Network Reconfiguration using Cops-and-Robber Games

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Abstract: The process number is the number of requests that have to be simultaneously disturbed during a routing reconfiguration phase of a connection oriented network. From a graph theory point of view, it is similar to the pathwidth. However they are not always equal in general graphs. Determining these parameters is in general NP-complete. In this paper, we propose a polynomial algorithm to compute an approximation of the process number of digraphs, improving the efficiency of the previous exponential algorithm.

Key-words: Rerouting, process number, vertex separation, pathwidth.

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Reconfiguration de routage à l'aide du jeu des gendarmes et du voleur

Résumé : Le *process number* est le nombre de requêtes simultanément perturbées lors du phase de reconfiguration de routage dans un réseau orienté connexions. Du point de vue de la théorie des graphes, ce problème est similaire à la *pathwidth*, mais pas toujours égal. Déterminer ce paramètre est en général NP-complet. Dans ce rapport, nous proposons un algorithme heuristique estimant en temps polynomial le process number d'un graphe orienté. Cet algorithme a de meilleures performances que les algorithmes existant.

Mots-clés : Reroutage, process number, vertex separation, largeur de chemins

1 Introduction

When designing a wavelength division multiplexing (WDM) network, designers have to take into account expected traffic patterns, class of services proposed for failure resilience, routing algorithms, various functionality of the components, and many other parameters. Such designs are extremely hard problems to solve, in particular due to unpredictable evolution of the traffic pattern during the lifetime of the network. Also, links are generally oversized to simplify the design of the network. However, with the rapid increase of bandwidth requirements it is more and more difficult to adapt the network usage to traffic variation and so to maintain a near-optimal usage of resources.

In WDM networks without wavelength conversion, the acceptation of a new connection request is subject to the availability of a lightpath (and so resources) from end to end in the network. Whatever the routing algorithm chosen, successive addition and removal of connections may lead to a poor usage of resources. Thus new connection requests might be rejected although the network has enough resources to handle the traffic pattern, up to the rerouting of some requests. For example, in Fig. 2(a), the network is a 9 nodes grid with one wavelength and 5 requests a, b, c, d, e. A new connection request r will be rejected in Fig. 2(a) although the routing of Fig. 2(b) is possible. Therefore, it is necessary to change from time to time the routing of established connections to improve the usage of resources, and so accept more traffic.

In this paper we concentrate on the reconfiguration phase, that is the problem of switching the set of connections from current routing, R_1, to a new pre-computed routing, R_2. To solve this problem, several strategies are possible. For example, one may interrupt all connections requests that have to be rerouted and after restart them with their new routes in R_2. This strategy is very simple, but requires to store a huge amount of traffic during the reconfiguration phase and so requires large and costly buffers.

Another strategy is to reroute connection requests one after the other, as soon as destination resources (lightpath in the new routing) are available, minimizing the number of simultaneous interrupted requests during the reconfiguration. It corresponds to compute the *process number* (Sec. 2) in the *dependency digraph* [6]. This requires to compute the scheduling of the rerouting, taking into account that destination resources might be currently used by other connections. More precisely, resources assigned to request r in R_2 might be used by some request r' in R_1, thus request r' has to be rerouted before r. We represent these constraints by a digraph $D = (V, A)$, the *dependency digraph* [6], in which each node corresponds to a request, and there is an arc from vertex u to vertex v if v must be rerouted before u.

For example to switch from routing R_1 of Fig. 2(a) to routing R_2 of Fig. 2(b), we construct the dependency digraph D of Fig. 2(c). It has one node per connection that has to be rerouted, and so, connection e is not represented in D since it is not affected. It has an arc from a to b since connection b has to be switched before connection a. Similarly, it has arcs from b to c and from b to d since c and d must be switched before b, and so on.

When the digraph D is a DAG (direct acyclic graph), the scheduling is straightforward, but in general, it may contain cycles. To break them, some requests have to be temporarily

interrupted, thus removing some incident arcs in D. So, the optimization problem is to find a scheduling minimizing the number of requests simultaneously interrupted.

This problem has first been considered in [6]. They have proposed a heuristic algorithm based on the minimum feedback vertex set (MFVS) of D (i.e. the smallest subset S of nodes of D such that $D - S$ is a DAG) that performs well on small instances. This problem has also been considered in [3, 4] with a modeling in terms of *cops-and-robber game* [8]: the *process number*, denoted pn. It allows to use more advance theoretical concepts and so to design more efficient heuristic algorithms. For example, the digraph of Fig. 1 is such that $|\text{MFVS}(D)| = n/2$ and pn(D) = 2. So it is possible to reroute corresponding requests with only two requests simultaneously interrupted.

This paper starts in Sec. 2 with the modeling of our reconfiguration problem as a cops-and-robber game in the dependencies digraph, and so the definition of the process number. Then, in Sec. 3 we present some previous results linking pathwidth, vertex separation and process number. We also recall the heuristic algorithm proposed in [6]. In Sec. 4, we propose a heuristic algorithm to compute the process number of digraphs (i.e. maximum number of simultaneously interrupted requests) and the corresponding network reconfiguration strategy. Finally, in Sec. 5 we present simulations results and analyze the efficiency of our heuristic algorithm compared to others.

2 Modeling

It has been proved in [3, 4] that the routing reconfiguration problem can be expressed as a cops-and-robber game [8], as for the *pathwidth* [10]. An interruption is represented by placing an *agent* on the corresponding node in D. A node is said *processed* when the corresponding request has been rerouted. If the node was occupied by an agent, then it can be reused. We call a *process strategy* a series of the three following actions (rules) allowing to reroute all requests with respect to the constraints represented by the digraph.

R_1. Put an agent on a node (*interrupt a connection*).

R_2. Remove an agent from a node if all its out-neighbors are either processed or occupied by an agent (*restart a connection on its final route when destination resources are available*). The node is now processed (*connection has been rerouted*).

R_3. Process a node if all its out-neighbors are either processed or occupied by an agent[1] (*connection has been rerouted because destination resources are available*).

A *p-process strategy* is a strategy which process the digraph using p agents and the *process number*, pn(D), is the smallest p such that a p-process strategy exists.

Clearly, pn(D) is upper bounded by the minimum feedback vertex set (MFVS) of D, that is the smallest subset S of nodes of D such that $D-S$ is a DAG. However, this bound is very

[1] In terms of cops and robber games in undirected graphs, rule R_3 expresses that the fugitive is forced to move at each step.

large. For example, the digraph represented in Fig. 1 is such that $|\text{MFVS}(D)| = n/2$, but it can be 2-processed. For that, we put an agent on node u (rule R_1). Then we apply rule R_3 to process node u'. After, we put a second agent on node v (rule R_1), process node v' (rule R_3) and then process node v (rule R_2). We repeat on the predecessor of v until processing of the out-neighbor w of u. Finally, we apply rule R_2 on u.

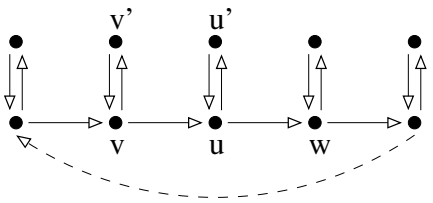


Figure 1: Digraph D such that pn(D) = 2 and $\text{MFVS}(D) = n/2$.

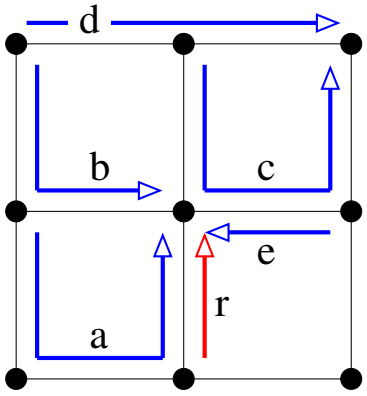
Notice also that when D is a DAG the scheduling is straightforward using rule R_3 from the leaves (nodes with out-degree 0), and we have pn(D) = 0.

3 Previous work

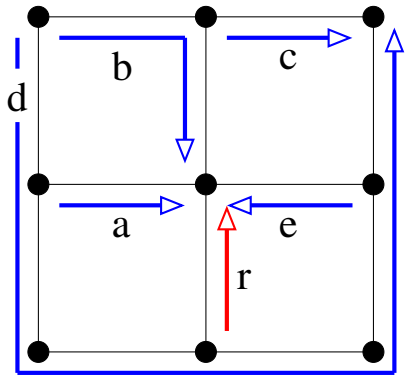
First of all, it has been proved in [4] that $\text{vs}(D) \leq \text{pn(D)} \leq \text{vs(D)} + 1$, where $\text{vs}(D)$ is the *vertex separation* of D. This implies that determining pn(D) is NP-complete in general, since it is also the case for $\text{vs}(D)$. Furthermore, when D is a symmetric digraph, the same result holds for the underlying graph G. We also have $\text{vs}(G) = \text{pw}(G)$, where $\text{pw}(G)$ is the *pathwidth* of G [7, 10], and determining the pathwidth is NP-complete in general, approximable within $O(\log^2 |V(G)|)$, and there is no polynomial-time algorithm with an absolute error guarantee of $|V(G)|^{1-\varepsilon}$ for any $\varepsilon > 0$ [8, 2]. The same holds for the process number and so for the routing reconfiguration problem considered in [6, 4] and in this paper, thus motivating the development of an efficient heuristic algorithm.

Notice that for specific topologies it is possible to determine the process number in polynomial time. In particular, a characterization of digraphs with process number 1 and 2 is given in [4] as well as $O(n+m)$ and $O(n^2(n+m))$ respectively time complexity recognition algorithms.

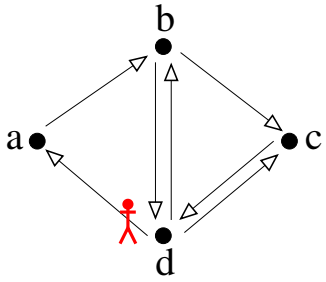
When the digraph is composed of several strongly connected components (SCCs), it is possible to process each SCC independently, and then obtain the processing of D. For that, let $\{\text{SCC}_i\}$ be the set of SCCs of D. Let also D_{SCC} be the DAG of SCCs of D, thus containing one node per component of $\{\text{SCC}_i\}$ and one arc from node i to node j iff D contains an arc from some node $u \in \text{SCC}_i$ to some node $v \in \text{SCC}_j$. Then the processing of D consists in processing the SCCs sequentially following the order given by D_{SCC}. Since SCCs are processed independently from each other the Lemma 1 follows. Notice that the computation of $\{\text{SCC}_i\}$ and D_{SCC} takes time $O(n + m)$ using standard algorithms.



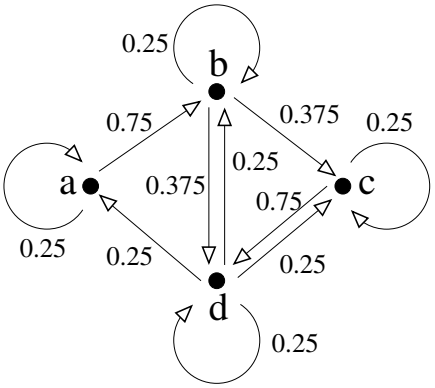
(a) Initial routing R_1 of requests a, b, c, d, e. The new request r can not be satisfied.



(b) The new routing R_2 with requests a, b, c, d, e and r.



(c) Dependency digraph D for switching the routing from R_1 to R_2.



(d) D_μ from D to apply *flow circulation* algorithm.

Figure 2: Dependency digraph D (c) and corresponding D_μ (d) to switch from routing R_1 (a) to routing R_2 (b).

Lemma 1. *Given a digraph D and the set $\{\text{SCC}_i\}$ of its strongly connected components, we have* $\text{pn(D)} = \max_i\{\text{pn}(\text{SCC}_i)\}$.

We now recall the principle of the heuristic algorithm proposed in [6], `HeurJS`, to obtain the rerouting ordering of a digraph $D = (V, A)$. It uses Lemma 1, and so we may assume that D is strongly connected. The main idea of `HeurJS` is to break elementary cycles. For that, it first computes the c elementary cycles of D using Johnson's algorithm [5] and assigns to each node u the number $\alpha(u)$ of cycles it belongs to. Then it places an agent on the node u_1 of maximum weight, thus breaking $\alpha(u_1)$ cycles, updates the values of each remaining nodes and repeats if $\alpha(u_i) > 0$, where u_i is the new node of maximum weight. Let L be the set of nodes covered by an agent. When all cycles have been broken, the digraph $D - L$ is a DAG and so can be 0-processed. Thus it processes $D - L$ in appropriate order and then the nodes of L. The number of agents simultaneously used is $|L|$, and the time complexity of `HeurJS` is dominated by the computation of elementary cycles which takes time $O((n+m)(c+1))$ [6]. Notice that this algorithm is exponential with the number c of elementary cycles, and so with the number n of nodes. Thus, `HeurJS` can only permit to solve the problem for small digraphs, as it has been designed for.

To the best of our understanding, `HeurJS` is in fact a heuristic algorithm for MFVS applied on each SCC of D, following Lemma 1. Although shown to be quite efficient in practice through a large number of experimentations [6], its result could be far from optimality (see the example of Fig. 1).

4 Process strategy based heuristic algorithm

We now propose a heuristic algorithm `HeurCM` for solving the reconfiguration problem, taking advantages of the modeling with the process number proposed in [4]. This heuristic algorithm has polynomial time complexity, and with high probability improves upon `HeurJS` on the number of agents needed simultaneously (corresponding to the number of requests disturbed at the same time) in resulting strategy.

4.1 Flow circulation algorithm

Our algorithm is based on a *flow circulation* algorithm which has the objective of choosing the best candidate node to receive an agent and so to break a large set of cycles.

The principle is the following. Given a dependency digraph $D = (V, A)$, each node $u \in V$ is initially assigned a weight $q_0(u) = 1/|V|$. Then, at each round t, each node u sends $(1-\mu)q_t(u)/d^+(u)$ to each of its out-neighbors, where $d^+(u) = |\Gamma^+(u)|$ with $\Gamma^+(u)$ the set of out-neighbors of u in D, and $\mu \in (0, 1)$. Furthermore u keeps $\mu q_t(u)$ in order to satisfy discrete Markov chain conditions. Indeed we transform the digraph $D = (V, A)$ into $D_\mu = (V_\mu, A_\mu)$, such that $V_\mu = V$ and $A_\mu = A \cup \{(u, u), \forall u \in V\}$. In other words, we add loops to each node of D. Clearly, the value of μ will influence the convergence time (i.e. number of rounds) of the flow circulation algorithm without significant impact on the

final weights. Thus, we choose it close to 0, and so we will only slightly slow down the convergence time. We have for all $u \in D_\mu$:

$$q_{t+1}(u) = \mu q_t(u) + \sum_{v \in \Gamma^-(u)} (1-\mu) q_t(v)/d^+(v) \tag{1}$$

where $\Gamma^-(u)$ is the set of predecessors of u.

After k rounds (the value of k will be discussed in Lemma 3), the node with maximum weight is the best candidate. With *flow circulation* algorithm, we expect that the node with maximum weight belongs to many cycles. So its removal from the digraph may break many cycles.

We will now evaluate the number k of rounds needed in the *flow circulation* algorithm. For that, we will first prove in Lemma 2 the convergence of the algorithm using a discrete Markov chain for which we define the set of states $\{u_1, u_2, ..., u_{|V_\mu|}\}$ corresponding to the nodes in V_μ and the transition matrix M associated to D_μ. We have for all $x, y \in V_\mu$

$$M[x,y] = \begin{cases} (1-\mu)/d^+(x) & \text{when } y \in \Gamma^+(x) \\ \mu & \text{when } x = y \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $\Gamma^+(x)$ is the set of out-neighbors of x and $d^+(x) = |\Gamma^+(x)|$. We also define the vector of probability $q_t(V_\mu) = (q_t(u_1), q_t(u_2), ..., q_t(u_{|V_\mu|}))$. $q_t(u_i)$ $(1 \leq i \leq |V_\mu|)$ is the probability that the state of the system is u_i at step $t \geq 0$.

Lemma 2. *Let D_μ be a strongly connected digraph with loops. Whatever $q_0(V_\mu)$, we have $\lim_{t \to +\infty} q_t(V_\mu) = \overline{q}(V_\mu)$, where $\overline{q}(V_\mu)$ is the unique stationary weights vector.*

Proof. The proof relies on the discrete time Markov chain described previously, where the vector $q_t(V_\mu)$ represents the probability of each state at step $t \geq 0$ and M, associated to D_μ, represents the transition matrix (see Equation 2).

From the Markov chains theory, we know that if the chain is irreducible and aperiodic (ergodic), then there exists a unique stationary distribution $\overline{q}(V_\mu)$ whatever the initial state $q_0(V_\mu)$ [9].

• The irreducibility of the Markov chain is obvious because D_μ is a strongly connected digraph. Indeed from each state x, there exists a positive probability to move to state y since there exists a path from x to y in D_μ.

• The Markov chain is aperiodic since D_μ is a strongly connected digraph with loops [9]. □

Fig. 2(c) consists on a digraph $D = (V, A)$ of 4 nodes a, b, c, d. We construct $D_\mu = (V_\mu, A_\mu)$ (Fig. 2(d)), adding loops in order to satisfy the Markov conditions described before, even if it is not necessary in this example. The probabilities on the edges in Fig. 2(d) represent the Markov transition matrix (see Equation 2). We have $q_t(V_\mu) = (q_t(a), q_t(b), q_t(c), q_t(d))$ such that $q_0(V_\mu) = (0.25, 0.25, 0.25, 0.25)$. After 4 steps, we have

$q_4(V_\mu) \approx (0.1257, 0.2473, 0.2503, 0.3767)$, that is to say almost the stable vector $\overline{q}(V_\mu) = (0.125, 0.25, 0.25, 0.375)$. Thus node d has maximum weight.

The Markov chain admits always a stable state due to the artificial loop transition on each node. We now have to choose k sufficiently large to get the stable vector $\overline{q}(V_\mu)$ with an allowed error ε, that is $\varepsilon_k \leq \varepsilon$, where $\varepsilon_k = ||q_k(V_\mu) - \overline{q}(V_\mu)||_\infty = O(|\lambda_2|^k)$, denoting λ_2 the second largest eigenvalue of the transition matrix M. Thus we will obtain the node with the maximum weight.

Lemma 3. *If $|\lambda_2| < \varepsilon^{1/k}$, then flow circulation algorithm computes $\overline{q}(V_\mu)$ with an allowed error ε.*

Proof. From Perron-Frobenius Theorem [1], we know that the convergence time from $q_t(V_\mu)$ to the stationary vector $\overline{q}(V_\mu)$ depends only of the second largest eigenvalue λ_2 of M (the first largest is always 1). Thus when $|\lambda_2|^k < \varepsilon$, the stable state is obtained with $||q_k(V_\mu) - \overline{q}(V_\mu)||_\infty \leq \varepsilon$. □

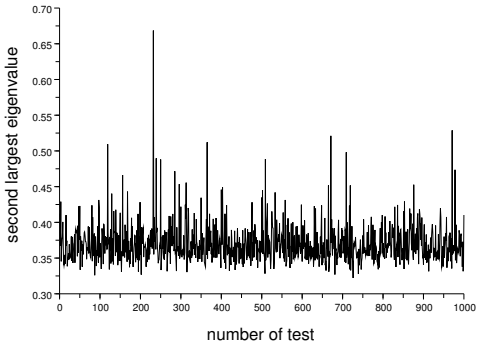
Since computing the second largest eigenvalue value λ_2 of M is time consuming, in our experiments we choose to fix arbitrarily $k = n$. For the example of Fig. 2(d), we thus have $||q_4(V_\mu) - \overline{q}(V_\mu)||_\infty \leq 0.003$. Another example is with $n = 500$ requests and an allowed error of $\varepsilon = 0.01$. Then, the flow circulation algorithm computes the stationary vector of the Markov chain if $|\lambda_2| < 0.99$ (remember that the second largest eigenvalue is strictly lower than 1, and that $0.99^{500} < 0.01$). Thus with a very high probability, we get the right stable vector and so a good candidate node for breaking cycles. See Sec. 5.1 for more details.

4.2 Heuristic algorithm

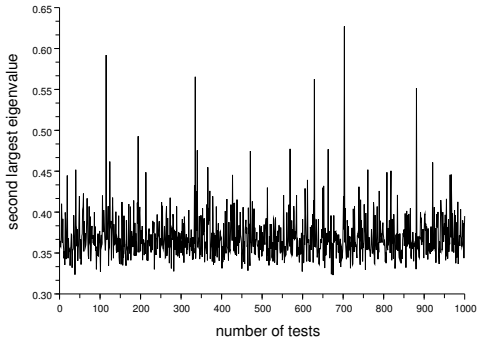
We assume that we are given a strongly connected digraph D. Otherwise, we apply our algorithm on each SCCs, according to Lemma 1. We also assume that D is different than a single node digraph (otherwise we can process it easily) and it has been transformed into D_μ, but we say D for simplicity.

The heuristic algorithm, `HeurCM`, is described in Algorithm 1. It consists in choosing a node to place an agent using the *flow circulation* algorithm, and then it decomposes the digraph into SCCs and repeats on each SCC_i. Algorithm `HeurCM` maintains the set of nodes covered by an agent, $\mathcal{S}$, sorted by insertion dates. Also $\mathcal{S}.last$ is the latest inserted node that has not yet been processed. After any step of `HeurCM`, $\mathcal{S}$ may change: insertion of a new node covered by an agent (rule R_1), and removal of nodes that can be processed (rule R_2). The number of agents used by `HeurCM` is thus the maximum size reached by $\mathcal{S}$ during the algorithm, $\mathcal{S}.\max$.

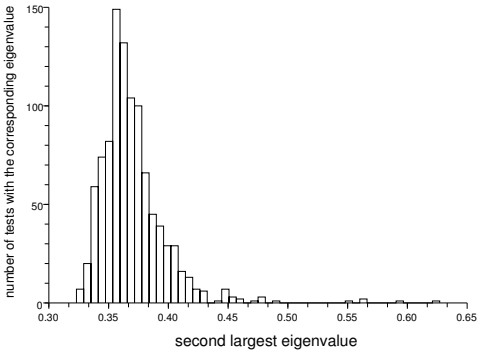
The dependency digraph D of Fig. 2(c) is strongly connected and we apply `HeurCM` on it. We first apply flow circulation algorithm to choose the node to be covered by an agent, i.e. the node with maximum weight. So we choose d and add it to $\mathcal{S}$ (Fig. 2(c)). Since $D - \{d\}$ is a DAG, we can now process all remaining nodes (line 1) and finally process d. Thus `HeurCM` uses 1 agent in this example.



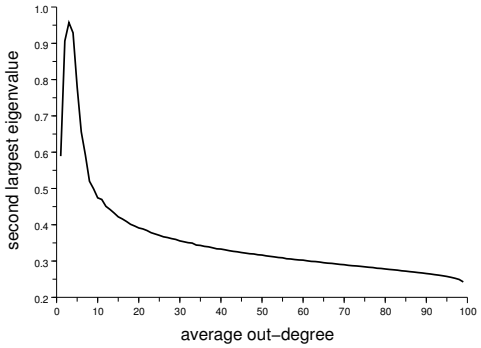
(a) λ_2 for a number of nodes $n = 100$, 1000 tests, and $\mu = 0.25$.



(b) λ_2 for a number of nodes $n = 100$, 1000 tests, and $\mu = 0.25$.

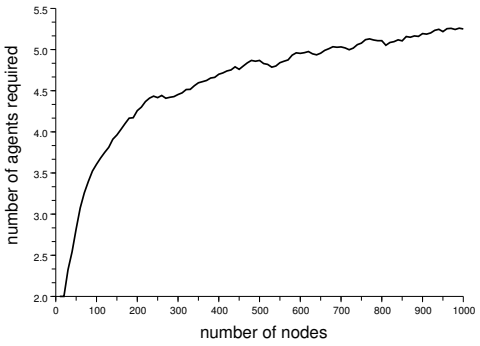


(c) Repartition of λ_2 for $n = 100$ and 100 tests.

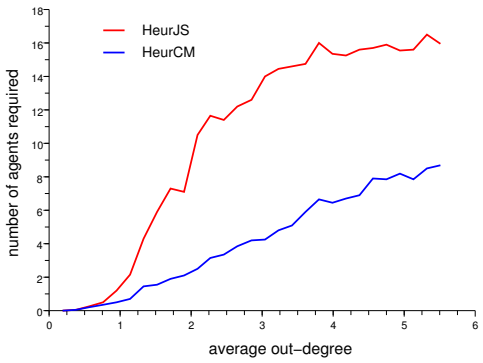


(d) λ_2 according to the average out-degree for graphs with $n = 100$ nodes and $\mu = 0.25$. 100 tests for each average out-degree.

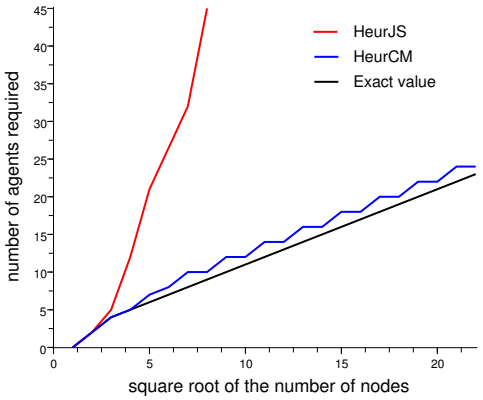
Figure 3: Simulations: second largest eigenvalue.



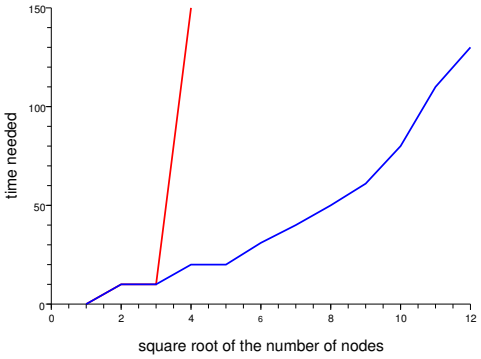
(a) Number of agents required by `HeurCM` to process digraphs of process number 2. 100 tests for each number of nodes $10k$, $k = 1..100$.



(b) Number of agents required to process a 20 nodes random graph function of the average out-degree.



(c) Number of agents required to process $n \times n$ grids.



(d) Computation time to process $n \times n$ grids.

Figure 4: Simulations: efficiency of `HeurCM`.

Algorithm 1 `HeurCM`

Require: A strongly connected digraph D, the set of nodes covered by an agent $\mathcal{S}$ and the set of active nodes $\mathcal{A}$
Ensure: The number of agents needed
1: Apply *flow circulation* on D.
2: Let u be one of the nodes of maximum weight. In case of equality we choose the closest node to $\mathcal{S}.last$.
3: Place an agent on u: add it to $\mathcal{S}$ and remove it from $\mathcal{A}$.
4: Process all the nodes whose can be processed updating $\mathcal{S}$ and $\mathcal{A}$.
5: Decompose $D \cap \{A\}$ into SCCs: $\{\text{SCC}_i\}$
6: **if** $\{\text{SCC}_i\}$ is not empty **then**
7: `HeurCM`$(\text{SCC}_i, \mathcal{S})$, $i = 1..|\{\text{SCC}_i\}|$
8: **end if**

With the dependency digraph of Fig. 1, and w.l.o.g., node u is chosen among the set of nodes of maximum weights (all belonging to the main cycle) and added to $\mathcal{S}$. Now, u' is processed and then the digraph is decomposed into SCCs, each being a 2-cycle, on which we apply recursively `HeurCM` (line 1). Thus, `HeurCM` will use only 2 agents.

When choosing the node of maximum weight at line 1 of Algorithm 1, it may happen that among the set of candidates, one of them allows to process a node of $\mathcal{S}$. This is typically the case when the digraph is a bidirectional path $u_0, u_1, u_2, u_3, \ldots, u_r, u_{r+1}$. W.l.o.g., we may assume that u_1 is chosen at line 1 and so added to $\mathcal{S}$. We process node u_0, and we repeat the algorithm on $u_2, u_3, \ldots, u_{r+1}$. Now, candidates will be u_3 and u_r (the symmetry of the graph give same weights for u_3 and u_r), but u_3 is clearly a better choice since is allows to process u_2 and after u_1 thus releasing one agent. It is why in case of equality, we choose the closest node to $\mathcal{S}.last$.

Lemma 4. *The worst case time complexity of `HeurCM` is* $O(kn(n+m))$.

Proof. Each flow circulation round takes time $O(k(n+m))$ and in the worst case $D \cap \{A\} = D - \{u\}$ is a single SCC. □

Finally, note that the rerouting strategy follows directly our algorithm. With `HeurCM`, it is sufficient to know entering and leaving dates in $\mathcal{S}$ and in $\mathcal{A}$ to know when a connection is suspended (enter in $\mathcal{S}$) or switched to its new route (leave $\mathcal{S}$ or $\mathcal{A}$).

5 Simulations

In this section, we first analyze further the convergence time of the *flow circulation* algorithm. Then, we analyze the performance of our heuristic algorithm `HeurCM`.

5.1 Convergence time of flow circulation

We present here some simulations showing that the second eigenvalue λ_2 of M respects almost surely the conditions given in Lemma 3. For that, we have designed different kinds of random Markov matrices $M_{n\times n}$.

• A fully random matrix, that is to say for each line of M (i.e. for each node), the out-degree and the out-neighbors are chosen randomly and uniformly. Fig. 3(d) and Fig. 3(b) show the distribution of λ_2 for $n = 100$ and 1000 tests. For these two simulations, λ_2 is always lower than 0.7. Furthermore, Fig. 3(c) describes that with high probability, λ_2 is small.

• We have designed a random matrix with constant average out-degree $\overline{d}$, that is to say for each line of M, $\overline{d}$ is a binomial distribution of parameters $\overline{d}/n$ and n. Then the out-neighbors are chosen randomly and uniformly. Fig. 3(d) shows the value λ_2 for each possible average out-degree for a digraph of 100 nodes. Note that for each average out-degree, 100 tests has been done. λ_2 is very small (lower than 0.5) except for very small average out-degree.

5.2 Simulations of `HeurJS` and `HeurCM`

We have implemented and analysed the efficiency of `HeurJS` and our algorithm `HeurCM` (in term of number of agents simultaneously required and in term of computation time).

• In [4], a characterization of the graphs with process number 2 is done and Fig. 4(a) shows the approximate process number computed by `HeurCM` for these graphs. The number of agents increases very slowly with the number of nodes (requests). For a digraph of 1000 nodes, the number of agents required is almost 5.

• For a symmetric grid $G_{n\times n}$, Fig. 4(c) shows that the approximate process number computed by `HeurCM` is closed to the exact value ($n + 1$ if $n > 2$) whereas the number of agents required by `HeurJS` increases exponentially. Furthermore, the computation time is very smaller for `HeurCM` compared to `HeurJS` (Fig. 4(d)), because a grid is a strongly connected symmetric digraph and there is an exponential number of cycles.

• For a 20 nodes random graph, starting with an average out-degree $\overline{d}$ closed to 0, increasing it until $\overline{d} = 6$, Fig. 4(b) shows that `HeurCM` requires less agents that `HeurJS`.

6 Conclusion

In this paper, we have proposed a new heuristic algorithm for the reconfiguration problem of switching the connections from one routing to another with objective of minimizing the number of connections simultaneously interrupted. This new heuristic algorithm is based on a modeling with cops-and-robbers games and improves upon previous proposal in both efficiency and computation time, as shown by our simulation results.

The next step is to design an exact and efficient exponential algorithm viable in practice for digraphs with a small number of nodes.

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