Efficient Inclusion Checking for Deterministic Tree Automata and DTDs

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LATA 2008, Tarragona, España
March 14, 2008

Tree Automata Inclusion

- Tree automata A and B
- Instance: $L(A) \subseteq L(B)$?
- Non deterministic B: DEXPTIME-complete [Seidl 1994]
- Deterministic B
  - B is complete: $O(|A| * |B|)$
  - Not complete: ?

Deterministic Tree Automata Inclusion

Usual algorithm

- Ranked automata, arity n
- Decide $L(A) \cap L(B)^c = \emptyset$

Complexity

- Running time: $O(|A| * |\Sigma| * |B|^n)$
- Involves computation of B complement
- Linear reduction to binary automata: $O(|A| * |\Sigma| * |B|^2)$
- Our goal: $O(|A| * |B|)$, independently of $|\Sigma|$

Inclusion into DTDs

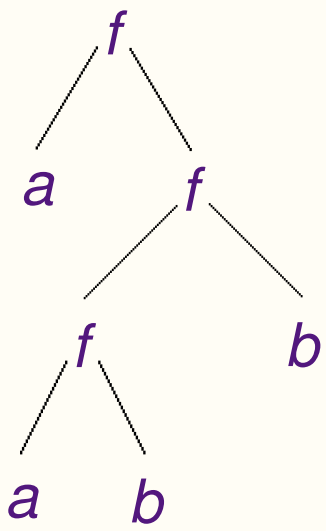
- $L(A)$: set of XML documents
  - Automata for unranked trees [TATA 2007]
  - Possibly non deterministic
- Representations for DTDs [Martens & Niehren 2007]
  - Hedge automata
  - Stepwise automata
- Algorithm for testing $L(A) \subseteq L(DTD)$

Outline

Tree Automata

Basic notions

- Binary tree t:



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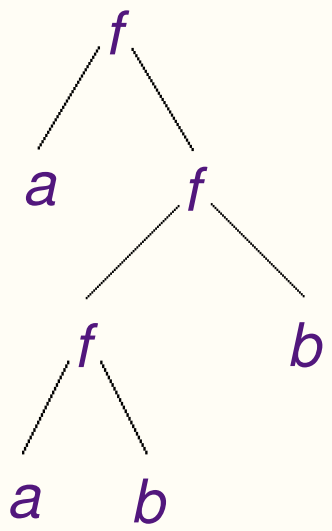

- Automaton A:
  - States: $\{1, 2\}$
  - Final states: $\{2\}$
  - Rules:$\{$
    $a \to 1,$
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    $f(1, 2) \to 1,$
    $f(1, 2) \to 2 \quad \}$

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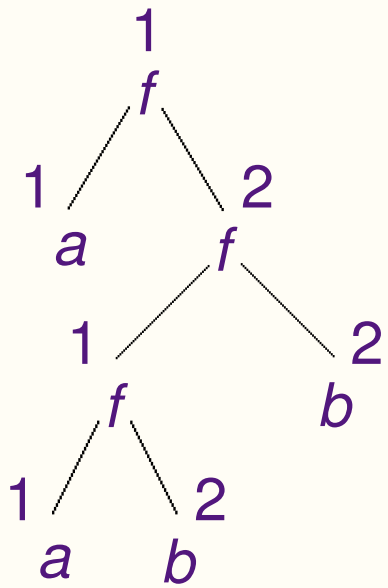

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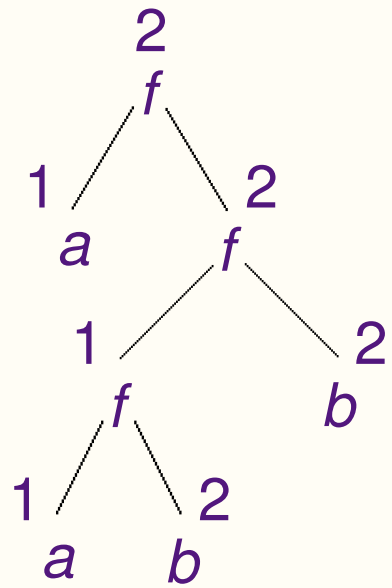

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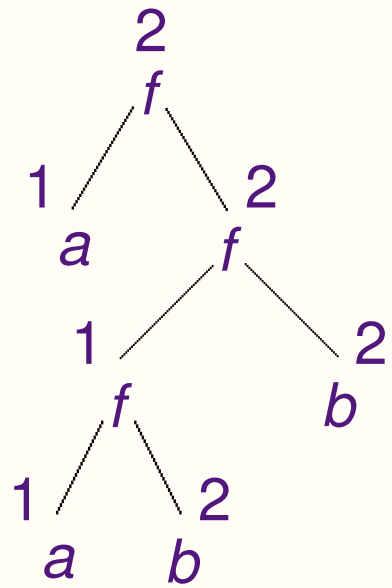

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$$L(A) = \{t | \text{eval}_A(t) \cap \text{final}(A) \neq \emptyset\}$$

Emptiness/Accessibility

Emptiness

Input: A, Σ
Output: $L(A) = \emptyset$
Algorithm: does A have accessible final states?

Accessibility

$\mathrm{acc}(p)$ in A if $\exists t$ such that $p \in \mathrm{eval}_A(t)$.

Efficiency of Ground Datalog

Inferring a ground Datalog program for accessibility

$$\frac{a \to p \in \text{rules}(A)}{\text{acc}(p).} \qquad \frac{f(p_1, p_2) \to p \in \text{rules}(A)}{\text{acc}(p) :- \text{acc}(p_1), \text{acc}(p_2).}$$

Number of clauses: $O(|A|)$

Theorem (Dantsin et al. 2001)

The least fixed point $\text{lfp}(P)$ *of a ground Datalog program* P *can be computed in time* $O(|P|)$*, even for unbounded predicate arities.*

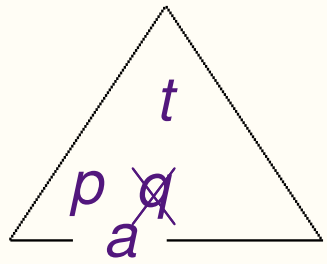
Guiding Ideas of Characterization

- Detection of inclusion failure $L(A) \not\subseteq L(B)$
- Find $t \in L(A)$ such that $t \notin L(B)$
- Accessibility on product $A \times B$
- Ground Datalog program $D_{A,B}$
- Conditions: B deterministic, A productive

Inclusion Failure

Constant rules

$$\frac{a \longrightarrow p \text{ in } A \qquad \nexists q.\ a \longrightarrow q \text{ in } B}{\texttt{fail.}}$$



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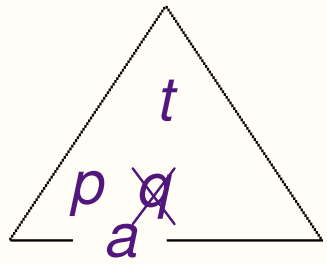
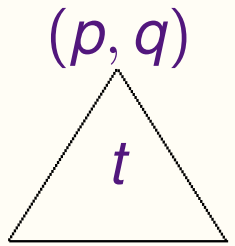


Final states

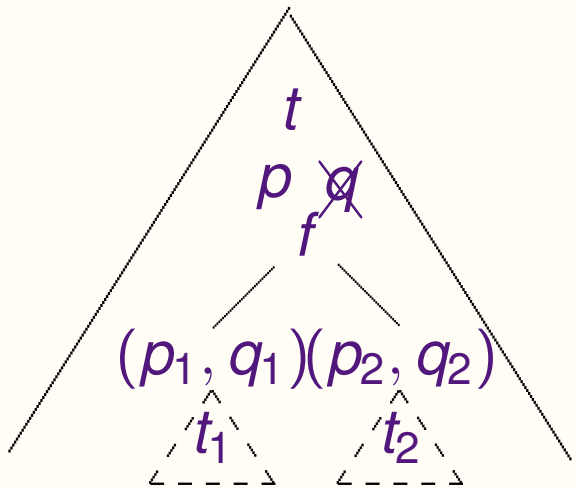
$$\frac{p \text{ final in } A \quad q \text{ not final in } B}{\texttt{fail} :\!-\ \text{acc}(p, q).}$$



Inclusion Failure (cont.)

Binary rules

$$\frac{f(p_1, p_2) \longrightarrow p \text{ in } A \qquad \nexists q.\ f(q_1, q_2) \longrightarrow q \text{ in } B}{\texttt{fail} :\!- \ \mathrm{acc}(p_1, q_1), \mathrm{acc}(p_2, q_2).}$$



Datalog Characterization

Proposition

For deterministic B and productive A, $L(A) \not\subseteq L(B)$ if and only if `fail` $\in \mathsf{lfp}(D_{A,B})$.

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Computing failures

- For constant rules and final states: $O(|A| * |B|)$
- For binary rules: $O(|A| * |B|^2)$

$$\frac{\begin{array}{rl} & f(p_1, p_2) \to p \text{ in } A \\ \nexists q. & f(q_1, q_2) \to q \text{ in } B \end{array}}{\texttt{fail} :- \mathrm{acc}(p_1, q_1), \mathrm{acc}(p_2, q_2).}$$

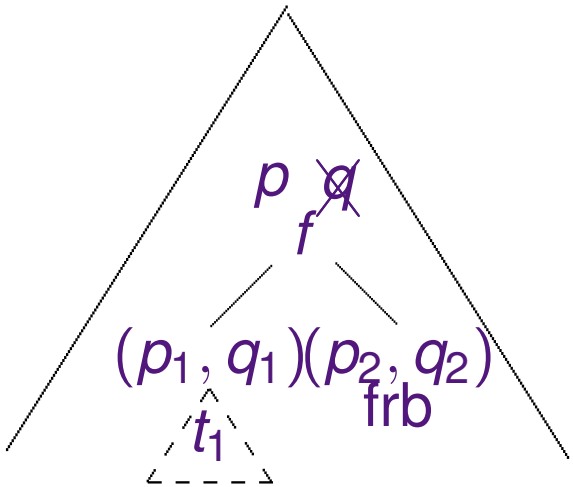
Forbidden States

$$\boxed{\mathrm{frb}(p,q) \Leftrightarrow (\mathrm{acc}(p,q) \Rightarrow L(A) \not\subseteq L(B))}$$

- Inferred Datalog clauses:

$$\frac{f(p_1,p_2) \to p \in \mathrm{rules}(A) \quad \nexists q.\ f(q_1,q_2) \to q \in \mathrm{rules}(B)}{\begin{array}{l}\mathrm{frb}(p_2,q_2) :- \mathrm{acc}(p_1,q_1).\\ \mathrm{frb}(p_1,q_1) :- \mathrm{acc}(p_2,q_2).\end{array}}$$

$$\frac{p \in \mathrm{states}(A) \quad q \in \mathrm{states}(B)}{\texttt{fail} :- \mathrm{acc}(p,q), \mathrm{frb}(p,q).}$$



- Number of clauses: $O(|A| * |B|^2)$

Grouping Forbidden States

For $Q \subseteq \text{states}(B)$:

$$\boxed{\text{frb}^c(p, Q) \Leftrightarrow \forall q \notin Q,\ \text{frb}(p, q)}$$

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- Size of the inferred Datalog program: $O(|A| * |B|)$

Testing Failure

Failure for binary rules

$$\frac{p \in \text{states}(A) \quad q \in \text{states}(B)}{\texttt{fail} :- \text{acc}(p, q), \text{frb}(p, q).}$$

Computing forbidden states

$$O(|A| * |B|) \left\{ \begin{array}{lcl} \text{frb}^c(p, Q_1) & \Rightarrow & \text{frb}(p, q_1),\ \forall q_1 \notin Q_1 \\ & \vdots & \\ \text{frb}^c(p, Q_m) & \Rightarrow & \text{frb}(p, q_m),\ \forall q_m \notin Q_m \end{array} \right\} O(|A| * |B|^2)$$

- Cannot enumerate frb predicates: need algorithmic solution

Efficient data structure

- Example: $\text{states}(B) = \{1, 2, 3, 4\}$

$$\left\{ \begin{array}{lcl} \text{frb}^c(p, \{1, 2\}) & \Rightarrow & \text{frb}(p, 3) \text{ and } \text{frb}(p, 4) \\ \text{frb}^c(p, \{2, 4\}) & \Rightarrow & \text{frb}(p, 1) \text{ and } \text{frb}(p, 3) \end{array} \right\}$$

- Array for state p:

q	1	2	3	4	$total$
$occurrence(q)$	1	2	0	1	2

- $\text{frb}(p, q)$ if $occurrence(q) < total$
- Overall time for testing failure: $O(|A| * |B|)$

Testing Inclusion

Theorem

$L(A) \subseteq L(B)$ can be tested in time $O(|A| * |B|)$ for binary tree automata A, deterministic B over Σ.

Outline

XML Trees

- Unranked trees
- Binary encoding *via* Currying
- Stepwise tree automata [Carme et al. 2004]

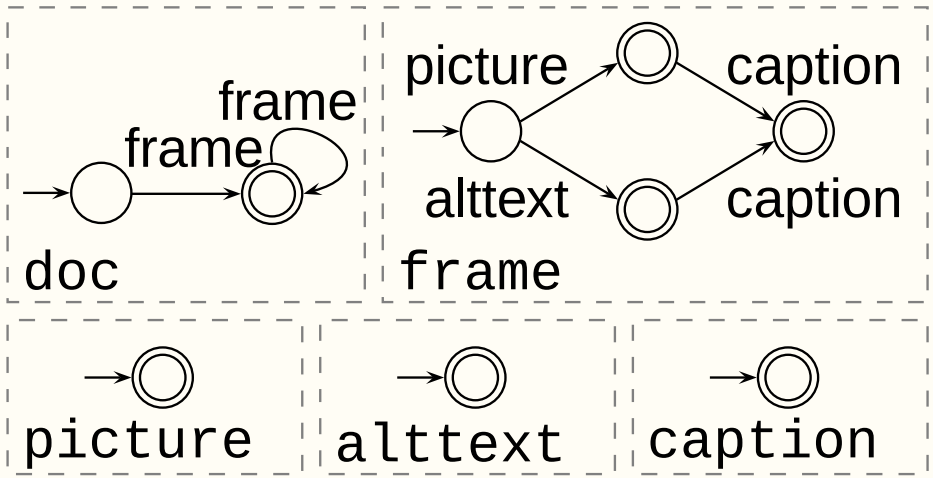
DTDs to Glushkov Automata

DTD

```
<!ELEMENT doc      (frame+)>
<!ELEMENT frame    (picture|alttext),caption?>
<!ELEMENT picture  (#PCDATA)>
<!ELEMENT alttext  (#PCDATA)>
<!ELEMENT caption  (#PCDATA)>
```

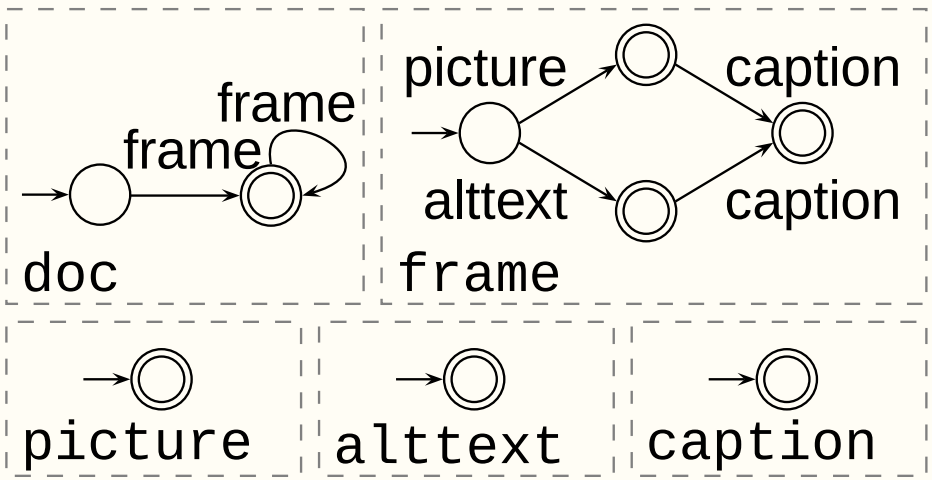
Glushkov automata

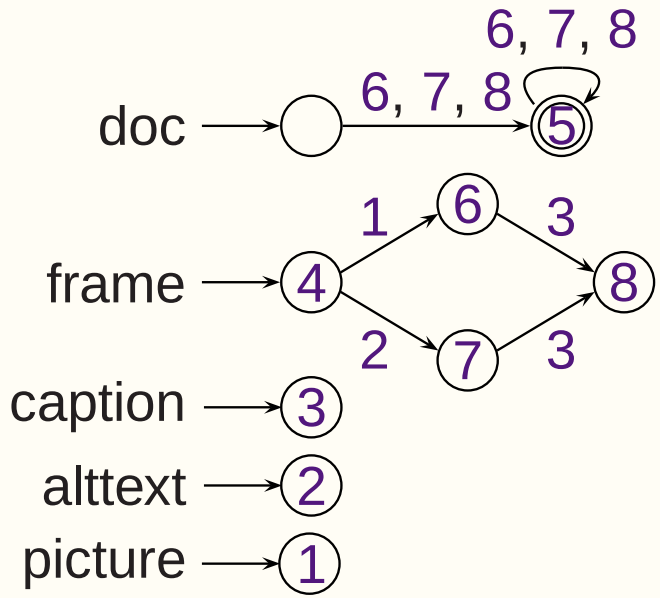
- One-unambiguous regular expressions (W3C)
- To deterministic word automata in $O(|\Sigma| * |DTD|)$ [Brüggemann-Klein & Wood 1998]



DTDs to Stepwise Tree Automata

Glushkov automata to stepwise tree automaton

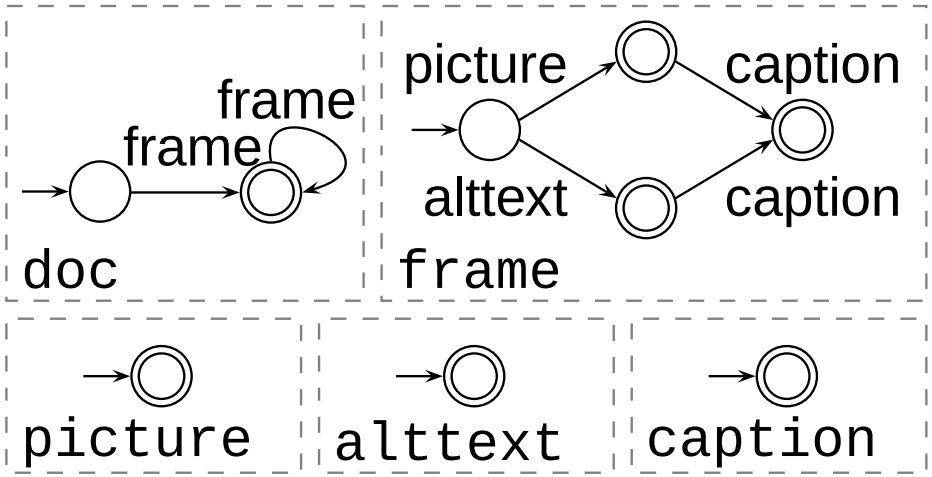




$O(|\Sigma| * |DTD|)$

$O(|\Sigma| * |DTD|^2)$

Factorized Tree Automata



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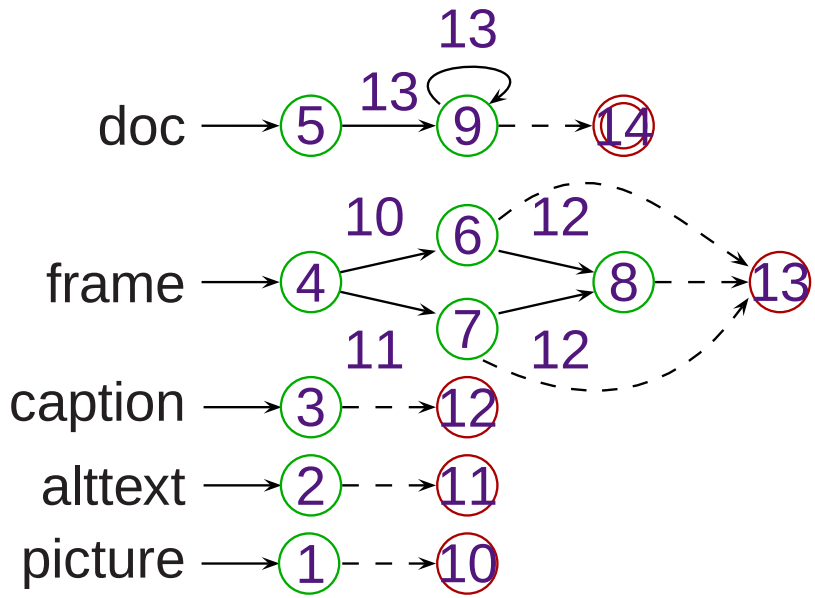

$O(|\Sigma| * |DTD|)$

Inclusion into Factorized Tree Automata

- More inference rules to consider
  - Epsilon-rules of factorized automaton
  - Distinct cases for each sort of states
  - Other stuff
- Complexity: $O(|A| * |\Sigma| * |DTD|)$

Summary

- Inclusion in deterministic binary tree automata in time $O(|A| * |B|)$
  - Characterization in terms of failure
  - Inferring a ground Datalog program
  - Efficient data structure
- Extension to inclusion in DTDs in time $O(|A| * |\Sigma| * |DTD|)$
  - Unranked tree automata to stepwise tree automata *via* Currying
  - DTDs to factorized tree automata
  - Adapt inclusion algorithm
  - Lower bound?