

EXTREME VALUE BASED ADAPTIVE OPERATOR SELECTION

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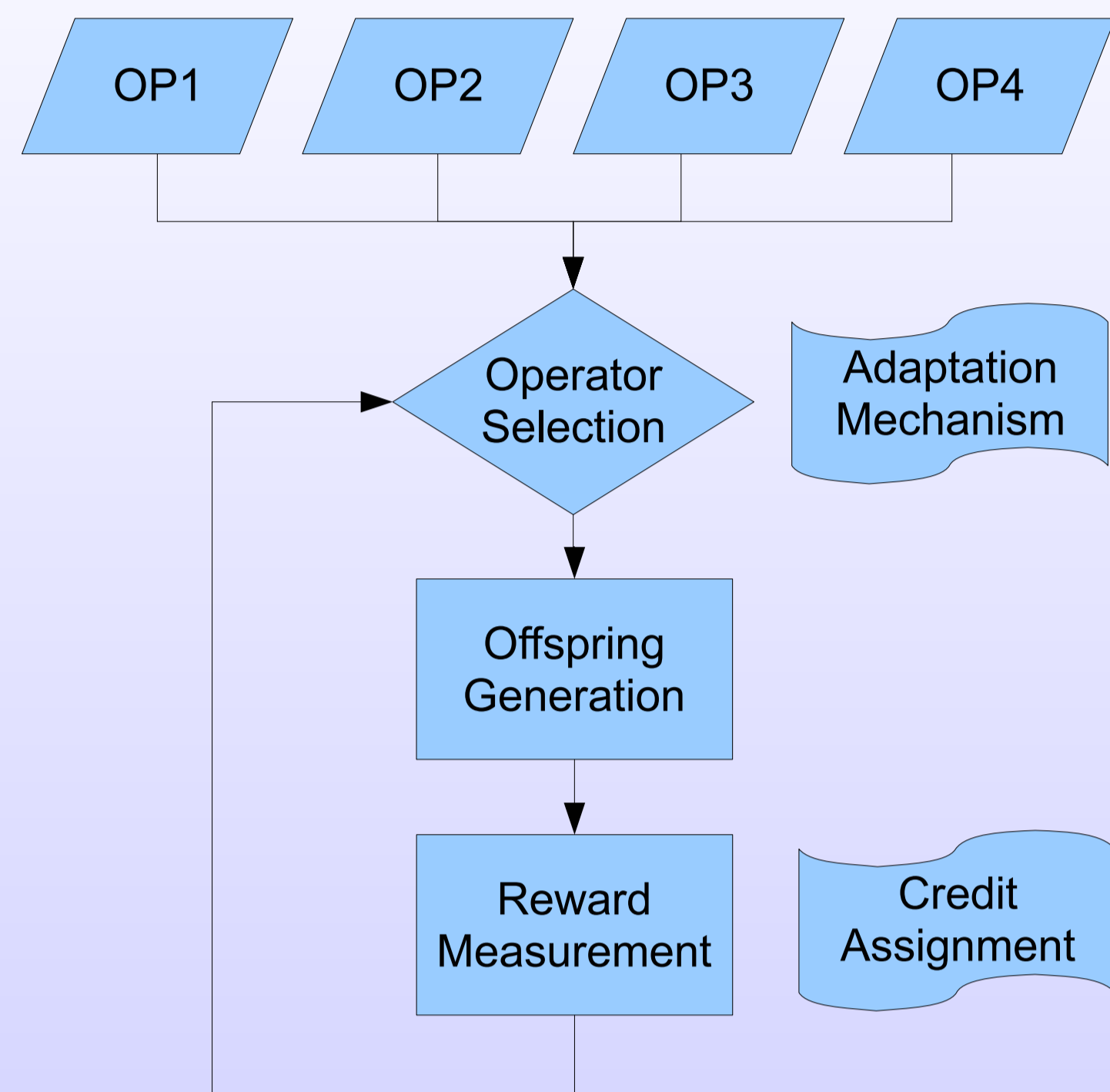
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We address the question of how to estimate the quality of the operators used on an evolutionary algorithm. We argue that operators giving rare but **extreme** improvements are very valuable, and we propose a mechanism to reward them.

Adaptive Operator Selection

Dynamically select “best” operator during evolution



- **Credit assignment:** What reward?
- **Adaptation mechanism:** How to adapt operator selection depending on reward?

Credit Assignment

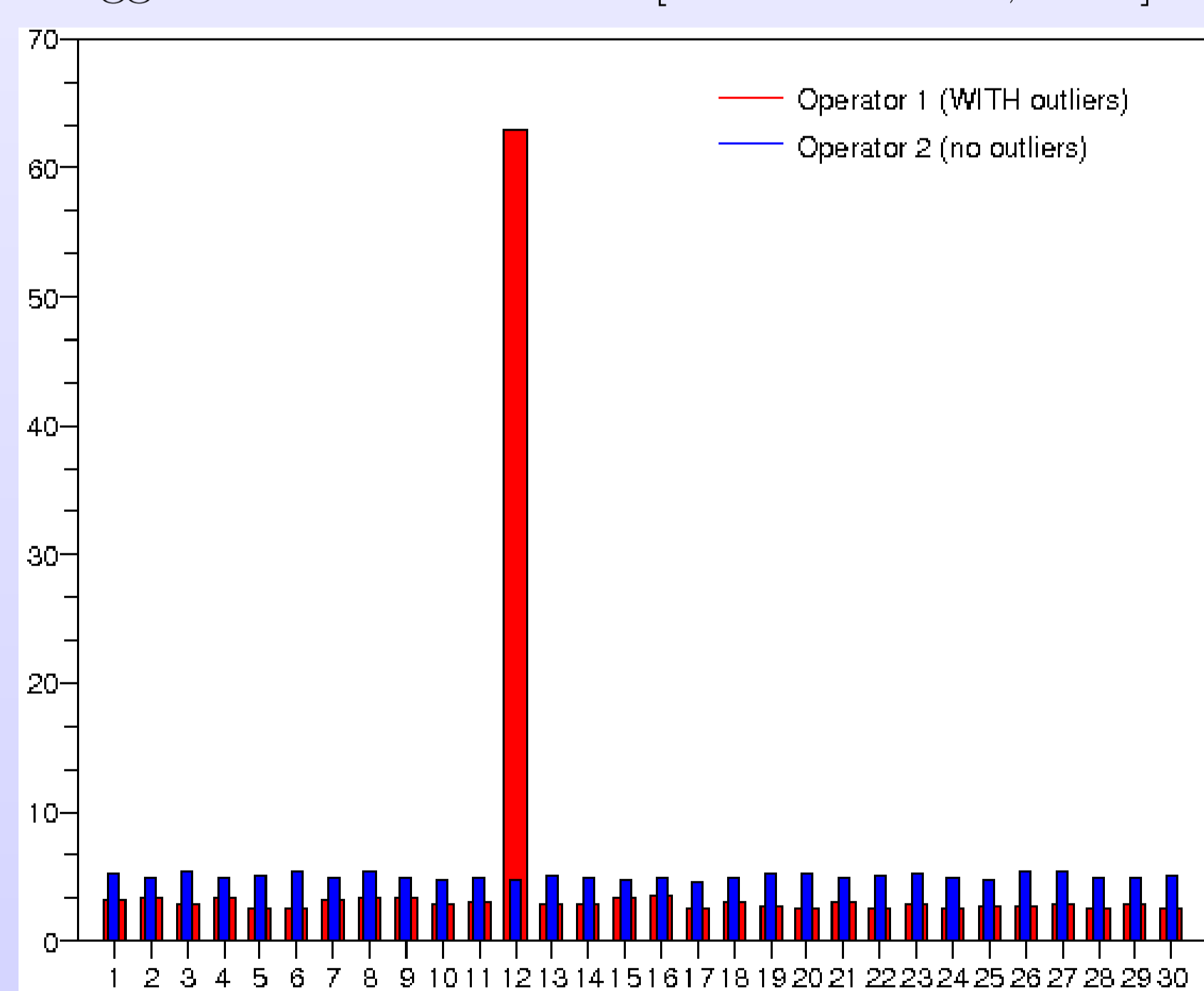
Previous work

- Reward \propto fitness improvement,
- possibly averaged with decay factor,
- eventually distributed among offspring's back lineage
Bucket Brigade-like algorithm

Claim: Outliers matter

Very rare but extremely beneficial events can be more important than averages

- Indications from other disciplines
e.g. finance, rogue waves, ...
- Suggested in AOS context [Whitacre *et al.*, 2006]



Two operators (“red” and “blue”), with same expectation. **Red** is extreme, **blue** is not

Proposed here: Extreme Reward

Best fitness improvement over a sliding window

Typically 50

Adaptation Mechanisms

Previous Work: Probability Matching and Adaptive Pursuit

see e.g. [Thierens, 2007]

Select an operator based on probabilities s_i ($\sum s_i = 1$)

- $\hat{Q}_{i,t}$ = estimated quality of operator i at time t
- Selecting operator $i \rightarrow$ reward $r_{i,t}$
- Estimated quality updated by relaxation

$$\hat{Q}_{i,t+1} = (1 - \alpha)\hat{Q}_{i,t} + \alpha r_{i,t}$$

- **Probability Matching:** s_i proportional to Q_i
- **Adaptive Pursuit:** increase s_i for operator with best estimated quality, decrease all other s_j 's evenly.
- Greediness: $s_i \geq P_{min}$ User-defined parameter

Dynamic Multi-Armed Bandits

proposed at GECCO 2008

- **Multi-Armed Bandit setting**



Reward for choosing arm j is

$$r_j = \begin{cases} 1 & \text{with prob. } = p_j \\ 0 & \text{with prob. } = 1 - p_j \end{cases}$$

- **Goal** Maximize cumulated reward
- **UCB1 algorithm** [Auer *et al.*, 2002]
Be optimistic when facing the unknown:
At time t , choose arm j maximizing

$$\hat{r}_{j,t} + \sqrt{\frac{2 \log \sum_k n_{k,t}}{n_{j,t}}}$$

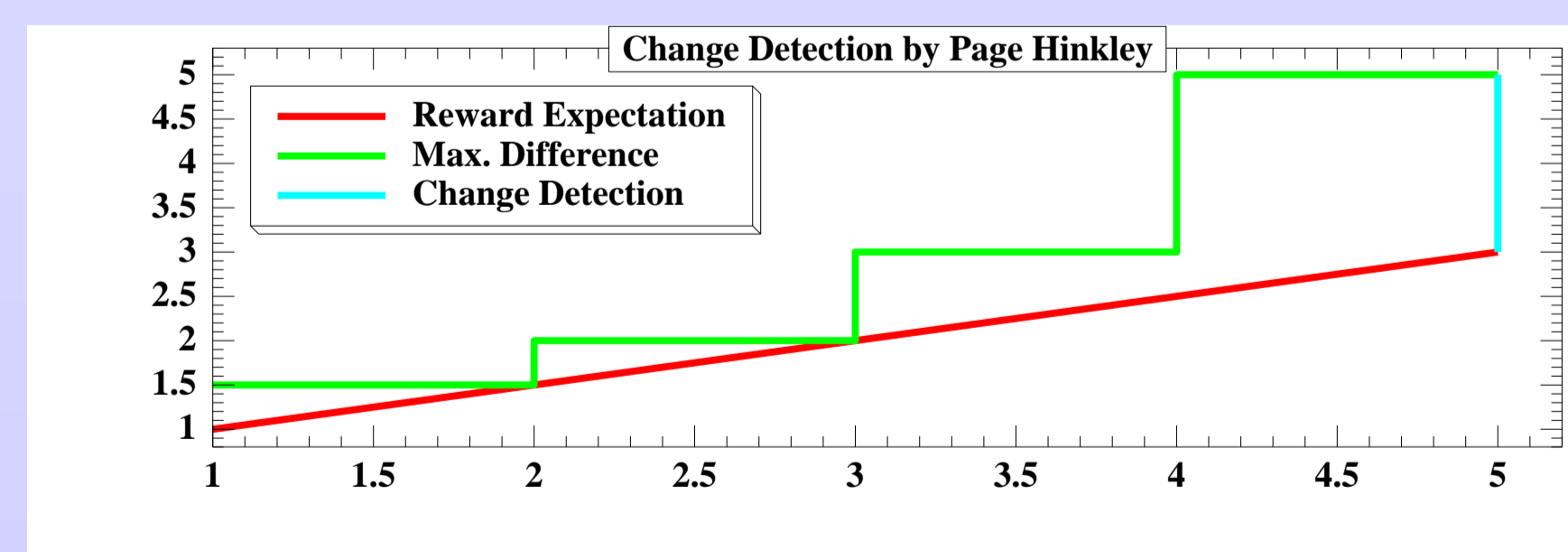
$\hat{r}_{j,t}$: empirical reward for arm j , favors exploitation
 $n_{j,t}$: # times arm j was chosen, favors exploration

- **Scaling** If $r_{j,t} \in [a, b] \neq [0, 1]$: balance between exploitation and exploration requires scaling of $\hat{r}_{j,t}$

- **Dynamic rewards**

- Operators' rewards likely to change during a run
- UCB1 algorithm reacts slowly
- should be restarted in case of change in rewards

- **Page-Hinkley statistics** [Page, 1954]
Detect changes in time-series distributions



- Monitor cumulated deviation of rewards m_t
- Trigger restart when $|\text{Max}_{i \leq t}(m_i) - m_t| > \lambda$
 λ user-defined parameter

Dynamic Multi-Armed Bandits (D-MAB):

UCB1 + Scaling + Page Hinkley

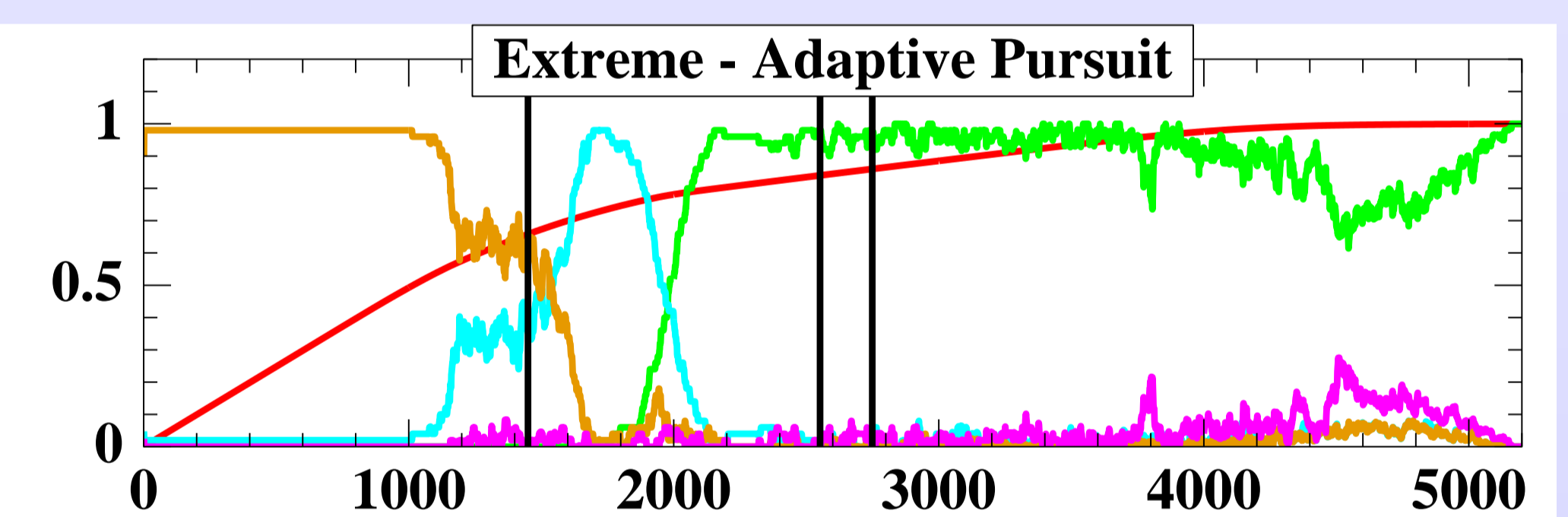
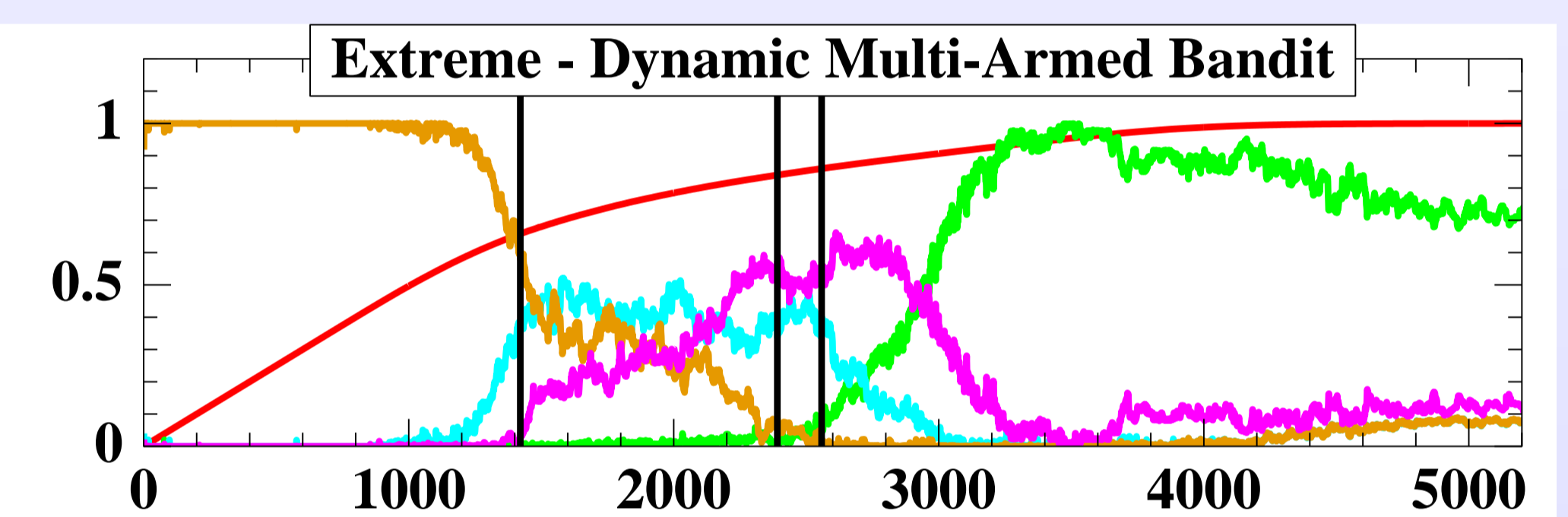
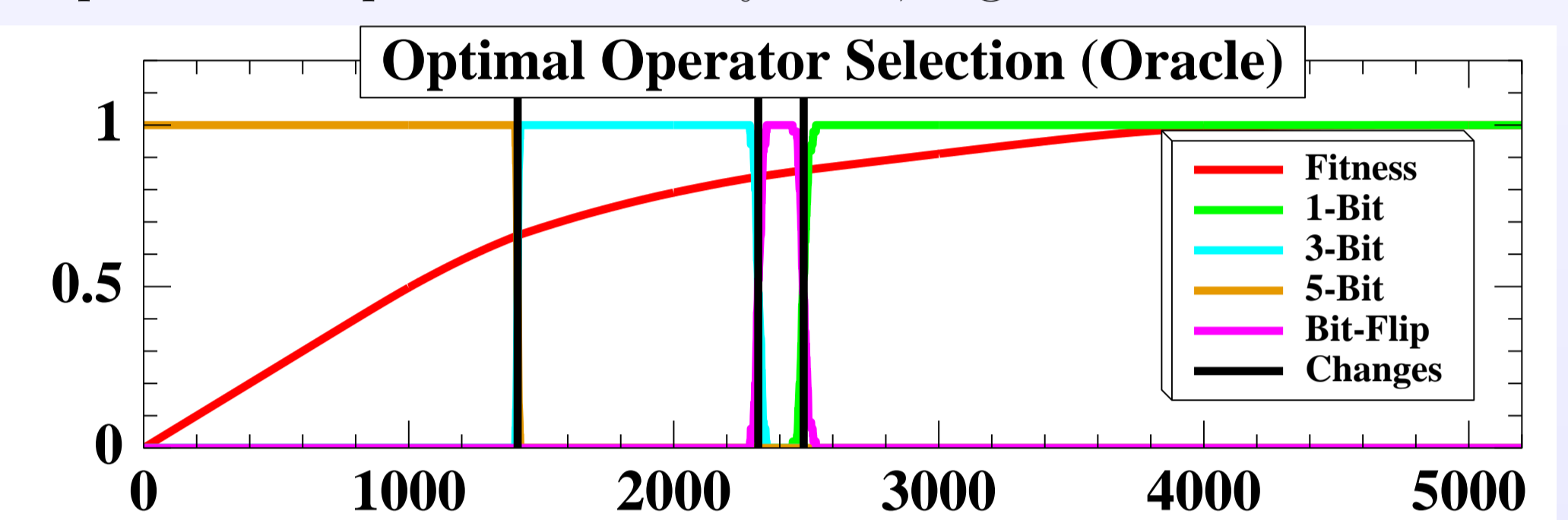
Winner Pascal EvE Challenge [Hartland *et al.*, 2007]

Experiments

- **10000-bits OneMax** the drosophila of EC!
- **4 mutation operators**, (1+50)-EA
 - k -bit mutation ($k=1,3,5$) flips exactly k uniformly chosen bits
 - standard **bit-flip** mutation flips all bits with probability $\frac{1}{n}$
- Which is the best operator only depends on the fitness value
5-bit when low, 1-bit close to optimum, ...

Typical Behaviors:

Proportion of operators actually used, together with **fitness**



Extreme vs Average?

Dynamic-MAB	
Credit Assignment	Generations to Optimum
Extreme	5467 ± 142
Average	7726 ± 178

Adaptive Pursuit	
Credit Assignment	Generations to Optimum
Extreme	5478 ± 82
Average	5829 ± 89

Baseline	
Non-Adaptive Strategy	Generations to Optimum
Optimal according to Oracle	5068 ± 80
Best Naive: $\mathcal{U}(1\text{-Bit}+5\text{-Bit})$	6792 ± 173

Open Issues

- How much exploration are we really doing?
- Can those results be generalized? need a sound test suite
- Make some meta-parameters (self-)adaptive? e.g. λ in PH test
- What about a rank-based reward? →scale-invariance