

# Exploring a type-theoretic approach to accessibility constraint modelling

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**Accessibility constraints** When dealing with anaphora resolution, anaphoric expressions are often considered to have access to a restricted set of discourse referents. The choice of the relevant discourse referent inside this set may depend on various kinds of information (morphosyntactic features, semantic features, salience...). But theories that give a dynamic interpretation of language also provide some ways to restrict the set of discourse referents by stating accessibility constraints.

The description of these constraints depends on the chosen formal theory of discourse interpretation. For instance, DRT [5] describes the accessibility constraints using DRS subordination. Taking into account the hierarchical structure of discourse, SDRT [2] uses DRS subordination together with an outscoping relation between discourse units. In both cases, the constraints are based on the *structure* of the discourse: for DRT, the structure arises from the negation while for SDRT it also arises from the nature of the relation between discourse units. In the latter case, it gives a formal definition of the Right Frontier Constraint (RFC) [1].

[3] proposes a modelling of DRT that fits Montague's semantic framework. In this approach, the logical quantifiers have their standard scoping definitions and free and bound variables also have their standard definitions. One of the advantages is to use standard notions (such as De Bruijn's indices for  $\lambda$ -calculus) for implementing variable renaming in semantic systems based on  $\lambda$ -calculus, instead of implementing specific and complex work-around. [3] also advocates its independence from any specific theory. While [3] exemplifies the DRT view on accessibility constraints, the work we propose here consists in exploring this approach in modelling two other kinds of accessibility constraints: the accessibility to proper nouns in DRT and the RFC.

**A Montagovian flavour of DRT** In [3], the syntactic type of sentences  $s$  is interpreted with the semantic type:  $\llbracket s \rrbracket = \gamma \rightarrow (\gamma \rightarrow t) \rightarrow t$ . It means that (the semantic interpretation of) a sentence, instead of being a proposition, requires two arguments to produce a proposition. The first argument is its left context (we can think about it as the current set of discourse referents). The second argument is its right context, or its *continuation*: something able to return a proposition if fed with a (potentially updated) set of discourse referents.

Two sentences  $s_1$  and  $s_2$  are combined in the following way:  $\llbracket s_1.s_2 \rrbracket = \lambda e\phi.\llbracket s_1 \rrbracket e (\lambda e'.\llbracket s_2 \rrbracket e' \phi)$ . It means that the result of the combination of  $s_1$  and  $s_2$  takes as input an environment  $e$  and a continuation  $\phi$ . This environment is the same as the one  $s_1$  has access to. So  $\llbracket s_1 \rrbracket$  takes  $e$  as first parameter. Then, the continuation of  $s_1$  is made of something taking  $e'$  as left environment<sup>1</sup> and the final result is the one we get by feeding  $s_2$  with the new left environment  $e'$  and the same continuation as the  $s_1.s_2$  combination.

With this interpretation and the right semantic recipes (see [3] for the detailed lexical semantics) we get the following interpretation for *John loves a woman*:  $\lambda e\phi.\exists y.\mathbf{woman} \mathbf{y} \wedge \mathbf{love} \mathbf{j} y \wedge \phi(y :: e)$  where  $::$  (of type  $e \rightarrow \gamma \rightarrow \gamma$ ) is a function that adds a discourse referent to a set of discourse referents. Note

<sup>1</sup>Note that the actual value of  $e'$  will be provided by the semantic recipe of  $s_1$ . That is  $s_1$  can choose to update or not  $e$  before it gives it as parameter to its continuation.

that the continuation of this sentence,  $\phi$ , will be provided with the  $y$  discourse referent, bound in the standard way by the existential quantifier.

**Modelling other accessibility constraints** To state as in DRT that negation blocks the access to discourse referents, [4] interprets:  $\llbracket(\text{doesn't VP})S\rrbracket = \lambda e\phi.\neg((VP S) e (\lambda e'.\top)) \wedge \phi e$ . The continuation of the sentence ( $\phi$ ) has no access to what  $VP$  and  $S$  could introduce as discourse referents, it only has access to the discourse referents of  $e^2$ .

Note that an alternative could have been:  $\llbracket(\text{doesn't VP})S\rrbracket = \lambda e\phi.\neg((VP S) e (\lambda e'.\phi e'))$ . But in this case, the continuation would also have been in the scope of the negation: the rest of the discourse would also be negated, which is not what we expect.

We propose to introduce the type of logical connectives  $\kappa = t \rightarrow t \rightarrow t$  and to interpret the syntactic type of sentences as  $\llbracket s \rrbracket = \kappa \rightarrow \gamma \rightarrow (\kappa \rightarrow \gamma \rightarrow t) \rightarrow t$ . Then negation can be interpreted as:  $\llbracket(\text{doesn't VP}) S\rrbracket = \lambda c e \phi.\neg((VP S) (\neg c) e (\lambda c' e'.\neg(\phi c' e)))$ . Without all the details, if we think as  $c$  being the logical connective  $\wedge$ , we have the result to be interpreted as  $\neg((VP S) \vee \neg(\phi e)) \equiv (\neg(VP S)) \wedge (\phi e)$  which is now the expected result. Interestingly, we see that it could also give access to the discourse referents introduced by  $VP S$  to  $\phi$  using the continuation  $\lambda c' e'.\neg(\phi c' e')$ . This, of course, is not expected, except for proper nouns (which are “always on top” in DRT). We then see how to add an additional left environment parameter for proper nouns (say  $e_1$ ) so that negation blocks the discourse referents introduced by existentials (in  $e_2$ ) but propagates the ones introduced by proper nouns. With the new value of  $\llbracket s \rrbracket = \kappa \rightarrow \gamma \rightarrow \gamma \rightarrow (\kappa \rightarrow \gamma \rightarrow \gamma \rightarrow t) \rightarrow t$  and of sentence composition:  $\llbracket s_1.s_2 \rrbracket = \lambda c e_1 e_2 \phi.\llbracket s_1 \rrbracket c e_1 e_2 (\lambda c' e'_1 e'_2.\llbracket s_2 \rrbracket c' e'_1 e'_2 \phi)$ <sup>3</sup>, the negation is now interpreted as:  $\llbracket \text{doesn't} \rrbracket = \lambda V S c e_1 e_2 \phi.\neg((V S) (\neg c) e_1 e_2 (\lambda c' e'_1 e'_2.\neg(\phi c' e'_1 e_2)))$ . Together with the following interpretations:

$$\begin{aligned} \llbracket \text{John} \rrbracket &= \lambda P c e_1 e_2 \phi.P \mathbf{j} c (\mathbf{j} :: e_1) e_2 \phi \\ \llbracket \text{own} \rrbracket &= \lambda O S.S(\lambda x.O(\lambda y c' e'_1 e'_2 \phi'.c'(\mathbf{own} x y)(\phi' c' e'_1 e'_2))) \\ \llbracket \text{car} \rrbracket &= \lambda x c e_1 e_2 \phi.c(\mathbf{car} x)(\phi c e_1 e_2) \\ \llbracket a \rrbracket &= \lambda P Q c e_1 e_2 \phi.\exists x.[\lambda \phi'.(P x c e_1 e_2 \phi') \wedge (Q x c e_1 e_2 \phi')](\lambda c' e'_1 e'_2.\phi c e'_1 (x :: e'_2)) \\ \llbracket it \rrbracket &= \lambda P.P(\mathbf{sel}(e_1 \cup e_2)) \\ \llbracket is \rrbracket &= \lambda A S.S(\lambda x c' e'_1 e'_2 \phi'.c'(A(\lambda y c'' e''_1 e''_2 \phi''.\top) x c' e'_1 e'_2 \phi'))(\phi' c' e'_1 e'_2)) \\ \llbracket red \rrbracket &= \lambda P x c e_1 e_2 \phi.(P x c e_1 e_2 \phi) \wedge (\mathbf{red} x) \end{aligned}$$

we can interpret:

$$\begin{aligned} \llbracket \text{own} \rrbracket(\llbracket a \rrbracket \llbracket \text{car} \rrbracket) &= \lambda S.S(\lambda x c e_1 e_2 \phi.\exists y.[\lambda \phi'.(c(\mathbf{car} y)(\phi' c e_1 e_2)) \wedge (c(\mathbf{own} x y)(\phi' c e_1 e_2))] \\ &\quad (\lambda c' e'_1 e'_2.\phi c e'_1 (y :: e'_2))) \\ &\equiv \lambda S.S(\lambda x c e_1 e_2 \phi.\exists y.[\lambda \phi'.c((\mathbf{car} y) \wedge (\mathbf{own} x y))(\phi' c e_1 e_2)] \\ &\quad (\lambda c' e'_1 e'_2.\phi c e'_1 (y :: e'_2))) \\ &= \lambda S.S(\lambda x c e_1 e_2 \phi.\exists y.c((\mathbf{car} y) \wedge (\mathbf{own} x y))(\phi c e_1 (y :: e_2))) \end{aligned}$$

thanks to the following equivalence ( $c$  is either  $\wedge$  or  $\vee$ ):

$$(c(\mathbf{car} y)(\phi' c e_1 e_2)) \wedge (c(\mathbf{own} x y)(\phi' c e_1 e_2)) \equiv c(\mathbf{car} y \wedge \mathbf{own} x y)(\phi' c e_1 e_2)$$

Finally, a discourse  $d$  such as *John doesn't own a car. It is red* with the empty context  $\mathbf{nil}$  and the empty continuation  $\phi_e = \lambda c e_1 e_2.\neg(c \top \perp)$  (which returns  $\top$  under the  $\wedge$  connective and  $\perp$  under the  $\vee$  connective) is interpreted as:

$$\begin{aligned} \llbracket d \rrbracket (\wedge) \mathbf{nil} \mathbf{nil} \phi_e &= \neg(\exists y.(\mathbf{car} y \wedge \mathbf{own} \mathbf{j} y) \vee (\neg(\mathbf{red}(\mathbf{sel}((\mathbf{j} :: \mathbf{nil}) \cup \mathbf{nil})))) \vee \perp)) \\ &= \neg(\exists y.(\mathbf{car} y \wedge \mathbf{own} \mathbf{j} y) \vee (\neg(\mathbf{red}(\mathbf{sel}(\mathbf{j} :: \mathbf{nil})))))) \\ &\equiv (\neg \exists y.(\mathbf{car} y \wedge \mathbf{own} \mathbf{j} y)) \wedge \mathbf{red}(\mathbf{sel}(\mathbf{j} :: \mathbf{nil})) \end{aligned}$$

<sup>2</sup> $\top$  stands for true. That is,  $\lambda e.\top$  is the continuation that always returns true.

<sup>3</sup>We use here a curried notation that introduces as many arrows as functions have parameters. We could of course use a product or record type to keep only one parameter.

It shows that whereas the discourse referent introduced by  $y$  is not accessible to the  $\text{sel}$  operator, the discourse referent introduced by  $\mathbf{j}$  would be accessible, in spite of the negation.

We extend this approach to model the RFC, introducing a parameter for the type of discourse relations: subordinating or coordinating, and two ways of combining sentences:  $s_1.\mathbf{c}s_2$  and  $s_1.\mathbf{s}s_2$  instead of just one  $s_1.s_2$ . The interpretation of these two combinations manage the set of discourse referents so that the environment given to  $s_2$  only includes the set of discourse referents defined by the RFC. As for now, we only deal with discourse structures that can be described by a formula (or syntactic tree) made of the  $\mathbf{.c}$  and of the  $\mathbf{.s}$  connectives. Discourse pops or discourse structure that are not trees, for instance, are not yet considered.

We consider a new type  $\kappa = \gamma \rightarrow \gamma \rightarrow \gamma$  and  $\llbracket s \rrbracket = \kappa \rightarrow \gamma \rightarrow \gamma \rightarrow (\kappa \rightarrow \gamma \rightarrow \gamma \rightarrow t) \rightarrow t$ , and we define **Coord** =  $\lambda e_1 e_2 . e_2$  and **Sub** =  $\lambda e_1 e_2 . e_1 \cup e_2$ . We give the following definition of sentence composition:

$$\begin{aligned} \llbracket s_1.\mathbf{s}s_2 \rrbracket &= \lambda c e_1 e_2 \phi . \llbracket s_1 \rrbracket c e_1 e_2 (\lambda c' e'_1 e'_2 . \llbracket s_2 \rrbracket \mathbf{Sub} e'_1 (c e_1 e_2) \phi) \\ \llbracket s_1.\mathbf{c}s_2 \rrbracket &= \lambda c e_1 e_2 \phi . \llbracket s_1 \rrbracket c e_1 e_2 (\lambda c' e'_1 e'_2 . \llbracket s_2 \rrbracket \mathbf{Coord} e'_1 (c e_1 e_2) \phi) \end{aligned}$$

Then

$$\begin{aligned} \llbracket s_1.\mathbf{c}(s_2.\mathbf{c}s_3) \rrbracket &= \lambda c e_1 e_2 \phi . \llbracket s_1 \rrbracket c e_1 e_2 (\lambda c' e'_1 e'_2 . \llbracket s_2 \rrbracket \mathbf{Coord} e'_1 (c e_1 e_2) \\ &\quad (\lambda c''' e'''_1 e'''_2 . \llbracket s_3 \rrbracket \mathbf{Coord} e'''_1 (c e_1 e_2) \phi)) \\ \llbracket s_1.\mathbf{c}(s_2.\mathbf{s}s_3) \rrbracket &= \lambda c e_1 e_2 \phi . \llbracket s_1 \rrbracket c e_1 e_2 (\lambda c' e'_1 e'_2 . \llbracket s_2 \rrbracket \mathbf{Coord} e'_1 (c e_1 e_2) \\ &\quad (\lambda c''' e'''_1 e'''_2 . \llbracket s_3 \rrbracket \mathbf{Sub} e'''_1 (c e_1 e_2) \phi)) \\ \llbracket s_1.\mathbf{s}(s_2.\mathbf{c}s_3) \rrbracket &= \lambda c e_1 e_2 \phi . \llbracket s_1 \rrbracket c e_1 e_2 (\lambda c' e'_1 e'_2 . \llbracket s_2 \rrbracket \mathbf{Sub} e'_1 (c e_1 e_2) \\ &\quad (\lambda c''' e'''_1 e'''_2 . \llbracket s_3 \rrbracket \mathbf{Coord} e'''_1 (e'_1 \cup (c e_1 e_2)) \phi)) \\ \llbracket s_1.\mathbf{s}(s_2.\mathbf{s}s_3) \rrbracket &= \lambda c e_1 e_2 \phi . \llbracket s_1 \rrbracket c e_1 e_2 (\lambda c' e'_1 e'_2 . \llbracket s_2 \rrbracket \mathbf{Sub} e'_1 (c e_1 e_2) \\ &\quad (\lambda c''' e'''_1 e'''_2 . \llbracket s_3 \rrbracket \mathbf{Sub} e'''_1 (e'_1 \cup (c e_1 e_2)) \phi)) \end{aligned}$$

which means that in addition to the environment introduced by their previous discourse unit ( $e'_1$  for  $s_2$  and  $e''_1$  for  $s_3$ ),  $s_2$  and  $s_3$  have access to  $(c e_1 e_2)$  whose value is: either  $e_2$  if the whole part of the discourse is in a coordinating relation with what comes before (and then only  $s_1$  should access  $e_1$ ), or  $e_1 \cup e_2$  if the whole part is in a subordinating relation with what comes before (and the previous unit dominates  $s_1$ ,  $s_2$  and  $s_3$  and each have access to  $e_1$  and  $e_2$ ).

**Conclusion** We show how [3]'s approach is flexible enough to model various accessibility constraints, such as the ones for discourse referents introduced by proper nouns or by the hierarchical structure of the discourse. Moreover, it proves to be able to combine the different constraints. We hope this could be helpful to give an account of the hierarchy of referential expressions and their adequacy to the RFC [1] by combining various constraints for pronouns or definite descriptions for instance.

## References

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