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On Minimizing the Average Reliability of Connections in Multilayer Networks under Shared Risk Groups and Costs Constraints*

David Coudert¹, Florian Huc¹, Fabrice Peix¹, Marie-Emilie Voge²

¹MASCOTTE, INRIA, I3S-CNRS, Univ. Nice Sophia, Sophia-Antipolis, France
e-mail: {firstname.lastname}@sophia.inria.fr

² IRCICA/LIFL, Univ. Lille 1, CNRS UMR 8022, INRIA Futurs, France.
e-mail: {Marie-Emilie.Voge}@lifl.fr

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Abstract

The notion of Shared Risk Resource Groups (SRRG) has been introduced to capture survivability issues when a set of resources may fail simultaneously. Applied to Wavelength Division Multiplexing Network (WDM), it expresses that some links and nodes may fail simultaneously. The reliability of a connection therefore depends on the number of SRRGs through which it is routed. Consequently, this number has to be minimized. This problem has been proved NP-complete and hard to approximate in general, even when routing a single request. Some heuristics using shortest paths have already been designed, however the cost (the usual routing cost, not in term of SRRG) was not part of the objective. In this paper we study the problem of minimizing a linear combination of the average number of SRRG per paths and the cost of the routing. The main result of our work is a column generation formulation that allows to solve the problem of maximizing the reliability of a set of connection requests in MPLS/WDM mesh networks with SRRGs while keeping the cost of the routing low.

Keywords: Reliability, Shared Risk Resource Groups, Integer Linear Programming, Column Generation.

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1 Introduction

A challenging issue for service providers operating connection oriented networks, such as MPLS/WDM mesh networks, is to provide reliable routes for accepted requests. This has been fostered by the growing use of virtual and overlay networks which are embedded, in a transparent way, on an underlying network. Therefore, a great deal of researchers efforts have been concentrated on the design of efficient protection mechanisms at the backbone level (see for example [11, 15, 16, 18]). At this level, the reliability of a single connection in the network might be measured as a function of the number of links it uses, each single link having its own probability of failure. However, in a physical network, several fibers, or links, may be packed together and hence physically cut simultaneously (earthquake, fire, malicious behaviour etc). Consequently, a single cause of breakdown induces several virtual link failures. The concept of Shared Risk Resource Group (SRRG) (or Shared Risk Group, SRG) has been introduced to capture network survivability issues in this situation, i.e. when a set of resources may fail simultaneously. This concept can be extended to various kind of correlated failures.

A typical example of SRG is given by MPLS multilayer networks as described in [14]. This is illustrated by Fig. 1 where links AH and EI of the virtual network are both routed into the underlying network through link FG and so belong to a same SRG, the one associated to FG. Indeed a failure on the physical link FG induce the disruption of both virtual links AH and EI. In such multilayer networks with SRGs, the reliability of a connection is measured as a function of the SRGs through which it is routed.

Given a network and a set of requests, the problem of finding a routing minimizing the average number of SRG through which the route of a request goes has already been studied. It has been proved to be NP-hard [16], hard to approximate [4], and heuristic algorithms have been designed. For example [16] studied a *Tabu Search* heuristic algorithm for routing requests with different requirements. However, if we consider a network with edge cost, such an objective may lead to a costly routing. To avoid this, the objective to be minimized has to take into account two cost criteria. First we want to minimize the average failure probability, that is the average number of risks of failure on which a connection is dependent, as previously. Secondly, since an operator is interested in both the safety of the connections and their operating costs, the objective has to take into account the cost of the edges used along the route of the connections.

Hence, we concentrate on the problem of computing under capacity constraints a set of paths minimizing a linear combination of their edge costs and the number of SRGs through which they are routed. To our knowledge, this is the first time in the SRG context that such an objective, considering both the quality of the service offered to customers and the cost of the services, is proposed. Our main result is the first integer linear program (ILP) with column generation for the Minimum Average Color Flow problem, even when we consider only the objective of minimizing the average number of SRGs.

In this paper we also discuss a way to generate “realistic” virtual networks with SRRG. Since the simultaneous failures of links are correlated, a random assignment of SRRG to virtual links does not necessarily correspond to a feasible routing of the virtual topology into a real physical network, as was pointed out in [6]. The solution we propose avoid this problem and hence the experiments are run on graphs presenting more characteristics of real instances.

Our paper is organized as follows. In the next Section, we present the notations and network models used in the rest of the paper. Sec. 3 is dedicated to the Minimum Average Color Flow problem: once the problem is defined, we study its complexity and present two ILP formulations.

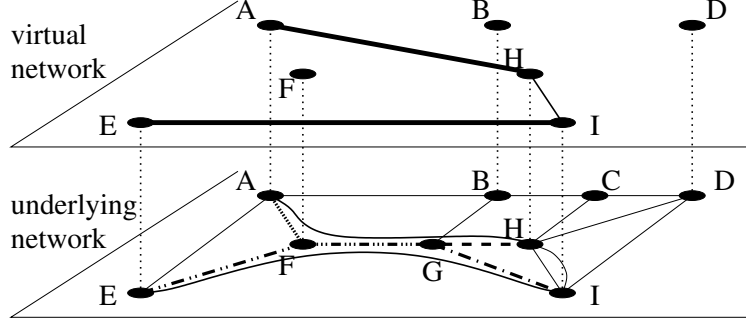


Figure 1: Example of SRLG: virtual links AH and EI share the same risk FG.

In Sec. 4.1 we remind some basics about column generation before applying it to Minimum Average Color Flow. We then detail in Sec. 5 the generation of our instances and the implementation of our solving process, and finally we analyze the computational results before concluding.

2 Network model and notations

The multilayer networks we consider are composed of two layers, a physical or underlying network and a virtual one.

The physical layer is modelled by a digraph $G_{PN} = (V_{PN}, L_{PN})$, where V_{PN} is the set of nodes, and $L_{PN} = \{e_1, \dots, e_m\}$ is the set of links (i.e. unidirectional fibers). Similarly, the virtual network embedded in the physical one is modelled by the digraph $G_{VN} = (V_{VN}, L_{VN})$. Each arc $e \in L_{VN}$ has a cost γ_e per flow unit and a capacity u_e . The construction of the virtual networks used in our simulations will be described more precisely in Sec. 5.

Each arc e_i of the physical network G_{PN} gives in the virtual layer of a single risk group that we represent by a color c_i . More precisely, color c_i is the subset of arcs of the virtual network routed through arc e_i on the underlying layer, that is the set of arcs sharing the risk of failure of arc e_i . The set $\mathcal{C} = \{c_1, \dots, c_m\}$ is the set of colors (i.e. SRG) of the network. Note at that point that an arc of the virtual network may belong to several SRGs (= colors). However, following [14, 16, 18], we apply the simple transformation on G_{VN} described in [4, 17]: each multi-colored arc is replaced with a chain of mono-colored arcs, so that each arc now belongs to exactly one color. For simplicity purpose we will denote also by G_{VN} the modified graph. Such graphs with SRG are referred to in the literature as *colored graphs*. Interested readers may refer to [9] for a model with multiple colors per edges, and to [17] for a preliminary comparison of both models.

Let K be a set of connection requests over G_{VN} , such that for each request $k \in K$, we are given its source s_k , its destination t_k and its integral bandwidth requirement d_k . We assume that each request can be split into unitary requests to be served independently. Consequently, in the rest of the paper all the requests are unitary. For each request $k \in K$, \mathcal{P}^k is the set of all the $s_k t_k$ -directed paths in G_{VN} and $\mathcal{P} = \cup_{k \in K} \mathcal{P}^k$

Finally, we denote respectively by $\Gamma^-(v)$ and $\Gamma^+(v)$ the set of incoming and outgoing links at a node v .

3 Minimum Average Color Flow Problem (MACF)

3.1 Problem definition

An instance of the Minimum Average Color Flow problem consists in a digraph $G_{VN} = (V_{VN}, L_{VN})$, representing the virtual network, with arc cost γ_e per flow unit and capacity u_e for each arc $e \in L_{VN}$. A set $\mathcal{C} = \{c_1, \dots, c_m\}$ of colors -or SRG- partitioning the arc set L_{VN} is given, as well as a set of unitary connection requests K and a real $\alpha \in [0, 1]$.

A solution is a routing of all the requests satisfying all the capacity constraints.

The objective to be minimized is a convex combination of two cost criteria whose respective contributions can be adjusted through the parameter $\alpha \in [0, 1]$. The first criterion, related to the operating cost, is the classical flow cost depending on the arc costs γ_e divided by the total flow request ($= |K|$). The second criterion, related to the safety of the connections, is the average number of colors - SRGs - to which belongs a path routing one unit of flow, that is the sum of the numbers of colors on which is routed each request divided by $|K|$. The objective function is then $\frac{\alpha}{|K|} \times (\text{classical flow cost}) + \frac{(1-\alpha)}{|K|} \times (\text{SRG flow cost})$.

Note that when $\alpha = 0$ the problem becomes the minimisation of the average number of SRG or in other word the average probability of failure for a connection, whereas if $\alpha = 1$ the problem is reduced to a classical integral multi-commodity flow.

3.2 Complexity

We now study the complexity of MACF under various hypothesis on the colored graph. We deduce these results for MACF from one of its subproblems: Minimum Color Path with Edge Costs problem (MCPwEC) which is the MACF problem with a single request, i.e. $|K| = 1$, and unitary arc capacities. Thus, it consists in finding a path from a source s to a sink t minimizing a convex combination of the number of colors used by the path and of the classical path cost.

3.2.1 Null arc costs

With null arc costs, this problem is the Minimum Color Path problem (MCP) which has been proved NP-hard in [18] and hard to approximate within a factor $2^{\log^{1-\delta} |\mathcal{C}|^{\frac{1}{2}}}$ in [4], with $\delta = (\log \log |\mathcal{C}|^{\frac{1}{2}})^{-\varepsilon}$ and $\varepsilon < \frac{1}{2}$. These results were proved for undirected graphs, but their adaptation to the directed case is straightforward and induces the same factor for MACF.

3.2.2 Span 1

MCP can be solved in polynomial time when some conditions are satisfied on the *span* of the colors of the graph [4, 7, 17]. The span of a color is the number of connected components in the subgraph induced by the edges of that color. The influence of this parameter on the MCP complexity has been studied in [4, 5]. However, we show that the span has no such an influence on the hardness of MCPwEC. We remind first a result from [4].

Theorem 1 [4] *When all colors have span 1, MCP is polynomial.*

The proof is based on the fact that within a connected component of a color c , each vertex is reachable from any other vertex of the component using only edges of color c .

The span definition and Theorem 1 can easily be extended to the *color symmetric digraph* settings, i.e. when an arc and its reverse arc belong to the same color: the span is simply the number of strongly connected components and the property is still verified.

Let us now prove that the adjunction of edge costs deeply modify the complexity properties of MCP since MCPwEC is as hard to approximate as in the general directed case in the aforesaid symmetric settings.

Theorem 2 *The MCPwEC problem on a color symmetric digraph is NP-hard and hard to approximate within a factor $2^{\log^{1-\delta} |\mathcal{C}|^{\frac{1}{2}}}$ with $\delta = (\log \log |\mathcal{C}|^{\frac{1}{2}})^{-\varepsilon}$ and $\varepsilon < \frac{1}{2}$ even when all colors have span 1.*

Proof. The proof is a reduction from MCP to MCPwEC. Let G be a color symmetric digraph on color set \mathcal{C} with unspecified span and two vertices s and t defining an instance of MCP. We construct an instance of MCPwEC on the digraph G' with convex coefficient α in the objective function. G' is the graph G with two additional symmetric arcs of color c and cost $X > \frac{1-\alpha}{\alpha}|\mathcal{C}|$ between each pair of strongly connected components of c . The cost of arcs of G' present in G is null and all the capacities are unitary.

An optimal st -path for the MCPwEC instance uses none of the added arcs since using only ones cost $\alpha X > (1 - \alpha)|\mathcal{C}|$, which is more than the cost of using all arcs and colors of G . Therefore, an optimal path in G' for MCPwEC is an optimal path in G for MCP too. Indeed the two costs of a same path for MCPwEC and MCP differ only by the multiplicative constant $1 - \alpha$. This last point concludes the proof. \square

From Theorem 2 we can deduce that in this special case MACF is as hard to approximate as its subproblem MCPwEC.

3.2.3 Color reduced to a single arc

Unlike MCPwEC and MCP, MACF remains NP-hard and hard to approximate when all colors are reduced to a single arc. Indeed in such a case, MCPwEC and MCP are merely polynomial shortest path problems while MACF still contains the Minimum Edge Cost Flow problem (MECF) as a subproblem. The MECF problem consists in routing a single integral request in a digraph with capacities and fixed arc costs, that is the cost of using an arc is the same whatever the flow value is on it, as long as it is positive. This problem is NP-hard and hard approximate within a factor $2^{\log^{1-\varepsilon} n}$, $\forall \varepsilon > 0$ [8].

Theorem 3 *The MACF problem is hard to approximate within a factor $\approx 2^{\log^{1-\varepsilon} n}$ $\forall \varepsilon > 0$ even when all colors are reduced to a single arc, unless $NP \subseteq DTIME(n^{\text{polylog}n})$.*

3.3 ILP Formulations

In this section we present two ILP formulations for MACF based on the two classical multi-commodity flow formulations: the compact node-arc formulation and the exponential size arc-path one (see [1]).

3.3.1 Node-Arc Formulation

The binary flow variable z_e^k for $e \in L_{VN}$ and $k \in K$ gets value 1 if the virtual link e is used to route the unitary request k and 0 otherwise. The binary color variable χ_k^c , for $k \in K$ and $c \in \mathcal{C}$, gets values 1 if the unitary request k is routed through an arc belonging to the risk group represented by color c .

$$\min \frac{1}{|K|} \sum_k \left(\alpha \sum_{c \in \mathcal{C}} \chi_k^c + (1 - \alpha) \sum_{e \in A} \gamma_e z_e^k \right)$$

$$\sum_{e \in \Gamma^+(v)} z_e^k - \sum_{e \in \Gamma^-(v)} z_e^k = \begin{cases} 1 & \text{if } v = s_k \\ -1 & \text{if } v = t_k \\ 0 & \text{otherwise} \end{cases} \quad \forall k \in K \quad (1)$$

$$\sum_{k \in K} z_e^k \leq u_e \quad \forall e \in L_{VN} \quad (2)$$

$$z_e^k \leq \chi_k^{c(e)} \quad \forall k, \forall e \in L_{VN} \quad (3)$$

Constraints (1) and (2) are the classical flow conservation and capacity constraints of a node-arc multi-commodity flow formulation. Constraint (3) ensures that whenever the flow for a request k on an arc e is positive, the color $c(e)$ of this arc is counted as used for this request, i.e. the value of $\chi_k^{c(e)}$ is forced to 1, where $c(e)$ is the color of arc e . There are $|K|(|L_{VN}| + |\mathcal{C}|)$ variables and $|K|(|V_{VN}| + |L_{VN}|) + |L_{VN}|$ constraints.

The objective to be minimized is the average over all requests of a convex combination of the number of colors used by a request and of the classical cost of the route of a request.

3.3.2 Arc-Path Formulation

We now use binary variable z_p^k which gets value 1 if the request $k \in K$ is routed on the path $p \in \mathcal{P}$, and 0 otherwise. As there might be a large number (exponential) of valid paths for each request, there might be a large number of variables in this family. As in the node-arc formulation, the binary variable χ_k^c has value 1 if the path used by request k uses an arc of color c and 0 otherwise.

$$\min \frac{1}{|K|} \sum_k \left(\alpha \sum_{c \in \mathcal{C}} \chi_k^{c(e)} + (1 - \alpha) \sum_{e \in L_{VN}} \sum_{P \in \mathcal{P}_k | e \in P} \gamma_e z_P^k \right)$$

$$\sum_{P \in \mathcal{P}_k} z_P^k = 1 \quad \forall k \in K \quad (\sigma_k) \quad (4)$$

$$\sum_k \sum_{P \in \mathcal{P}_k | e \in P} z_P^k \leq u_e \quad \forall e \in L_{VN} \quad (\omega_e) \quad (5)$$

$$\sum_{P \in \mathcal{P}_k | e \in P} z_P^k \leq \chi_k^{c(e)} \quad \forall k \in K, \forall e \in L_{VN} \quad (\pi_e^k) \quad (6)$$

Constraints (4) ensures the satisfaction of all requests: for a given request k there must be a path used to route it; we call σ_k the associated dual variable. The capacity constraint (5) on each

arc expresses that there can not be more paths using an arc e than there is bandwidth to fit them; we call ω_e the associated dual variables. As in the node-arc formulation, constraints (6) takes into account the colors used to serve each request. The objective function is also similar. There are $|K|(|L_{VN}| + 1) + |L_{VN}|$ constraints and empirically at most $10|K|$ variables, which is a lot smaller (for both variables and constraints) than in the node-arc formulation.

Note that to apply the column generation technique, or more precisely to define the auxiliary problem, we need the dual constraint associated to the large size family of variables z_p^k .

4 Solving Minimum Average Color Flow

4.1 Principle of column generation

A column generation formulation is a transformation of a linear program containing a very large size variable family (e.g., exponential) into two subproblems: the *master problem* and the *auxiliary problem*. This decomposition relies on the Dantzig-Wolf decomposition [1]. It allows to consider only a subset of variables at a time.

Basically, starting with an initial reduced set of variables, we solve the master problem and obtain dual variable values that will be used by the auxiliary problem. Then, we solve the auxiliary problem to obtain new variables for the master problem, and so on. We repeat this loop until the auxiliary problem finds no more variables for the master problem. The solution of the master problem is then optimal (see [3]).

Column generation can be combined with branch-and-bound techniques to solve integral linear programs with a large number of variables, see [2] for a nice overview.

4.2 Column generation applied to MACF

4.2.1 Master Problem

The master problem corresponds to the original arc-path formulation in which only a subset of the variables from the large size families $\{z_p^k\}_{p \in \mathcal{P}^k}$ is considered, all other variables of these families simply do not exist.

4.2.2 Auxiliary Problem

The auxiliary problem is deduced from the dual constraints of the master problem associated to the large size family of variable z_p^k :

$$\sigma_k \leq \sum_{e \in P} \left(\frac{(1-\alpha)}{|K|} \gamma_e + \omega_e + \pi_e^k \right) \quad \forall k, \forall P \in P_k$$

A solution of the auxiliary problem is a path P violating this constraint. In our case the auxiliary problem is a shortest path problem on G_{VN} with arc cost $\frac{(1-\alpha)}{|K|} \gamma_e + \omega_e + \pi_e^k$. We have to solve the auxiliary problem for each request k . Notice that the arc costs depend on the request, indeed the *color cost* ($\chi_k^{c(e)}$) is not the same for each request.

4.2.3 Generation of the initial solution

The initial set of variables of the master program has to contain the non null variables of a feasible solution otherwise the column generation process can not start. To find this initial subset of

variables we solve a classical minimum cost multi-commodity flow problem: from MACF we keep only the arc costs and capacities and we try to satisfy all the requests under the capacity constraints (see [1] for references on the minimum cost multi-commodity flow problem).

4.3 Obtaining integral solutions:

When looking for an integral solution, a lot of computational time can be saved by considering the χ_k^c as fractional variables¹. Indeed, since all requests are unitary, once the flow variables are integral, the constraints on the χ_k^c variables force them to be integral too.

5 Experiments

We have used CPLEX with a Concert Java interface and the open source MASCOPT library [10] to implement our model.

5.1 Generation of instances

To give a realistic significance to the colors, we assigned them to the virtual links according to a real routing of the virtual layer on a physical network as follows.

We used two networks as physical layers : NSFNET (14 nodes and 21 links) and BRAZIL (27 nodes and 70 links) [12]. Each physical link and node is associated to a distinct color. If the network is already precolored with meaningful groups of risk, we may use them as well.

The virtual layer was generated using the following steps.

1. We generate an undirected graph on the physical vertex set using a *pseudo random* graph generator, implemented in MASCOPT, which accepts a set of constraints on the resulting graph. We choose the following ones: diameter at most 3, minimum cut at least 2, and number of links around 3 times the number of nodes.
2. We replace each edge by two symmetrical arcs.
3. We randomly generate unitary requests: 140 requests for NSFNET and 200 for BRAZIL.
4. We compute a *double multi-commodity flow*: the demands are routed on the generated graph whose arcs are themselves routed on the physical network simultaneously.
5. Each arc of the generated graph corresponds to a set of parallel virtual links: an arc is routed on a set of paths, each one giving a distinct virtual link. To each virtual link we assign the colors of the physical resources through which is routed its associated path and a weight corresponding to the total request flow routed on it.
6. The capacity given to each virtual link v_l is the minimum over each physical link p_l of the proportion of utilisation of p_l by v_l among all virtual links multiplied by the capacity of p_l , that is: $\min_{p_l \ni v_l} \frac{\text{weight}(v_l)\text{capacity}(p_l)}{\sum_{v'_l \ni p_l} \text{weight}(v'_l)}$
7. We transform the network given by the virtual links into a mono-colored graph [4] since at that point it may be multi-colored.

¹this optimization is also valid for the node-arc formulation

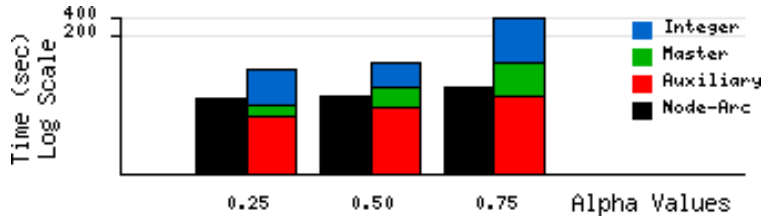


Figure 2: NSFNET

5.2 Implementation

We now briefly describe our implementation to allow a better understanding of next section 5.3.

5.2.1 Columns generation

As explained in 4.1, the column generation is based on two problems (*master* and *auxiliary*). In our implementation both problems are solved using CPLEX. During the process, we only add columns to the master problem without removing any.

5.2.2 Obtaining an integral solution

We first implemented a branch-and-price algorithm using pseudo-cost to order the variables [2] and depth-first-search to visit the branch-and-price tree. However, our implementation is not yet efficient enough due to the lack of significant cuts. Therefore, we decided to obtain an integral solution via CPLEX using only the set of variables selected during the column generation process. The obtained solution might not be optimal, but close to it in our experimentations. We asked CPLEX to guarantee a gap with optimality of less than 5%, and in practice the gap was less than 5%.

5.3 Computational results

We made all our tests on a *Intel* Core 2 2.4 GHz with 4Mb of L1 cache and 2 Gb of memory. We present our results in two graphics, one for NSFNET (Fig. 2) and one for BRAZIL (Fig. 3). We solved each problem ten times using both formulations asking for an integer solution at most 5% of the optimal (except when $\alpha = 0.75$ in BRAZIL, where we ask for 10%). The height of the rectangle represents the total time spent to find the solution. For the arc-path formulation, the rectangle is splitted accordingly to the time spent to solve the auxiliary problems, the master problems and to find an integral solution.

5.3.1 MACF: column generation vs node-arc formulation

Computational time Surprisingly, the node-arc formulation is always faster than the column generation formulation whereas there is a lot more variables in the first one. This can be partly explained by our implementation (we used a Java interface to call CPLEX which slows down the process) but also by the constraints on the color variables.

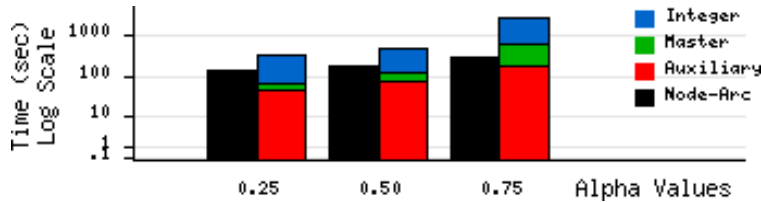


Figure 3: BRAZIL

In addition in the column generation formulation, each time we solve the master problem, we have to solve an auxiliary problem for each request (cf 4.2.2).

Size of the instances solved The column generation formulation allows to solve larger instances than the node-arc formulation, where the number of variables is too large.

Influence of the colors The resolution time increases with α for both formulations due to the increasing importance of the color variables. Indeed, the color cost of an edge depends of the path while the classical cost of an edge is fixed. It is worth noting that in the column generation formulation, the quality of the initial solution decreases when α increases since it does not take the colors into account at all.

6 Conclusion

In this paper we have investigated the problem of maximizing the reliability of a set of connection requests in mesh networks with SRRG. We have shown that MACF is NP-hard and hard to approximate, and we have proposed a column generation formulation to solve it that allows to obtain optimal or near optimal solutions in reasonable time.

We will now pursue our implementations and extend our work to the multiple-path problem where each connection request is protected by a SRRG-disjoint path and to other classical protection schemes. Note that SRRG-disjoint path in multilayer networks and with a different objective function currently under investigation in [13].

Acknowledgments

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