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Network Reconfiguration using Cops-and-Robber Games

David Coudert — Dorian Mazauric

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Network Reconfiguration using Cops-and-Robber Games

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Abstract: The process number is the number of requests that have to be simultaneously disturbed during a routing reconfiguration phase of a connection oriented network. From a graph theory point of view, it is similar to the pathwidth. However they are not always equal in general graphs. Determining these parameters is in general NP-complete. In this paper, we propose a polynomial algorithm to compute an approximation of the process number of digraphs, improving the efficiency of the previous exponential algorithm.

Key-words: Rerouting, process number, vertex separation, pathwidth.

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Reconfiguration de routage à l'aide du jeu des gendarmes et du voleur

Résumé : Le *process number* est le nombre de requêtes simultanément perturbées lors du phase de reconfiguration de routage dans un réseau orienté connexions. Du point de vue de la théorie des graphes, ce problème est similaire à la *pathwidth*, mais pas toujours égal. Déterminer ce paramètre est en général NP-complet. Dans ce rapport, nous proposons un algorithme heuristique estimant en temps polynomial le process number d'un graphe orienté. Cet algorithme a de meilleures performances que les algorithmes existant.

Mots-clés : Reroutage, process number, vertex separation, largeur de chemins

1 Introduction

When designing a wavelength division multiplexing (WDM) network, designers have to take into account expected traffic patterns, class of services proposed for failure resilience, routing algorithms, various functionalities of the components, and many other parameters. Such designs are extremely hard problems to solve, in particular due to unpredictable evolution of the traffic pattern during the lifetime of the network. Also, links are generally oversized to simplify the design of the network. However, with the rapid increase of bandwidth requirements it is more and more difficult to adapt the network usage to traffic variation and so to maintain a near-optimal usage of resources.

In WDM networks without wavelength conversion, the acceptance of a new connection request is subject to the availability of a lightpath (and so resources) from end to end in the network. Whatever the routing algorithm chosen, successive addition and removal of connections may lead to a poor usage of resources. Thus new connection requests might be rejected although the network has enough resources to handle the traffic pattern, up to the rerouting of some requests. For example, in Fig. 2(a), the network is a 9 nodes grid with one wavelength and 5 requests a, b, c, d, e . A new connection request r will be rejected in Fig. 2(a) although the routing of Fig. 2(b) is possible. Therefore, it is necessary to change from time to time the routing of established connections to improve the usage of resources, and so accept more traffic.

In this paper we concentrate on the reconfiguration phase, that is the problem of switching the set of connections from current routing, R_1 , to a new precompute routing, R_2 . To solve this problem, several strategies are possible. For example, one may interrupt all connections requests that have to be rerouted and after restart them with their new routes in R_2 . This strategy is very simple, but requires to store a huge amount of traffic during the reconfiguration phase and so requires large and costly buffers.

Another strategy is to reroute connection requests one after the other, as soon as destination resources (lightpath in the new routing) are available, minimizing the number of interrupted requests during the reconfiguration. It corresponds to compute the *process number* (Sec. 2) in the *dependency digraph* [6]. This requires to compute the scheduling of the rerouting, taking into account that destination resources might be currently used by other connections. More precisely, resources assigned to request r in R_2 might be used by some request r' in R_1 , thus request r' has to be rerouted before r . We represent these constraints by a digraph $D = (V, A)$, the *dependency digraph* [6], in which each node corresponds to a request, and there is an arc from vertex u to vertex v if v must be rerouted before u .

For example to switch from routing R_1 of Fig. 2(a) to routing R_2 of Fig. 2(b), we construct the dependency digraph D of Fig. 2(c). It has one node per connection that has to be rerouted, and so, connection e is not represented in D since it is not affected. It has an arc from a to b since connection b has to be switched before connection a . Similarly, it has arcs from b to c and from b to d since c and d must be switched before b , and so on.

When the digraph D is a DAG (direct acyclic graph), the scheduling is straightforward, but in general, it may contain cycles. To break them, some requests have to be temporarily

interrupted, thus removing some incident arcs in D . So, the optimization problem is to find a scheduling minimizing the number of requests simultaneously interrupted.

This problem has first been considered in [6]. They have proposed an heuristic algorithm based on the minimum feedback vertex set (MFVS) of D (i.e. smallest subset S of nodes of D such that $D - S$ is a DAG) that performs well on small instances. This problem has also been considered in [3, 4] with a modelization in terms of *cops and robber game* [8]: the *process number*, denoted pn . It allows to use more advance theoretical concepts and so to design more efficient heuristic algorithms. For example, the digraph of Fig. 1 is such that $|MFVS(D)| = n/2$ and $pn(D) = 2$. So it is possible to reroute corresponding requests with only two requests simultaneously interrupted.

In this paper we describe the modelisation proposed by Somani in [6] and we propose in Sec. 2 a new point of view of this problem using cops and robber game. Indeed minimizing the number of simultaneously disrupted requests is exactly the same that compute the process number in the dependency digraph. We present in Sec. 3 some previous results linking pathwidth, vertex separation and process number. Furthermore, we recall the heuristic algorithm proposed in [6] We propose and analyse in Sec. 4 a polynomial algorithm to approximate the process number of digraphs (the maximum number of simultaneously disturbed requests) and the corresponding strategy to reconfigure the network, improving the exponential heuristic proposed in [6]. Finally we present simulations (Sec. 5) to prove the efficiency of our heuristic algorithm as well as in term of simultaneously disrupted requests (process number) and in term of complexity.

2 Modelisation

It has been proved in [3, 4] that the routing reconfiguration problem can be expressed as a cops and robber game [8], as for the *pathwidth* [10]. An interruption is represented by placing an *agent* on the corresponding node in D . A node is said *processed* when the corresponding request has been rerouted. If the node was occupied by an agent, then it can be reused. We call a *process strategy* a series of the three following actions (rules) allowing to reroute all requests with respect to the constraints represented by the digraph.

R_1 . Put an agent on a node (*interrupt a connection*).

R_2 . Remove an agent from a node if all its outneighbors are either processed or occupied by an agent (*restart a connection on its final route when destination resources are available*). The node is now processed (*connection has been rerouted*).

R_3 . Process a node if all its outneighbors are either processed or occupied by an agent¹ (*connection has been rerouted because destination resources are available*).

¹In terms of cops and robber games in undirected graphs, rule R_3 expressed that the fugitive is forced to move at each step.

A p -process strategy is a strategy which process the digraph using p agents and the process number, $\text{pn}(D)$, is the smallest p such that a p -process strategy exists.

Clearly, $\text{pn}(D)$ is upper bounded by the minimum feedback vertex set (MFVS) of D , that is smallest subset S of nodes of D such that $D - S$ is a DAG. However, this bound is very large. For example, the digraph represented in Fig. 1 is such that $|\text{MFVS}(D)| = n/2$, but it can be 2-processed. For that, we put an agent on node u (rule R_1). Then we apply rule R_3 to process node u' . After, we put a second agent on node v (rule R_1), process node v' (rule R_3) and then process node v (rule R_2). We repeat on the predecessor of v until processing of the outneighbor w of u . Finally, we apply rule R_2 on u .

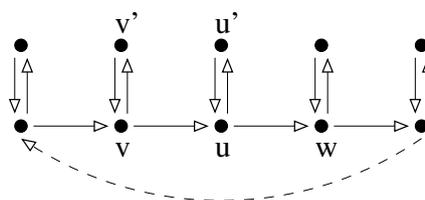


Figure 1: Digraph such that $\text{pn}(D) = 2$ and $\text{MFVS}(D) = n/2$.

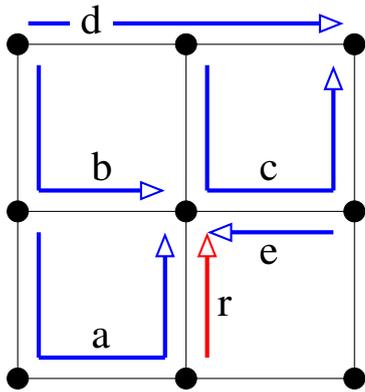
Notice also that when D is a DAG the scheduling is straightforward using rule R_3 from the leaves (nodes with outdegree 0), and we have $\text{pn}(D) = 0$.

3 Previous work

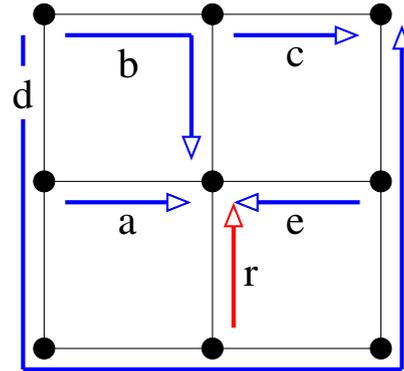
First of all, it has been proved in [4] that $\text{vs}(D) \leq \text{pn}(D) \leq \text{vs}(D) + 1$, where $\text{vs}(D)$ is the *vertex separation* of D . This implies that determining $\text{pn}(D)$ is NP-complete in general, since it is also the case for $\text{vs}(D)$. Furthermore, when D is a symmetric digraph, the same result holds for the underlying graph G . We also have $\text{vs}(G) = \text{pw}(G)$, where $\text{pw}(G)$ is the *pathwidth* of G [7, 10], and determining the pathwidth is NP-complete in general, approximable within $O(\log^2 |V(G)|)$, and there is no polynomial-time algorithm with an absolute error guarantee of $|V(G)|^{1-\varepsilon}$ for any $\varepsilon > 0$ [8, 2]. The same holds for the process number and so for the routing reconfiguration problem considered in [6, 4] and in this paper, thus motivating the development of an efficient heuristic algorithm.

Notice that for specific topologies it is possible to determine the process number in polynomial time. In particular, a characterization of digraphs with process number 1 and 2 is given in [4] as well as $O(n+m)$ and $O(n^2(n+m))$ respectively time complexity recognition algorithms.

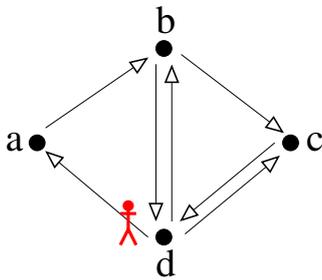
When the digraph is composed of several strongly connected components (SCCs), it is possible to process each SCC independently, and then obtain the processing of D . For that, let $\{\text{SCC}_i\}$ be the set of SCCs of D . Let also D_{SCC} be the DAG of SCCs of D , thus containing one node per component of $\{\text{SCC}_i\}$ and one arc from node i to node j iff D contains an arc



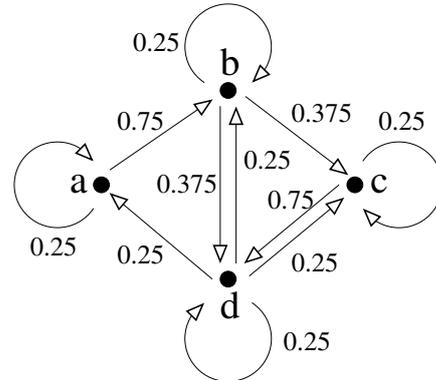
(a) Initial routing R_1 of requests a, b, c, d, e . The new request r can not be satisfied.



(b) The new routing R_2 with requests a, b, c, d, e and r .



(c) Dependency digraph D for switching the routing from R_1 to R_2 .



(d) D_μ from D to apply *flow circulation* algorithm.

Figure 2: Dependency digraph D (c) and corresponding D_μ (d) to switch from routing R_1 (a) to routing R_2 (b).

from some node $u \in \text{SCC}_i$ to some node $v \in \text{SCC}_j$. Then the processing of D consists in processing the SCCs sequentially following the order given by D_{SCC} . Since SCCs are processed independently from each other the Lemma 1 follows. Notice that the computation of $\{\text{SCC}_i\}$ and D_{SCC} takes time $O(n + m)$ using standard algorithms.

Lemma 1. *Given a digraph D and the set $\{\text{SCC}_i\}$ of its strongly connected components, we have $\text{pn}(D) = \max_i \{\text{pn}(\text{SCC}_i)\}$.*

We now recall the principle of the heuristic algorithm proposed in [6], **HeurJS**, to obtain the rerouting ordering of a digraph $D = (V, A)$. It uses Lemma 1, and so we may assume that D is strongly connected. The main idea of **HeurJS** is to break elementary cycles. For that, it first computes the c elementary cycles of D using Johnson's algorithm [5] and assigns to each node u the number $\alpha(u)$ of cycles it belongs to. Then it places an agent on the node u_1 of maximum weight, thus breaking $\alpha(u_1)$ cycles, updates the values of each remaining nodes and repeats if $\alpha(u_i) > 0$, where u_i is the new node of maximum weight. Let L be the set of nodes covered by an agent. When all cycles have been broken, the digraph $D - L$ is a DAG and so can be 0-processed. Thus it processes $D - L$ in appropriate order and then the nodes of L . The number of agents simultaneously used is $|L|$, and the time complexity of **HeurJS** is dominated by the computation of elementary cycles which takes time $O((n + m)(c + 1))$ [6]. Notice that this algorithm is exponential with the number c of elementary cycles, and so with the number n of nodes. Thus, **HeurJS** can only permit to solve the problem for small digraphs, as it has been designed for.

To the best of our understanding, **HeurJS** is in fact an heuristic algorithm for MFVS applied on each SCC of D , following Lemma 1. Although shown to be quite efficient in practice through a large number of experimentations [6], its result could be far from optimality (see the example of Fig. 1).

4 Process strategy based heuristic algorithm

We now propose a heuristic algorithm **HeurCM** for solving the reconfiguration problem, taking advantages of the modelization with the process number proposed in [4]. This heuristic algorithm has polynomial time complexity, and with high probability improves upon **HeurJS** on the number of agents needed simultaneously (corresponding to the number of requests disturbed at the same time) in resulting strategy.

4.1 Flow circulation algorithm

Our algorithm is based on a *flow circulation* algorithm which has the objective of choosing the best candidate node to receive an agent and so to break a large set of cycles.

The principle is the following. Given a dependency digraph $D = (V, A)$, each node $u \in V$ is initially assigned a weight $q_0(u) = 1/|V|$. Then, at each round t , each node u sends $(1 - \mu)q_t(u)/d^+(u)$ to each of its outneighbors, where $d^+(u) = |\Gamma^+(u)|$ with $\Gamma^+(u)$

the set of outneighbors of u in D , and $\mu \in (0, 1)$. Furthermore u keeps $\mu q_t(u)$ in order to satisfy discrete Markov chain conditions. Indeed we transform the digraph $D = (V, A)$ into $D_\mu = (V_\mu, A_\mu)$, such that $V_\mu = V$ and $A_\mu = A \cup \{(u, u), \forall u \in V\}$. In other words, we add loops to each node of D . Clearly, the value of μ will influence the convergence time (i.e. number of rounds) of the flow circulation algorithm without significant impact on the final weights. Thus, we choose it close to 0, and so we will only slightly slow down the convergence time. We have for all $u \in D_\mu$:

$$q_{t+1}(u) = \mu q_t(u) + \sum_{v \in \Gamma^-(u)} (1 - \mu) q_t(v) / d^+(v) \quad (1)$$

where $\Gamma^-(u)$ is the set of predecessors of u .

After k rounds (the value of k will be discussed in Lemma 3), the node with maximum weight is the best candidate. With *flow circulation* algorithm, we expect that the node with maximum weight belongs to many cycles. So its removal from the digraph may break many cycles.

We will now evaluate the number k of rounds needed in the *flow circulation* algorithm. For that, we will first prove in Lemma 2 the convergence of the algorithm. The proof relies on a discrete Markov chain for which we define the state vector $q_t(V_\mu) = (q_t(u_1), q_t(u_2), \dots, q_t(u_{|V_\mu|}))$ corresponding to the weights of the nodes in V_μ and the transition matrix M associated to D_μ . We have for all $x, y \in V_\mu$

$$M[x, y] = \begin{cases} (1 - \mu) / d^+(x) & \text{when } y \in \Gamma^+(x) \\ \mu & \text{when } x = y \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where $\Gamma^+(x)$ is the set of outneighbors of x and $d^+(x) = |\Gamma^+(x)|$.

Lemma 2. *Let D_μ be a strongly connected digraph with loops. Whatever $q_0(V_\mu)$, we have $\lim_{t \rightarrow +\infty} q_t(V_\mu) = \bar{q}(V_\mu)$, where $\bar{q}(V_\mu)$ is the unique stationary weights vector.*

Proof. The proof relies on a discrete time Markov chain in which weights of nodes represent the states of the system (thus the state space is finite) and edges represent the transition probability between two states. In other words, we use the state vector $q_t(V_\mu)$ and the transition matrix M (see Equation 2) associated to D_μ .

From the Markov chains theory, we know that if the chain is irreducible and aperiodic (ergodic), then there exists a unique stationary distribution whatever the initial state $q_0(V_\mu)$ [9].

- The irreducibility of the Markov chain is obvious because D_μ is a strongly connected digraph. Indeed from each state x , there exists a positive probability to move to state y since there exists a path from x to y in D_μ .

- The Markov chain is aperiodic since D_μ is a strongly connected digraph with loops [9].

□

Fig. 2(c) consists on a digraph $D = (V, A)$ of 4 nodes a, b, c, d . We construct $D_\mu = (V_\mu, A_\mu)$ (Fig. 2(d)), adding loops in order to satisfy the Markov conditions described before, even if it is not necessary in this example. The probabilities on the edges in Fig. 2(d) represent the Markov transition matrix (see Equation 2). We have $q_t(V_\mu) = (q_t(a), q_t(b), q_t(c), q_t(d))$ such that $q_0(V_\mu) = (0.25, 0.25, 0.25, 0.25)$. After 4 steps, we have $q_4(V_\mu) \approx (0.1257, 0.2473, 0.2503, 0.3767)$, that is to say almost the stable vector $\bar{q}(V_\mu) = (0.125, 0.25, 0.25, 0.375)$. Thus node d has maximum weight.

The Markov chain admits always a stable state due to the artificial loop transition on each node. We now have to choose k sufficiently large to get the stable vector $\bar{q}(V_\mu)$ with an allowed error ε , that is $\varepsilon_k \leq \varepsilon$, where $\varepsilon_k = \|q_k(V_\mu) - \bar{q}(V_\mu)\|_\infty = O(|\lambda_2|^k)$, denoting λ_2 the second largest eigenvalue of the transition matrix M . Thus we will obtain the node with the maximum weight.

Lemma 3. *If $|\lambda_2| < \varepsilon^{1/k}$, then flow circulation algorithm computes $\bar{q}(V_\mu)$ with an allowed error ε .*

Proof. From Perron-Frobenius Theorem [1], we know that the convergence time from $q_t(V_\mu)$ to the stationary vector $\bar{q}(V_\mu)$ depends only of the second largest eigenvalue λ_2 of M (the first largest is always 1). Thus when $|\lambda_2|^k < \varepsilon$, the stable state is obtained with $\|q_k(V_\mu) - \bar{q}(V_\mu)\|_\infty \leq \varepsilon$. \square

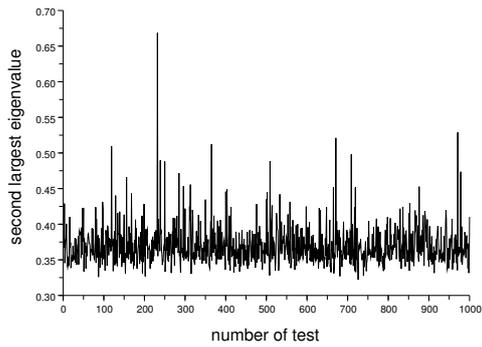
Since computing the second largest eigenvalue value λ_2 of M is time consuming, in our experiments we choose to fix arbitrarily $k = n$. For the example of Fig. 2(d), we thus have $\|q_4(V_\mu) - \bar{q}(V_\mu)\|_\infty \leq 0.003$. Another example is with $n = 500$ requests and an allowed error of $\varepsilon = 0.01$. Then, the flow circulation algorithm computes the stationary vector of the Markov chain if $|\lambda_2| < 0.99$ (remember that the second largest eigenvalue is strictly lower than 1, and that $0.99^{500} < 0.01$). Thus with a very high probability, we get the right stable vector and so a good candidate node for breaking cycles. See Sec. 5.1 for more details.

4.2 Heuristic algorithm

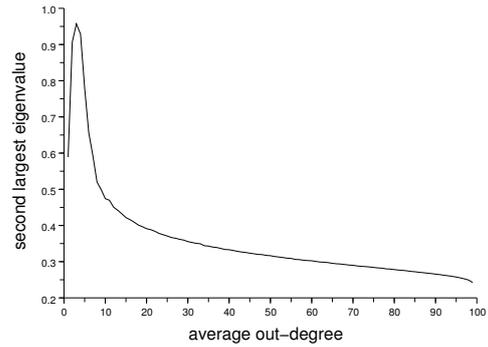
We assume that we are given a strongly connected digraph D . Otherwise, we apply our algorithm on each SCCs, according to Lemma 1. We also assume that D has been transformed into D_μ , but we say D for simplicity.

The heuristic algorithm, **HeurCM**, is described in Algorithm 1. It consists in choosing a node to place an agent using the *flow circulation* algorithm, and then it decomposes the digraph into SCCs and repeats on each SCC $_i$. Algorithm **HeurCM** maintains the set of nodes covered by an agent, \mathcal{S} , sorted by insertion dates. Also $\mathcal{S}.last$ is the latest inserted node that has not yet been processed. After any step of **HeurCM**, \mathcal{S} may change: insertion of a new node covered by an agent (rule R_1), and removal of nodes that can be processed as soon as rule R_2 applies. The number of agents used by **HeurCM** is thus the maximum size reached by \mathcal{S} during the algorithm, $\mathcal{S}.max$.

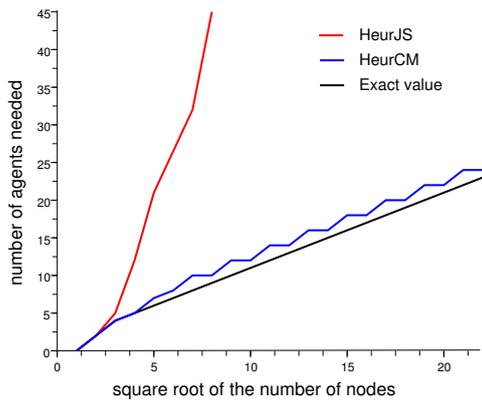
The dependency digraph D of Fig. 2(c) is strongly connected and we apply **HeurCM** on it. We first apply flow circulation algorithm to choose the node to be covered by an agent,



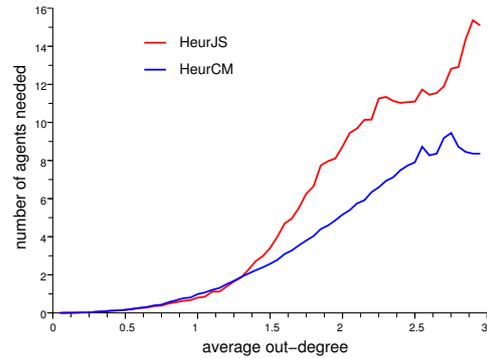
(a) λ_2 for a number of nodes $n = 1000$, 1000 tests, and $\mu = 0.25$.



(b) λ_2 in according to the average out-degree for a number of nodes $n = 100$, 100 tests for each average out-degree, and $\mu = 0.25$.



(c) Number of agents needed to process the grid function of the larger of the grid.



(d) Number of agents needed to process the random graph function of the average out-degree.

Figure 3: Simulations: second largest eigenvalue (a and b) and efficiency of HeurCM (c and d).

Algorithm 1 HeurCM

Require: A strongly connected digraph D , the set of nodes covered by an agent \mathcal{S} and the set of active nodes \mathcal{A}

Ensure: The number of agents needed

- 1: **if** D is a single node u **then**
- 2: Process u : remove it from \mathcal{A} .
- 3: **else**
- 4: Apply *flow circulation* on D .
- 5: Let u be one of the nodes of maximum weight. In case of equality we choose the closest node to $\mathcal{S}.last$.
- 6: Place an agent on u : add it to \mathcal{S} and remove it from \mathcal{A} .
- 7: Process all the nodes whose can be processed updating \mathcal{S} and \mathcal{A} .
- 8: Decompose $D - \{u\}$ into SCCs: $\{SCC_i\}$
- 9: HeurCM(SCC_i, \mathcal{S}), $i = 1.. |\{SCC_i\}|$
- 10: **end if**

i.e. the node with maximum weight. So we choose d and add it to \mathcal{S} (Fig. 2(c)). Since $D - \{d\}$ is a DAG, we can now process all remaining nodes (line 7) and finally process d . Thus HeurCM uses 1 agent in this example.

With the dependency digraph of Fig. 1, and w.l.o.g., node u is chosen among the set of nodes of maximum weights (all belonging to the main cycle) and added to \mathcal{S} . Now, u' is processed and then the digraph is decomposed into SCCs, each being a 2-cycle, on which we apply recursively HeurCM (line 9). Thus, HeurCM will use only 2 agents.

When choosing the node of maximum weight at line 5 of Algorithm 1, it may happen that among the set of candidates, one of them allows to process a node of \mathcal{S} . This is typically the case when the digraph is a bidirectionnal path $u_0, u_1, u_2, u_3, \dots, u_r, u_{r+1}$. W.l.o.g., we may assume that u_1 is chosen at line 5 and so added to \mathcal{S} . We process node u_0 , and we repeat the algorithm on u_2, u_3, \dots, u_{r+1} . Now, candidates will be u_3 and u_r (the symmetry of the graph give same weights for u_3 and u_r), but u_3 is clearly a better choice since it allows to process u_2 and after u_1 thus releasing one agent. It is why in case of equality, we choose the closest node to $\mathcal{S}.last$.

Lemma 4. *The worst case time complexity of HeurCM is $O(kn(n+m))$.*

Proof. Each flow circulation round takes time $O(k(n+m))$ and in the worst case $D - \{u\}$ is a single SCC. \square

Finally, note that the rerouting strategy follows directly our algorithm. With HeurCM, it is sufficient to know entering and leaving dates in \mathcal{S} and in \mathcal{A} to know when a connection is suspended (enter in \mathcal{S}) or switched to its new route (leave \mathcal{S} or \mathcal{A}).

5 Simulations

In this section, we first analyze further the convergence time of the *flow circulation* algorithm. Then, we analyze the performance of our heuristic algorithm **HeurCM**.

5.1 Convergence time of flow circulation

We present here some simulations showing that the second eigenvalue λ_2 of M respects almost surely the conditions given in Lemma 3. For that, we have designed two kinds of random Markov matrices $M_{n \times n}$:

- a fully random matrix, that is to say for each line of M (i.e. for each node), the out-degree and the out-neighbors are chosen randomly and uniformly. Fig. 3(a) shows the distribution of λ_2 for $n = 100$ and 1000 tests. λ_2 is always lower than 0.5.
- a random matrix with constant average out-degree \bar{d} , that is to say for each line of M , \bar{d} is a binomial distribution of parameters \bar{d}/n and n . Then the out-neighbors are chosen randomly and uniformly. Fig. 3(b) shows the value λ_2 for each possible average out-degree. Note that for each average out-degree, 100 tests has been done. λ_2 is very small (lower than 0.5) except for very small average out-degree. But even if that case, the second largest eigenvalue is lower than 0.99.

5.2 Simulations of HeurJS and HeurCM

We have implemented and analysed the efficiency (in term of number of agents simultaneously needed and in term of process time) of **HeurJS** and our algorithm **HeurCM**:

- for a symmetric grid $G_{n \times n}$, Fig. 3(c) shows the approximate process number computed by **HeurJS** and **HeurCM** compared to the exact value $n + 1$ if $n > 2$. **HeurCM** computes an approximate process number closed to the exact value whereas the value computed by **HeurJS** is far when n increases. Furthermore, the computation time is very smaller for **HeurCM** compared to **HeurJS**, because a grid is a strongly connected symmetric digraph and there is an exponential number of cycles.
- for a 50 nodes random graph, starting with an average out-degree \bar{d} closed to 0, increasing it until $\bar{d} = 3$, Fig. 3(d) represents the approximate process number computed by **HeurJS** and by our algorithm **HeurCM** (doing 1000 tests for each $\bar{d} \in (0, 3]$). The average out-degree increasing, we show that **HeurCM** requires less agents than **HeurJS**.

6 Conclusion

We proposed in this paper a polynomial algorithm to approximate the process number of a digraph, corresponding to the number of requests simultaneously interrupted during a network reconfiguration. We used the first rigid modelisation of this problem done by Somani in [6], linking it with graph searching problems like cops and robber games. We improved the efficiency and the complexity of previous heuristic algorithm.

An interesting topic will be to design an exact and efficient exponential algorithm viable in practice for digraphs with a small number of nodes.

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References

- [1] O. Axelsson. *Iterative solution methods*. Cambridge University Press, 1994.
- [2] H.L. Bodlaender, J.R. Gilbert, H. Hafsteinsson, and T. Kloks. Approximating treewidth, pathwidth, frontsize, and shortest elimination tree. *Journal of Algorithms*, 18(2):238–255, March 1995.
- [3] D. Coudert, S. Perennes, Q.-C. Pham, and J.-S. Sereni. Rerouting requests in wdm networks. In *AlgoTel'05*, pages 17–20, Presqu'île de Giens, France, mai 2005.
- [4] D. Coudert and J.-S. Sereni. Characterization of graphs and digraphs with small process number. Research Report 6285, INRIA, September 2007.
- [5] Donald B. Johnson. Finding all the elementary circuits of a directed graph. *SIAM Journal on Computing*, 4(1):77–84, 1975.
- [6] N. Jose and A.K. Somani. Connection rerouting/network reconfiguration. *Design of Reliable Communication Networks (DRCN)*, pages 23–30, October 2003.
- [7] N. G. Kinnarsley. The vertex separation number of a graph equals its pathwidth. *Information Processing Letters*, 42(6):345–350, 1992.
- [8] M. Kirovski and C.H. Papadimitriou. Searching and pebbling. *Theoretical Computer Science*, 47(2):205–218, 1986.
- [9] S.P. Meyn and R.L. Tweedie. *Markov Chains and Stochastic Stability*. Springer-Verlag, London, 1993.
- [10] N. Robertson and P. D. Seymour. Graph minors. I. Excluding a forest. *J. Combin. Theory Ser. B*, 35(1):39–61, 1983.



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