

Efficient GPU-based Construction of Occupancy Grids Using several Laser Range-finders

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Systems, 2006

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Occupancy
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Bayesian Model
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From range
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GPU Imple-
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General Algorithm
Architecture

Dirac model

Gaussian model

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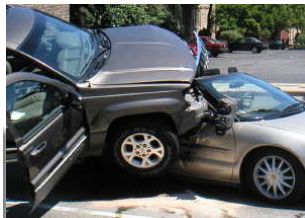
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Enforce vehicle safety by

- percieving the whole robot surroundings,
- being robust to false measurements,
- having precise map,
- providing real-time map.



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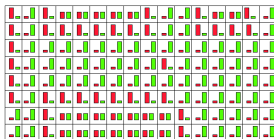
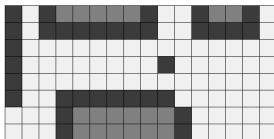
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Definition

An OG is a stochastic tessellated representation of spatial information that maintains probabilistic estimates of the occupancy state of each cell in a lattice [1].

Features:

- no assumption about environment geometry,
- simple fusion process,
- occultation information,
- each cell is independent.



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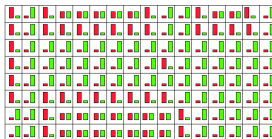
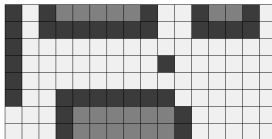
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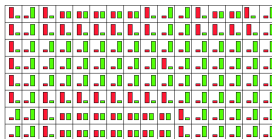
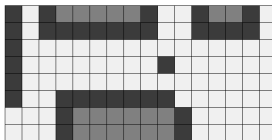
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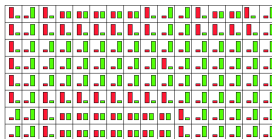
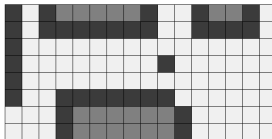
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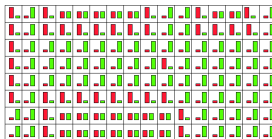
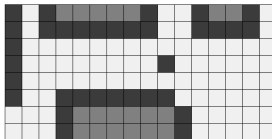
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Grids have image-like structures:

- huge amount of data
e.g. 100mx100m grid with cell side of 5cm
→ 4M cells
- enlarging the field of view
→ increases the amount of data
- increasing precision
→ increases the amount of data

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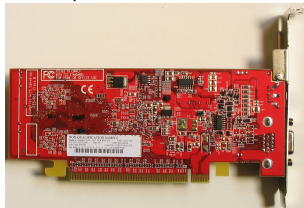
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Grids have image-like structures:

- huge amount of data
e.g. $100m \times 100m$ grid with cell side of $5cm$
→ **4M cells**
- enlarging the field of view
→ **increases the amount of data**
- increasing precision
→ **increases the amount of data**

Graphical processor units (GPUs):



- dedicated to work with images,
- high level of parallelism,
- easy to program with shading languages,
- cheap.

Objective:

accurate implementation of OG fusion with the GPU

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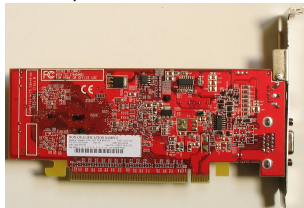
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- Variables:

- $\vec{Z} = (Z_1, \dots, Z_n)$ a vector of n random variables,
- $O_{x,y} \in \mathcal{O} \equiv \{\text{occ}, \text{emp}\}$. $O_{x,y}$ is the state of the cell (x, y) ,

- joint probability distribution:

$$P(O_{x,y}, \vec{Z}) = P(O_{x,y}) \prod_{i=1}^n P(Z_i | O_{x,y})$$

- inference:

let $\vec{Z} = (z_1, \dots, z_n)$ a vector of sensor measurements:

$$p(o_{x,y} | \vec{Z}) = \frac{p(o_{x,y}) \prod_{i=1}^n p(z_i | o_{x,y})}{p(\text{occ}) \prod_{i=1}^n p(z_i | \text{occ}) + p(\text{emp}) \prod_{i=1}^n p(z_i | \text{emp})}$$

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Values of sensor model characteristics for a beam:

position	ρ dependent	z dependent
if $z \ll \rho$	--	×
if z around ρ	×	×
of $z \gg \rho$	--	×

where:

- 1 ρ is the cell range,
- 2 z the measured range,
- 3 gaussian sensor uncertainty,

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where:

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- 2 z the measured range,
- 3 gaussian sensor uncertainty,
- 4 very weak occupancy *a priori* : world almost empty.

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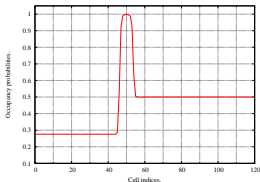
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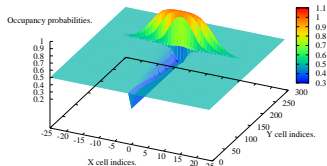
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(a)



(b)

Extension from 1D to 2D OG. (a) 1D OG (b) 2D OG of a sensor beam.

The sensor is positioned in (0,0).

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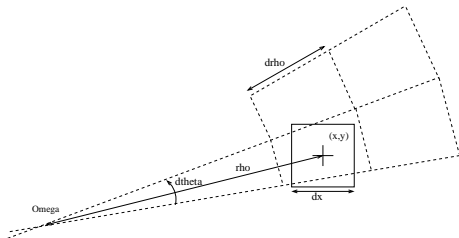


Figure: Polar and Cartesian grids parameters.

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Figure: Grid based slam (from Dirk Hähnel home page).

Moirage problem with Bresenham algorithm

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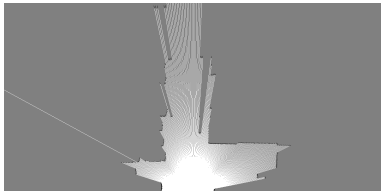
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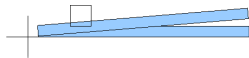
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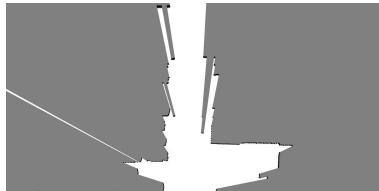
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Bresenham algorithm (log plot)



superposition problem



exact algorithm

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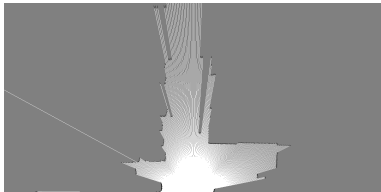
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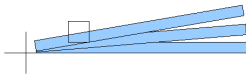
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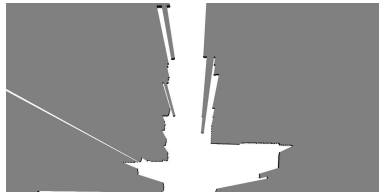
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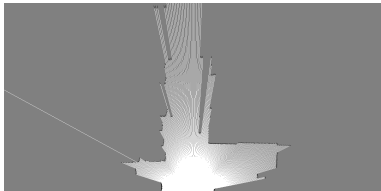
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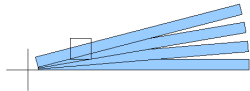
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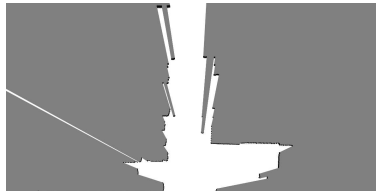
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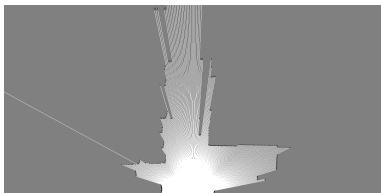
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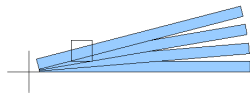
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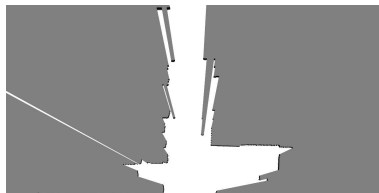
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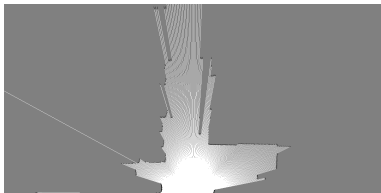
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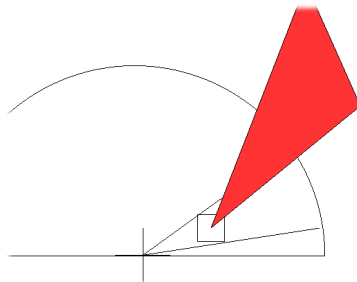
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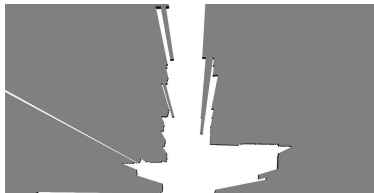
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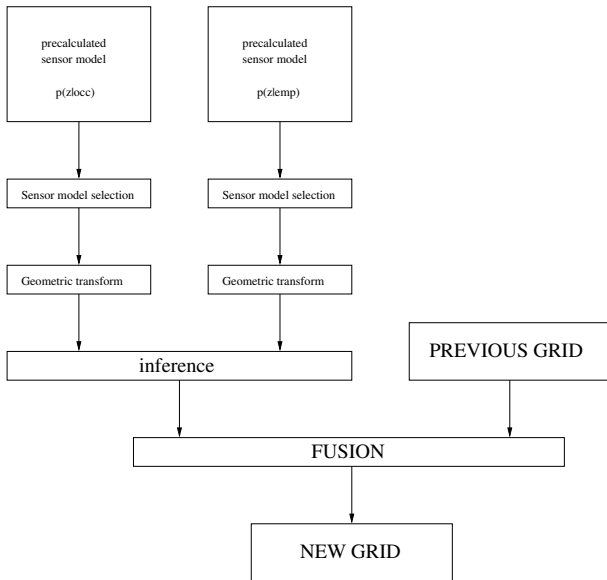
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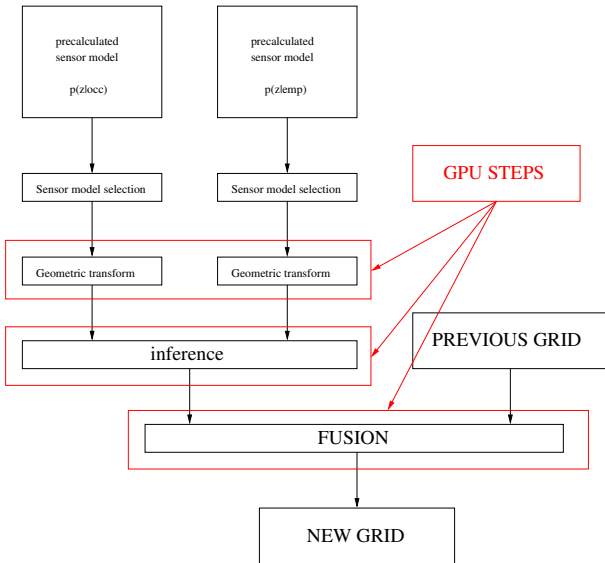
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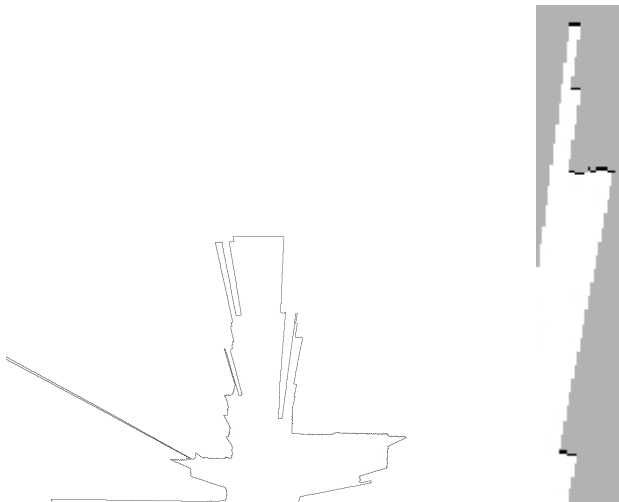
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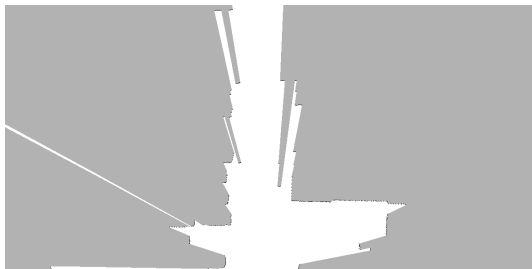


Figure: Occupancy grid generated by the GPU, polygon mode

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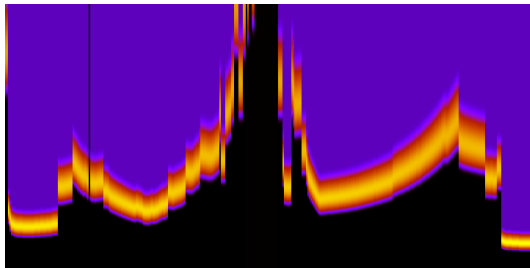
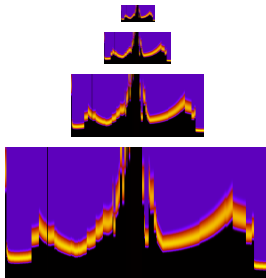
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Angle beam in abscissa and ρ in ordinate at different scales



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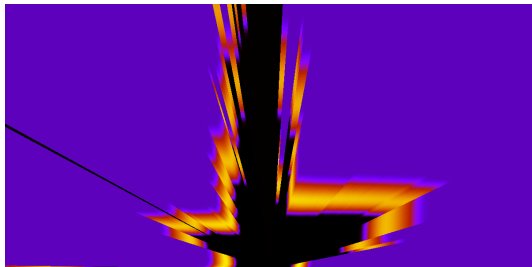


Figure: Occupancy grid generated by the GPU.

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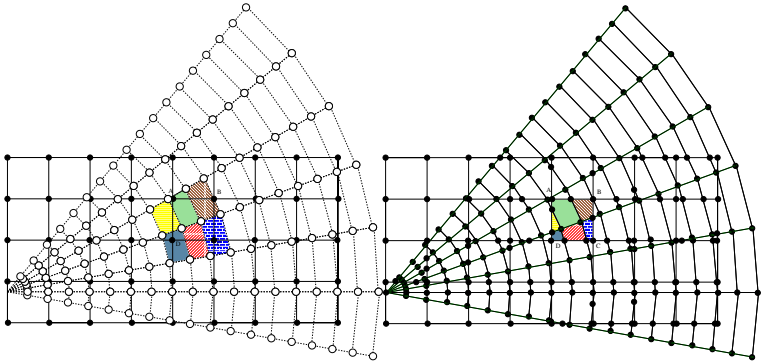
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- 1 exact,
- 2 Bresenham,
- 3 adaptative sampling,
- 4 GPU (polygon mode)

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Method	Avg. Error	Max. Error	avg. time
exact	0	0	1.23s (CPU)
line drawing	0.98	25.84	0.22s (CPU)
sampling	0.11	1.2	1.02s (CPU)
GPU	0.15	1.8	0.049s on MS 0.0019s on board

Table: Comparison of change of coordinate system methods (MS: master storage).

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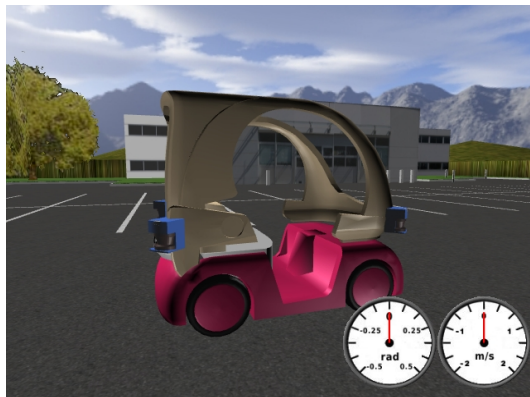
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Fusion of four SICK laser range-finders surrounding a vehicle.

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- 1 correct sampling is necessary,
- 2 graphical hardwares for sensor geometry,
- 3 graphical hardwares for multiple sensor fusion and grid handling,
- 4 on board: fusion of 50 sensors is possible in real-time.

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Handling with GPU:

- 1 angle uncertainty,
- 2 sensor position uncertainty,
- 3 scan matching.



Alberto Elfes.

Occupancy grids: a probabilistic framework for robot perception and navigation.

PhD thesis, Carnegie Mellon University, 1989.