifier (after non-maximal suppr.).

### 3 Experiments

We used two datasets: LFW [5] and MNIST [6]. MNIST is a simple optical digit recognition dataset; we use it to illustrate certain properties of the algorithms presented. The LFW dataset contains images of faces in the media. Although primarily intended for evaluating face recognition, we use it within a detection setting. For ‘hard’ non-face images we used false-positives returned by a Viola-Jones face detector. Fig. 7 shows example images from each datasets. In all experiments we use random Haar features [25] as the weak classifiers.

### 3.1 Multiple Instance Learning

As shown in [4], MIL-BOOST is well suited for discovering translational alignment. In fact, Viola *et al.* argue that delineating object boundaries is an inherently ambiguous task. Instead, we place several overlapping bounding boxes in a bag and use MIL to simultaneously train a classifier and select a good alignment.



**Fig. 8. MIL Results.** *Top: MNIST, Bottom: LFW.* (a) Comparison of the different softmax functions. (b) Comparing the discrete and real versions of MIL-BOOST. (c) Equal error rate (EER) versus the number of weak classifiers. (d) MIL-BOOST compared to standard learning (AdaBoost) on aligned and non-aligned data.

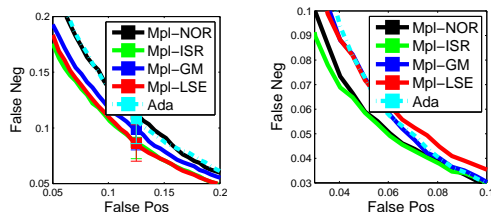
**LFW** The LFW faces are aligned and centered. For the purpose of this experiment we ‘corrupted’ this alignment by randomly shifting the face 1-10 pixels within each  $63 \times 63$  image. The same process was applied to negatives to avoid artifacts. Example images are shown in Figure 7. In each trial we used 400 images for training and 400 for testing; results are averages over 10 trial. For all algorithms tested we used boosting to select  $T = 64$  weak stump classifiers from a pool of 1000 random Haar features.

Our results can be divided into 3 groups. First, we compare variations of MIL-BOOST. We show results obtained using different softmax function in Fig. 8(a) and using the real and discrete variations in 8(b). Second, we compare MIL-BOOST with standard supervised learning. We trained two additional classifiers using AdaBoost [22]: one using the unaligned data we used for MIL and one using cropped and aligned data. Results are shown in Fig. 8(c-d).

Finally, we seek to measure how well MIL-BOOST aligns the data. Specifically, we measure the *consistency* of the alignment w.r.t. to the ground truth (a constant offset in the predictions can easily be corrected). We define the alignment error  $\epsilon$  as the standard deviation of the true and predicted locations  $l_i^*$

and  $l_i$ :  $\epsilon = \mathbf{std}(l_i^* - l_i)$ . We compute  $l_i$  by smoothing the classifier response and recording the maximum location (but only for  $i$  s.t.  $p_i > 25\%$ ).

For MIL-BOOST (the Real-LSE version) the alignment error averaged over 10 trials was  $\epsilon = 5.07$ . For comparison, we evaluated how well AdaBoost trained with (the small amount of) aligned data can perform alignment. The alignment error was  $\epsilon = 5.97$ , which was actually higher. The classifier response was rather diffuse, most likely due to the small amount of training data (note that AdaBoost with aligned data still had lower classification error). Nevertheless, this experiment suggests that MIL-BOOST does indeed recover a good alignment.



**Fig. 9. MPL Results.** ROC plots comparing four variants of MPL-BOOST and standard learning (AdaBoost) with: *left*: MNIST; *right*: LFW.

**MNIST** We arbitrarily chose the digit ‘3’ to be the positive and the rest negative. We pad each  $28 \times 28$  image with an 8 pixel border and randomly translated the digit within the resulting  $44 \times 44$  image. All results are averaged over 10 trials and use 1000 images, 10% for training each time. We performed identical experiments as for the LFW data. Results are shown in Figure 8, top; the average alignment errors were  $\epsilon = 3.08$  for AdaBoost and  $\epsilon = 1.40$  for MIL-BOOST.

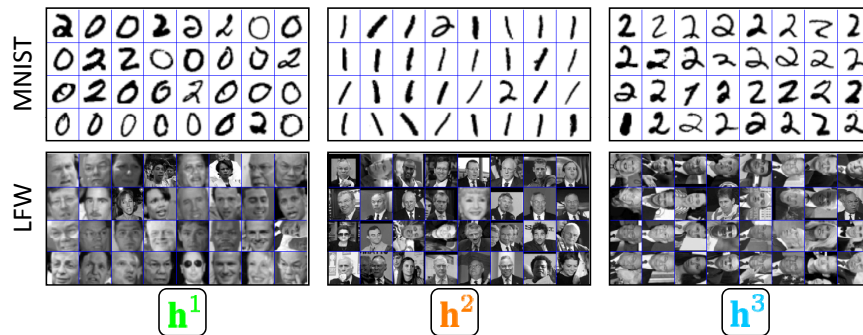
### 3.2 Multiple Pose Learning

The goal of MPL is to simultaneously group data and train a classifier for each group. Therefore a natural application of MPL is to train detectors for object categories that have several distinct views. *E.g.*, out of plane rotations make it difficult to place faces into correspondence, which can make it difficult to train a precise face detector. On the other hand, if we can accurately group data during training, we not only achieve higher accuracy but also potentially more meaningful output (*e.g.* a face detector that also predicts facial pose).

**LFW** We took 2000 LFW faces and synthesized three distinct groups: (1) faces kept as is, (2) faces rescaled by a factor of 2.5, and (2) faces rescaled by 2 and rotated  $90^\circ$ . In each case we use boosting to select  $T = 64$  weak classifiers from 1000 candidates. We randomly split 4000 images into equal sized training and testing sets, and average all experiments over 10 trials. Figure 9(right) shows ROC curves comparing AdaBoost with four versions of MPL-BOOST. In Fig. 10 we

show randomly selected test images grouped according to the learned classifiers. We see that MPL-BOOST recovers the groups reasonably well. Since we have the ground truth memberships  $y_i^k$ , we can quantify the quality of the grouping (up to a permutation of the group ids). For the faces, the grouping accuracy for the four softmax models were: NOR 0.81%, ISR 0.84%, GM 0.83%, LSE 0.88%.

**MNIST** Our final experiment is similar to the one described above. We chose the digits 0-2 as positives and the others as negatives. The setup varied only slightly (1500 positives, 1500 negatives, 40% used for training, averaged over 10 trials). ROC curves are shown in Fig. 9 (left). We computed membership accuracy by assuming each positive digit forms its own group: NOR 0.83%, ISR 0.89%, GM 0.87%, LSE 0.87%.



**Fig. 10. MPL Alignment Results.** Randomly selected test images grouped according to the 3 classifiers trained with MPL. *Top:* MNIST; *Bottom:* LFW.

## 4 Conclusion and Future Work

In this paper we discussed two simultaneous learning and alignment strategies that can potentially lead to higher accuracy detection systems. Specifically, we derived a novel framework we call Multiple Pose Learning and a unified boosting approach that encompasses both MPL and MIL. The results we presented are preliminary but promising. Significant testing remains to evaluate if these approaches can improve state of the art detection systems. We also described a number of extensions to MPL and MIL that warrant further investigation.

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