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# Scheduling Hybrid Flowshop with Parallel Batching Machines and Task Compatibilities

Adrien Bellanger \*

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## 1 Introduction

We consider a two-stage hybrid flowshop scheduling problem where each of  $n$  tasks is to be processed first at stage 1 and then at stage 2. The first stage contains  $m_1$  identical parallel discrete machines and the second stage contains  $m_2$  identical parallel batching machines. Each discrete machine can process no more than one task at time and each batching machine can process up to  $k$  ( $k < n$ ) tasks simultaneously in batch. The processing time of task  $j$  on any machine of stage one requires  $p_j$  time units and on any machine of stage two requires  $q_j$  time units, which is given by the interval  $[a_j, b_j]$ . On the second stage the tasks are processed in batch and all tasks of the same batch start and finish together with the additional constraint that the tasks of the same batch have to be compatible. A compatibility is a symmetric binary relation in which a pair  $(i, j)$  of tasks is compatible if they share a similar processing time on the second machine (i.e,  $[a_i, b_i] \cap [a_j, b_j] \neq \emptyset$ ). The batch processing time on the batching machine is determined as the maximum initial value  $a_j$  of compatible tasks. The objective is to find a schedule such that the completion time of the latest batch is minimized. This problem is motivated by scheduling in tire manufacturing industry [2]. A typical tire is built in a two-stage process. In the first stage (tire building), all components (sidewalls and tread) are assembled and radial tires are built on a round drum that is part of the tire building machine. The end result is called a *green tire* or *uncured tire*. In the second stage (tire curing), curing occurs through a series of chemical reactions. Tire curing is a high-temperature and high-pressure batch operation in which a pair of uncured tires is placed into a mold at a specified temperature. Each type of tire must be cured for a total duration in an interval. Two kinds of tires can be cured together if they share a same value of total curing duration. After the mold is closed it cannot be opened until the curing reaction is completed for both green tires on the same mold.

Although extensive research has been carried out on batch scheduling problems [1], [3], [4], [5], [6], to the best of our knowledge, the problem of hybrid flowshop involving batching machines and task compatibilities has not been considered before.

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Since this problem is NP-hard, we focus on developing a heuristic algorithms with a performance guarantee. First we consider the general case with  $m_1$  machines at stage one and  $m_2$  machines at stage two, abbreviated in the following as  $FH2(m_1, m_2)$ . We propose three heuristics  $H_1$ ,  $H_2$  and  $H_3$ . The heuristic  $H_1$  schedules tasks at the first stage following LPT rule (i.e. sort the tasks in non-increasing order of their processing time at the first stage, and assign each task not yet scheduled to a machine with minimum load), and on the second stage after constructing a list  $L$  of batches following the FBCLPT rule [2] (i.e. sort the tasks in non-increasing order of their initial interval value  $a_j$ , and let  $L$  be the list of tasks. At iteration  $i$ , construct a new batch  $B_i$  and add to  $B_i$  the first task  $j$  not yet placed ; starting with  $j$  put into  $B_i$  the next  $k - 1$  tasks not yet placed that are compatible with  $j$ , remove from  $L$  the tasks of  $B_i$  and go to the next iteration until  $L$  become empty), each batch is scheduled as soon as possible (i.e when all tasks that compose a batch are available for processing on second stage) on batching machine with minimum load.  $H_1$  gives a schedule in  $O(n \log n)$  with a performance guarantee of  $\frac{8}{3} - \frac{2}{3m}$ , where  $m = \max\{m_1, m_2\}$  and this bound is tight. Heuristic  $H_2$  constructs a list  $L$  of batches using the FBCLPT rule, then for each batch of  $L$  not yet placed on the second stage, schedules its tasks on the first stage according to LPT rule, and on the second stage the batch is scheduled on a machine with minimum load.  $H_2$  gives a schedule in  $O(n \log n)$  with a performance guarantee of  $\frac{10}{3} - \frac{4}{3m}$ . Heuristic  $H_3$  starts as  $H_2$  by constructing a list  $L$  of batches using FBCLPT rule, then the obtained batches are scheduled following johnson's algorithm by considering each stage as one machine (i.e. the processing time of each batch  $B_i$  is given as  $p_1(B_i) = \sum_{j \in B_i} p_{1,j}/m_1$  and  $p_2(B_i) = a_i(B_i)/m_1$  at first and second stage, respectively).  $H_3$  gives a schedule in  $O(n \log n)$  with performance guarantee of  $4 - \frac{2}{m}$ . We conduct an extensive computational experiments to compare these heuristics to lower bounds of problem, we observe that even the heuristic  $H_1$  is the best one from the theoretical point of view, the experiments show the superiority of heuristics  $H_2$  and  $H_3$ .

We consider the special cases  $FH2(1, m)$  and  $FH2(m, 1)$  in which a stage 1 consists of one discrete machine and a stage 2 consists of one batching machine, respectively. For the case  $FH2(1, m)$ , heuristics  $H_2$  and  $H_3$  give performance guarantee of  $\frac{7}{3} - \frac{1}{3m}$  and  $3 - \frac{1}{m}$ , respectively. For the case  $FH2(m, 1)$ ,  $H_1$  and  $H_3$  give performance guarantee of  $\frac{7}{3} - \frac{1}{3m}$  and  $3 - \frac{1}{m}$ , respectively.

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