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► **To cite this version:**

Rodolphe Charrier, Christine Bourjot, François Charpillet. Deterministic Nonlinear Modeling of Ant Algorithm with Logistic Multi-Agent System. International Conference on Autonomous Agents and Multiagent Systems - AAMAS'07, May 2007, Honolulu Hawaii, United States. pp.920-922, 10.1145/1329125.1329293 . inria-00168313

**HAL Id: inria-00168313**

**<https://inria.hal.science/inria-00168313>**

Submitted on 16 Mar 2011

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# Deterministic Nonlinear Modeling of Ant Algorithm with Logistic Multi-Agent System

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May 2007

## Abstract

Ant algorithms are one of the main programming paradigms in swarm intelligence. They are built on stochastic decision functions, which can also be found in other types of bio-inspired algorithms with the same mathematical form. However, though this modeling leads to high-performance algorithms, some phenomena, like symmetry break, are still not well understood or modeled at the ant level. This paper proposes an original analysis of the problem : we establish a reactive multi-agent system based on logistic nonlinear decision maps, and designed according to the influence-reaction scheme. Our proposition is an entirely novel approach to the mathematical foundations of ant algorithms : contrary to the current stochastic approaches, we show that an alternative deterministic model exists, which has its origin in deterministic chaos theory. The rewriting of the decision functions leads to a new way of understanding and visualizing the convergence behavior of ant algorithms. We apply our approach on a concrete example, namely the binary bridge problem.

## 1 Introduction

Bio-inspired algorithms are important in Swarm Intelligence, giving rise to many paradigms and questioning conventional artificial intelligence approaches. Intelligence is not considered only as an individual characteristic, but also as a swarm emer-

gent phenomenon. The challenge of swarm intelligence is to understand these mechanisms and build algorithms which will artificially reproduce this emergence without any global centralized controller. The paradigm of ant colony is one of the most studied and applied on optimization problems [3]. Nevertheless, convergence proofs have been recently given for these algorithms [5,7] after many years of experimentations. The stochastic approach succeeds in simulating the global phenomenon but is not sufficient to really explain specific features of ant colony mechanisms, like the symmetry break which occurs in the binary bridge experiment for example, or like the specific involved dynamics of the colony.

In this paper, we consider ant colonies as a paradigm of complex systems as Boccaro's book presents this phenomenon [1]. A complex system is a system with many interacting entities or agents. A frequent added assumption is that nonlinearities within the system are the main cause of the complexity. This paper is based on a more precise assumption: nonlinearities are within entities and govern their internal behavior. This is the root principle of the Logistic Multi-Agent System (LMAS), described in section 2. The LMAS is composed of logistic agents, that is agents with internal logistic (nonlinear) decision maps. The LMAS has thus its origin in deterministic chaos theory and is completely described in the dynamical system theory.

To demonstrate the relevance of the LMAS in simulating swarm phenomena, we resume the modelling of

the binary bridge experiment, since this experiment shows the basis of the collective choice mechanism of ant colonies. It is also the simplest instance of a graph-based ant algorithm from which all other instances derive.

## 2 The Logistic Multi-Agent System

### 2.1 System design

The logistic multi-agent system is a MAS composed of a set  $A$  of  $N$  agents and an environment  $Env$  which is the medium of all the interactions between agents. Our interaction model is indeed an entirely indirect one. The environment has a multi-field structure, needed for the indirect interaction processes, which is specific to the handled problem.  $s_i$  is the state of agent  $i$  and  $\sigma$  the state of the environment. We denote by  $t$  the time step variable. The state transition equation of the system is governed by the following dynamical system derived from the influence-reaction model of Ferber-Müller [4]:

$$\begin{cases} s_i(t+1) &= F(s_i(t), \sigma(t)) \quad \forall i \in \{1, \dots, N\} \\ \sigma(t+1) &= G(\sigma(t), s_1(t+1), \dots, s_N(t+1)) \end{cases} \quad (1)$$

The first equation expresses the fact that agents perceive their environment before changing their internal state. The second one expresses the change of the environment state through the combined influences of agents, induced by their state update. The explicit coupling between these two parts of the system makes a natural distinction between local and global level and gives the schedule of the mechanism.

### 2.2 The logistic agent model

The logistic agent is a reactive agent whose internal behavior is totally determined by the iterations of a logistic map. Its internal state  $s = \langle x, a \rangle$  includes the following variables:

- $x \in D = [0, 1] \subset \mathbb{R}$  is the decision variable.
- $a \in D$  is the internal control variable. It governs the chaotic level of the agent behavior.

The internal state transition of agents can be written as a coupled dynamical system expressed by:

$$\begin{cases} x(t+1) &= h_x(x(t), a(t), p_x(\sigma(t))) \\ a(t+1) &= h_a(p_a(\sigma(t))) \end{cases} \quad (2)$$

where the  $p_x$  and  $p_a$  components represent respectively agent perception functions for the internal state variables  $x$  and  $a$ .

$h_x$  is defined as a compound of the logistic map  $f$  and a  $\epsilon$ -coupling operator  $I$ :

$$h_x(x(t), a(t), p_x(t)) = f(I_\epsilon(x(t), p_x(t)), a(t)) \quad (3)$$

where  $\forall (u, v) \in D^2, I_\epsilon(u, v) = (1 - \epsilon)u + \epsilon v$

The meaning of  $I_\epsilon$  is relative to the level of individual interactions between agents and distinguish mass recruitment behavior from individual recruitment behavior in ant modeling. In this latter case,  $\epsilon$  is a shared characteristic of the whole colony.

### 2.3 The logistic map

The logistic map is a well known polynomial mapping, very simple to compute but very complex in the produced sequences. We use a particular form

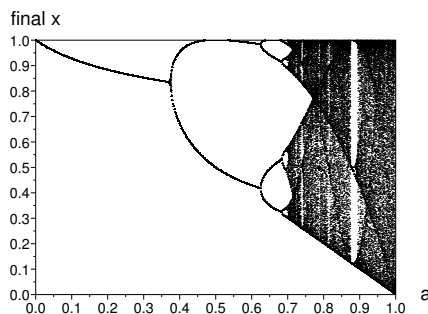


Figure 1: Bifurcation diagram for the logistic map  $x_{n+1} = 1 - a(1 - 2x_n^2)$  over  $[0, 1]$  with 500 iterations.

of the logistic map whose characteristics are summarized in its bifurcation diagram in fig.1. The logistic map for a parameter  $a$  is the following recursive relation :

$$x_{n+1} = f(x_n, a) = 1 - a(1 - 2x_n^2) = f^{n+1}(x_0, a) \quad (4)$$

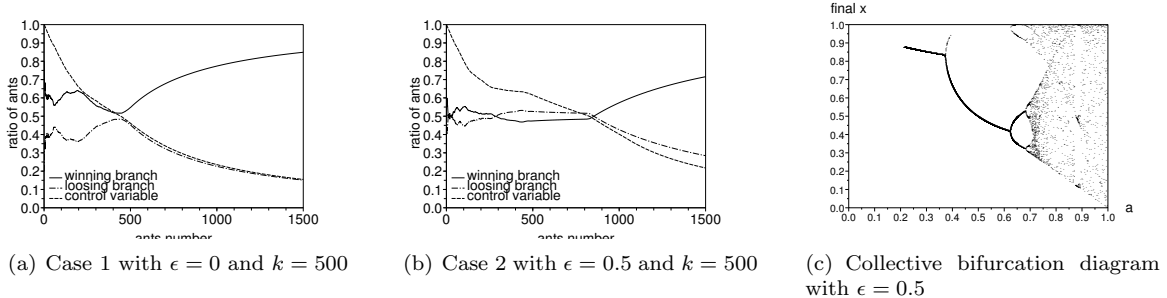


Figure 2: Simulation results of the binary bridge : mean curves over 10 runs with 1500 ants

$x_n$  is the iteration variable after  $n$  iterations of  $f$  from the initial value  $x_0$ . For a given  $a \in [0, 1]$ , the interval  $[0, 1]$  is stable through  $f$ . The bifurcation diagram shows the limit values of  $\lim_{n \rightarrow \infty} x_n$  according to the values of the parameter  $a$  in  $[0, 1]$ . In practice, hundreds or thousands of iterations are needed to get a good view of the diagram. It reveals by numerical means, the fixed points, cycles, bifurcation points and chaos areas (dark bands) of the map (for further information, see for example [6]). When  $a = 1$ , chaos is complete, that is the map corresponds statistically to a random sequence between 0 and 1. This specific behavior from complete randomness to complete order according to a single parameter is used in LMAS for generating internal agent states and consequently agent behaviors. It is the root principle of the model.

### 3 A concrete ant problem: the binary bridge

#### 3.1 Original model

Deneubourg and al. published their experiment and mathematical model in [2] explaining the trail following behavior of an argentine ant species when foraging. They studied the self-organization process that achieves the collective selection of a path to a food source. More precisely, the experimental protocol was made of a binary bridge  $A$  and  $B$  connecting the ant-nest to the food source. Along the way to the food source, and while returning back to the nest,

they lay down a constant amount of pheromone. The underlying assumptions are: the bridge has got two equal long branches, the pheromone does not evaporate, and the amount of pheromone on a bridge is proportional to the number of ants crossing the bridge one by one. The experiments are of short duration so that the amount of pheromone evaporation is negligible.

This leads to the following model [2]: we denote by  $A_i$  (respectively  $B_i$ ) the number of ants choosing the branch  $A$  (respectively  $B$ ) after the  $i$ th ant crosses the bridge. It represents the amount of pheromone on each branch as well. Let  $P_A$  (respectively  $P_B$ ) be the probability the  $i+1$ th ant chooses  $A$  (respectively  $B$ ).  $P_A$  is expressed by:

$$P_A = 1 - P_B = \frac{(K + A_i)^n}{(K + A_i)^n + (K + B_i)^n} \quad (5)$$

The best fit to the experimental data is given with parameter values  $K = 20$  and  $n = 2$ . This stochastic model produces expected results : all the ants end up choosing the same branch after a while, called the winning branch, even if neither branch is favored *a priori*. One can wonder what is the cause of the global symmetry break at the local level of ants. In the following, we will show how the LMAS implements the binary bridge problem and why it is relevant to analyse the symmetry break.

### 3.2 LMAS implementation

As stated before, the time step of the problem corresponds to the event of an agent crossing the binary bridge. The environment is a small graph, composed of only three nodes and two edges: one of the nodes is the nest and the others correspond to the binary bridge destinations  $A$  and  $B$ . We will distinguish in our model two types of agent influences on the environment, each relating to an indirect type of interaction: a pheromone deposit on the edges of the graph, stored in the field  $\tau$ , has a long range interaction effect, whereas, an internal state deposit on the current node, stored in the field  $X$  has a short range interaction effect. We describe the algorithm adapted to the current problem in terms of perception-decision-action scheme:

- Perception. An agent perceives the amount of pheromone, positions itself toward the bridge with this maximum perceived value of pheromone; the agent perceives as well the internal value of the preceding agent on the current node. At time  $t$  for an agent  $i$ , the perception functions and the agent state variables transitions are therefore:

$$\begin{cases} p_x(t) = X_i(t) = x_{i-1}(t) \\ p_a(t) = \max\{\tau_a, \tau_b\} \\ a_i(t+1) = \frac{\alpha}{\alpha + p_a(t)} \\ x_i(t+1) = f^k((1-\epsilon)x_i(t) + \epsilon x_{i-1}(t)) \end{cases} \quad (6)$$

where  $\alpha$  is a constant parameter of the model, which controls the initial exploration period of the agent, and  $k$  is the iteration number for computing the logistic map ( $\alpha \approx 200$  and  $k \approx 500$  in the simulation).

- Decision. If  $x_i(t+1) > \frac{1}{2}$  choose the branch with  $\tau_{max}$  else choose the other. This rule is relative to the branch where the pheromone is maximal.
- Action. Agent actions consist in the agent move to the chosen branch, in storing then its own internal  $x$  variable on the current node, and finally in updating the field pheromone. Only the amount of pheromone of the chosen edge is updated in a cumulative process without evapora-

tion as stated in the binary bridge experiment:  
 $\tau_{chosen}(t+1) = \tau_{chosen}(t) + x_i(t+1)$

- the initial conditions correspond to the experimental ones: the initial amount of pheromone is null and the initial control variable is close to 1 for each agent, that is the random behavior is the default behavior.

### 3.3 Results and discussion

The results of the above implemented algorithm can be analyzed in figure 2. On the same graph, the ratio of ants traversing the branch averaged over ten runs is plotted for the winning and loosing branch as well as the control variable average over these ten runs. In the cases 2(a) and 2(b), one can notice that the symmetry break occurs exactly when the control variable reaches the value 0.5, in other words when final computed  $x$  are always above the decision threshold. The variable  $a$  decreases as the ant number increases because of the formula (6). Before reaching  $a = 0.5$ , both branches are explored, implying that the agent system oscillates quite identically between values under and above 0.5. The ants ratio curves in the graph 2(a) fits well with the experimental data curves [2], even if this comparison remains a qualitative one. The graph 2(b) shows a long term symmetry break in a mixed recruitment case ( $\epsilon = 0.5$ ), which fits well with experimental observations on particular ant species. An explanation of this effect appears by considering the bifurcation diagram (2(c)). This diagram is not an individual one like the diagram in fig.(1), but a collective one: we have plotted every  $(x, a)$  pair of values for each agent crossing the bridge at time  $t$ . The graph is to read from right to left because  $a$  is decreasing in time, and it demonstrates that the system only selects the lower branch of the diagram, which delays the symmetry break. The long term symmetry break is also caused by the control variable  $a$  which levels off at the bifurcation points.

## 4 Conclusion

The most salient contribution of this paper is the use of the same deterministic nonlinear logistic maps within the internal state of the agents, which provides theoretical tools and visualization tools to follow the dynamics of the agent system. The internal control variable is a novel feature as well. It allows the distribution of control inside agents without removing their autonomy. We have thus defined a new MAS called the Logistic MAS, specially designed for swarm intelligence. The results of this model on the binary bridge problem demonstrate that our approach is relevant in terms of collective behavior analysis and visualization of the whole dynamics. However, this specific problem is not really considered as an optimization problem. Future work consists therefore of a more detailed study of optimization problems. Our first tests on small instances of the symmetric TSP show that this is a promising application of the model. We also intend to exploit this model to build other simulations, especially of biological phenomena, so as to show its general applicability.

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