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Discoideum

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Decentralised grouping on a discrete field:
preliminary study of a bio-inspired model from
Dictyostelium Discoideum

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Abstract

We address the question to know which are the minimal ingredients to obtain a decentralised grouping of agents that move on grid according to discrete local rules. The agents and their environment are described with a stochastic discrete model inspired from an example taken in biology: the aggregation of the cellular slime mold *Dictyostelium Discoideum*. The environment transmits information according to a reaction-diffusion mechanism and the agents move by following excitation fronts. We show that despite its simplicity the model exhibits interesting properties of self-organisation. Moreover, first observations show that the system is robust to introduction of noise on the moves of the amoebae and robust to the introduction of obstacles on the grid.

keywords:

bio-inspired modeling, cellular automata, multi-agent systems, self-organisation, aggregation, reaction-diffusion, cellular slime mold, *Dictyostelium Discoideum*, phase transitions.

1 Introduction

1.1 Decentralised Grouping

The purpose of this paper is to consider the problem of forming a group by using only local rules. Imagine a group of real or virtual robots that move on a grid and need to group to form a compact pattern. The robots, all identical, have no idea of their own position, nor do they have on the position of the other robots. All they can do is send messages which can be relayed, possibly with

errors, by the cells of the grid. The robots have limited perception and actions: they can only perceive the state of the neighbouring cells and can only move to these cells.

The main question is to determine what are the minimal ingredients involved to achieve a decentralised grouping with these constraints. More precisely, we wish to know whether we can produce a simple discrete model that will be able to achieve this grouping by using a small number of states for the propagation of messages and a small number of rules for controlling the moves of the robots.

This problem is somehow similar to the leader election problems [10] where all cells are initially in the same state and where the goal is to attain a configuration in which a single cell is in a particular distinguished state. In our problem, the agents are initially in a random location and the objective is to make them move until they form a cluster. In both problems, the difficulty stems from the fact that the information has to be transmitted only by local rules. The initial random dispersion of the agents and the introduction of noise in the system also relates our work to the *amorphous computing* paradigm [1].

1.2 Biological Inspiration

The cellular slime mold *Dictyostelium Discoideum* is a fascinating organism that has the ability to live as a mono-cellular organism and to transform into a multi-cellular organism when needed. In normal conditions, the cells live as single individuals by eating decaying logs, humus and bacteria (*e.g.*, [5]). However, when the environment becomes depleted of food, a grouping phenomenon is triggered and single cells aggregate to form a complex organism that will move and react with coordinated changes. The transformation from a group of individual amoebae into a multi-cellular aggregate is a complex phenomenon that involves different stages. In this work, we only take inspiration from the first stage of the multi-cellular organisation process, the aggregation stage, which consists in the grouping of all the cells at the same place.

Observations of *in vitro* experiments show that this aggregation is caused by the spontaneous emergence of “pacemakers” or “signalling centres” (*e.g.*, [6]). These pacemakers are formed by groups of cells that attract other cells that are located in their vicinity. Once the first pacemakers are formed, they are in a meta-stable situation: under normal conditions, they struggle one against another and merge until they ultimately form one single signalling centre that is neither “too big” nor “too small”.

Our proposition is to take the essential ingredients of this aggregation mechanism to propose a first solution to the Decentralised Grouping Problem.

2 Existing models

It is out of scope of this article to draw a precise review of the models that have been proposed to study the dynamics of *Dictyostelium D*. We will here simply cite some works as entry points to the literature on the subject.

Among the first models proposed to model chemotaxis, the proposition of Keller and Segel [11] inspired many works on *Dictyostelium D.* Many other models followed and one of the most authoritative reference was the proposition of Martiel and Goldbeter [13] from which numerous other models were derived. The question of knowing if whether a single cell is sufficient for triggering the whole aggregation process was examined in [6]. However, the model produced is composed of no less than forty variables or functions and this makes it difficult to understand for the non-specialist.

The techniques of modeling evolved, along with computation power and impressive simulation results were obtained in the years 1990. We may refer to the work by Weijer *et al.* (*e.g.*, [14]) or Marée *et al.* (*e.g.*, [12]) to see examples of successful modelling by using partial differential equations.

The greatest part of the literature is composed of continuous models. To our knowledge, no fully discrete model was proposed for capturing *Dictyostelium D.*'s behaviour. As far as the environment is concerned, the reaction-diffusion process is well understood and the links between the discrete and continuous models can be explicitly formulated (*e.g.*, [16]). The computing complexity of such models have also been studied : Golès and Matamala have shown that three states are sufficient for obtaining universality of the model [9] ; Adamatzky studied some problem-solving abilities of reaction-diffusion models [2].

3 Our model

Our model belongs to the category of stochastic discrete dynamical systems as time, space and state are discrete. It is composed of two “layers”: the first layer is a cellular automaton that models a reaction-diffusion process. The second layer describes the moves of virtual amoebae (or simply *amoebae* in the following).

3.1 The environment layer

Space is modelled by a regular grid $\mathcal{L} = \{1, \dots, X\} \times \{1, \dots, Y\}$ in which each cell $c = (c_x, c_y) \in \mathcal{L}$ is associated to a state. The set of possible states for each cell is $Q = \{0, \dots, M\}$, the state of cell c at time t is $\sigma_c(t)$.

The state 0 is the *neutral* state, the state M is the *excited* state. A cell may evolve from the neutral state to the excited state if at least one of its neighbours is excited (R1). To model the uncertainty on this transition, we will consider that it happens with a given probability p_T , called *excitability*. The states 1 to $M - 1$ are the *refractory states*. A cell in a refractory state evolves in an autonomous way by decrementing its state of 1 (R2) until it reaches the neutral state. A neutral cell surrounded by neutral cells stays neutral (R3). In an abuse of language, we will call neutral or excited, refractory, a cell of a given configuration that is respectively in the neutral, excited, or refractory state.

Formally, for a cell $c \in \mathcal{L}$, let us denote by $N(c)$ the neighbourhood of this cell, it is supposed to be fixed over time. Let $E_c(t)$ be the set of excited cells in

the neighbourhood of c at time t : $E_c(t) = \{c' \in N(c), \sigma_{c'}(t) = M\}$.

To model the stochastic phenomena, let us denote by $\mathcal{B}(p)$ the Bernoulli probabilistic function that takes the value 1 with probability p and the value 0 with probability $1 - p$. The rules governing the environment can then be written:

$$\sigma_c(t+1) = \begin{cases} M & \text{if } \sigma_c(t) = 0 \text{ and } |E_c(t)| > 0 \text{ and } \mathcal{B}(p_T) = 1 & (R1) \\ \sigma_c(t) - 1 & \text{if } \sigma_c(t) \in [1 \dots M] & (R2) \\ 0 & \text{otherwise} & (R3) \end{cases}$$

A series of adjacent cells all that are in the excited state M is called an *excitation front*.

3.2 The agent layer

The amoebae are supposed to be in constant number, let K be their number. The position of the k -th amoeba at time t is represented by $P_k(t) \in \mathcal{L}$. For a cell $c \in \mathcal{L}$, let $A_c(t)$ be the number of amoebae located on this cell. We arbitrarily allow at most two amoebae to share a single cell. A cell which has less than two amoebae on it is *free* cell.

At each time step, an amoeba can either:

- move to an adjacent free cell by following an excitation front (R4),
- move randomly to an adjacent free cell (R5),
- remain on the same cell (R6).

In order to move towards an excitation front, an amoeba has to select one cell among the excited free cells in its neighbourhood. In a first step, we will consider that this choice is made at random uniformly. However, this mechanism can be refined, for example by assigning smaller weights to diagonal cells in order to overcome discretisation effects.

Let $\Sigma_k(t) = \sigma_{P_k(t)}(t)$ be the state of the cell on which the k -th amoeba is located. Let $\tilde{N}_k(t)$, respectively $\tilde{F}_k(t)$, be the set of free cells, respectively excited free cells, in the neighbourhood of k -th amoeba. For a finite set S , we denote by $\mathcal{R}(S)$ the operation of selecting one element in S with uniform probability. The moves of the amoebae are described by the following rules:

$$P_k(t+1) = \begin{cases} \mathcal{R}[\tilde{F}_k(t)] & \text{if } |\tilde{F}_k(t)| > 0 \text{ and } \Sigma_k(t) = 0 & (R4) \\ \mathcal{R}[\tilde{N}_k(t)] & \text{if } |\tilde{F}_k(t)| = 0 \text{ and } \mathcal{B}(p_R) = 1 & (R5) \\ P_k(t) & \text{otherwise} & (R6) \end{cases}$$

The rule (R4) models moves towards an excitation front or *chemotaxis*: it can only occur by a transition from a neutral cell to an excited cell. The rules (R5) and (R6) model the possible application of a noise on the moves with probability p_R , called the *agitation rate*.

3.3 Coupling of environment and agents

Amoebae act on the environment by emitting excitations with probability p_E called the *emitting rate*. This interaction is modelled by the following rule:

$$\Sigma_k(t+1) = M \quad \text{if } \Sigma_k(t) = 0 \text{ and } \mathcal{B}(p_E) = 1 \quad (R7)$$

As rules (R1) and (R7) may interfere, we need to clear this ambiguity of formulation. A first possibility consists in according priority to rule (R7) over rule (R1). A second possibility, that better respects the cellular automata paradigm, which is cell-centred, is to re-write rule (R1) by rule (R1'):

$$\begin{aligned} \sigma_c(t+1) = M & \quad \text{if } \sigma_c(t) = 0 \\ & \quad \text{and } [E_c(t) > 0 \text{ or } (A_c(t) > 0 \text{ and } \mathcal{B}(p_E) = 1)] \\ & \quad \text{and } \mathcal{B}(p_T) = 1 \end{aligned} \quad (R1')$$

where $A_c(t)$ gives the number of amoebae on cell c at time t .

We wish to call the reader's attention on the fact that our description does not specify any type of neighbourhood. In the following, we will arbitrarily use the eight-cell neighbourhood, *i.e.*: $N(c) = \{c' \in U, \max\{|c'_x - c_x|, |c'_y - c_y|\} = 1\}$. Note that it is not strictly equivalent to the Moore neighbourhood as : a) each cell is excluded from its own neighbourhood b) cells in the border have a smaller neighbourhood. We also arbitrarily set the excitation level to $M = 4$; the set of states Q is thus composed of 5 states.

4 Small number of agents

As our purpose is to examine the behaviour from a qualitative point of view, we begin our study by simulating the evolution of a system containing a small number of amoebae.

4.1 Fully deterministic case

For the first experiment, we begin by examining what happens to a system composed of two amoebae. The emitting rate and the excitability are set to 1, random moves are forbidden: $(p_T, p_E, p_R) = (1, 1, 0)$. Figure 1 shows the evolution of the system for these settings.

We observe that no move ever occurs as excitation fronts collide with each other. Contrarily to classical diffusion waves, it is a well-known phenomenon that reaction-diffusion fronts annihilate when they meet. This property is respected by our model. The effect of collision implies that the transmission of information from an emitting cell to other cells is limited only by other emitting cells. Let us define informally an *influence region* of an emitting cell as the set of cells that will receive the information emitted by this cell. Intuitively, we see that in the fully deterministic case with k amoebae, the regions of influence of the amoebae correspond to the discrete Voronoi diagram of the grid with amoebae as centre points (*e.g.*, [2]).

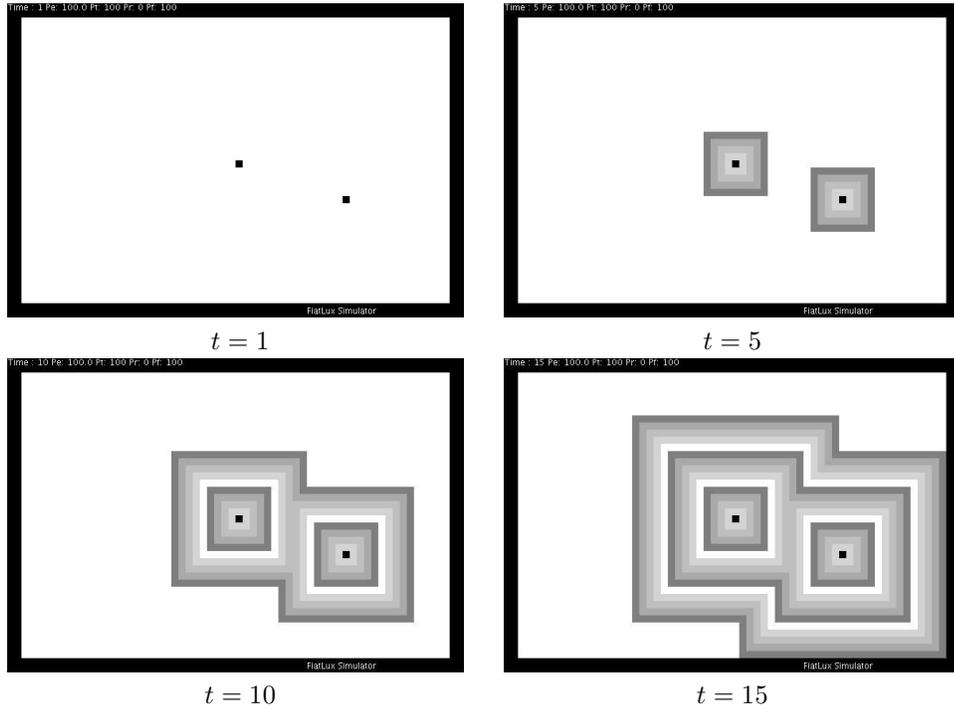


Figure 1: Four views of the evolution of the model with two amoebae in the fully deterministic case: $p_T = 1$, $p_E = 1$, $p_R = 0$. Amoebae are represented by black squares, white squares are neutral cells, darkest grey squares are excited cells, other grey squares are refractory cells. This colour code is kept in the following. All simulations are made with the *FiatLux* CA-simulator [7].

4.2 Transmission is no longer perfect

We now set the excitability to $p_T = 0.90$, *i.e.*, we introduce a 10% chance that a unit fails to transmit the excitation to its neighbours. The evolution of the system for these settings is shown on Figure 2.

We see that for small times the system behaves qualitatively in the same way as for $p_T = 1$. However, as time advances and transmission errors accumulate, the waves become more round. After a few hundred steps, stable spirals appear and become independent of the waves issued from the amoebae. As p_T decreases, the situation becomes more confused, and the waves lose their coherent aspect even more rapidly. For example, for $p_T = 0.80$, the loss of coherence is observed only after $T = 200$ steps. We call this behaviour where waves can develop independently of the position of the amoebae, the *non-coherent regime*.

When excitability is small enough, *i.e.*, for $p_T < 0.20$, we observe that the wave are no longer persistent and the probability of extinction of excitation fronts become important. We call this regime the *extinction regime*. The transition from the non-coherent regime to the extinction regime is abrupt and is located around $p_T \sim 0.23$. This abrupt change is further studied in the next paragraph.

4.3 Quick study of the transition between extinct and non-coherent regimes

In order to examine how the model behaves when excitability p_T is varied, we conducted two experiments to monitor the evolution of the density of excited cells $e(t) = \{c \in \mathcal{L}, \sigma_c(t) = M\}/X.Y$.

We took a grid of size $(100, 100)$ with no amoebae on it. The system was initialised by setting cells with an arbitrary random probability of 10% to the *excited* state, leaving all the other cells to the neutral state. Excitability was varied by 1% steps from $p_T = 0.01$ to $p_T = 1$, and the evolution of the system was monitored during $T = 10^4$ time steps. For each value of p_T , this experiment was repeated 10 times and for each sample, we computed the average value of $e(t)$ for $t \in [10^3, 10^4]$.

The average, the minimum and maximum values of $e(t)$ observed in the 10-sized sample are plotted in Fig.3. The shape of the curve suggests that the change of behaviour is a second-order phase transition that occurs for $p_T \sim 0.23$. It is interesting to make an analogy with the observations of second order phase-transitions in asynchronous cellular automata for which it was shown that the effect of changing the synchrony rate could also produce this type of curves [3, 8].

In order to analyse this change near criticality, we monitored the evolution of $e(t)$ for the three excitability values $p_T = 0.228$, $p_T = 0.230$ and $p_T = 0.232$. The average value of $e(t)$ was obtained by using 100 samples and for a grid size $(X, Y) = (200, 200)$. As the extinct regime can be characterised by $e(t) = 0$, we also plotted the fraction of samples which attained this regime at time t . Again, the results presented in Fig. 4 are strongly similar to those observed in

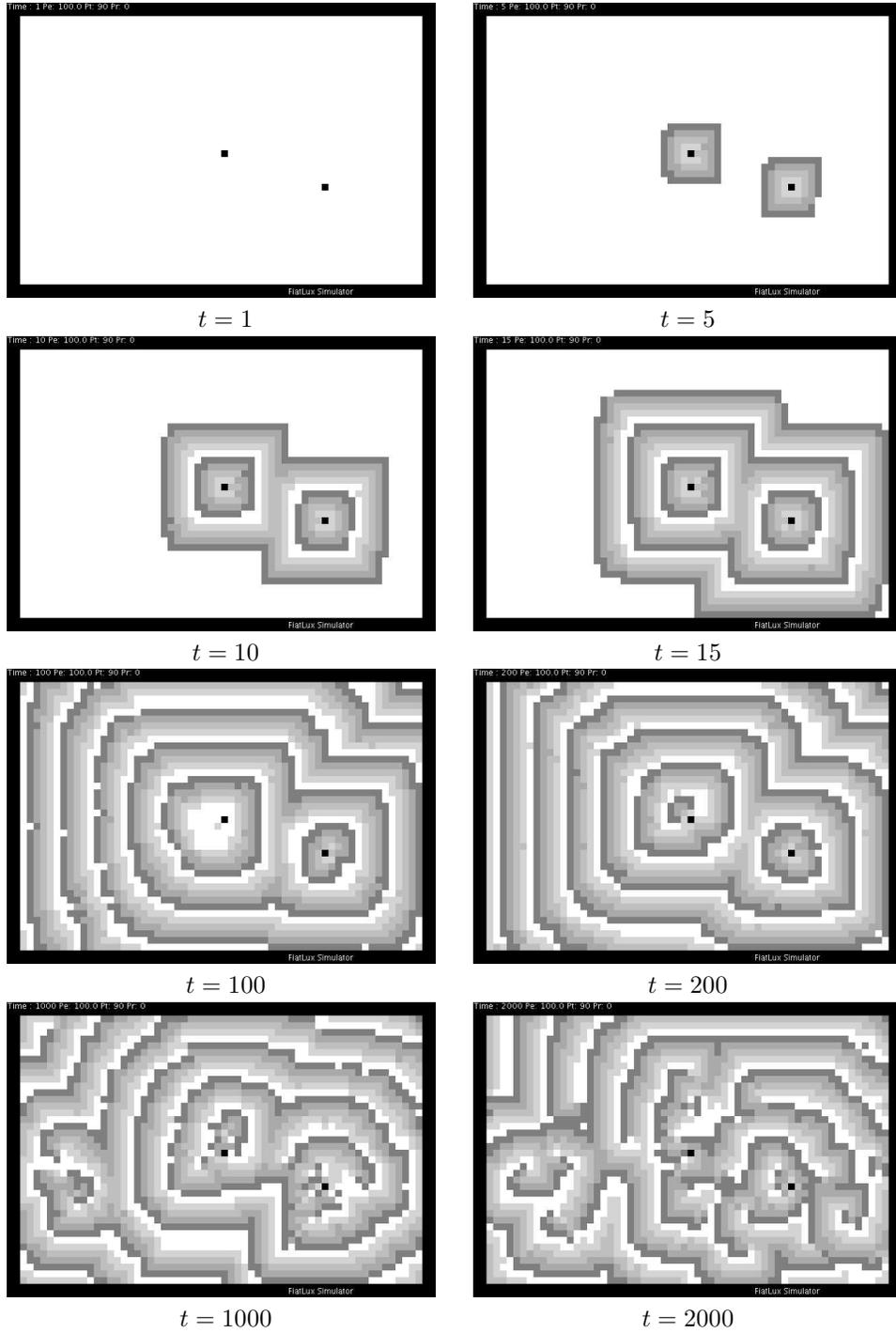


Figure 2: Evolution of the model with non-perfect transmission rate: $p_T = 0.90$, $p_E = 1$, $p_R = 0.10$.

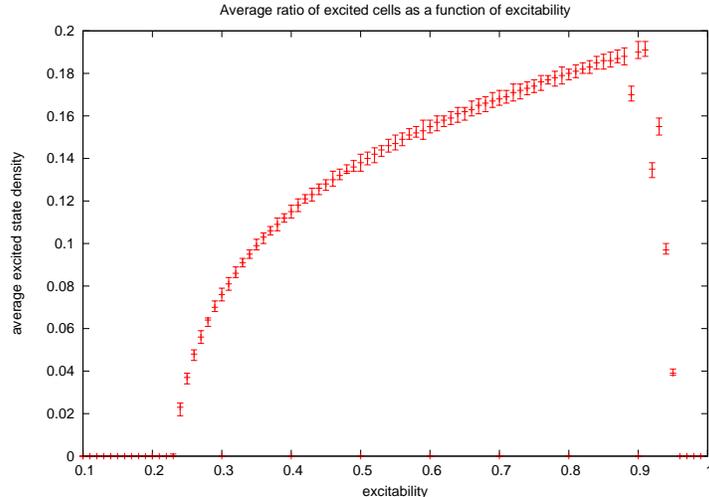


Figure 3: Evolution of the reaction-diffusion part for a $(100, 100)$ grid size, 10 samples for averaging and time interval $t \in [10^3, 10^4]$. Bars around each plot shows the minimal value and the maximum of the 10 samples.

asynchronous ECA as the evolution of $e(t)$ near the critical point is close to a power-law in the time interval $[10, 10^3]$.

Of course, these experiments have to be conducted with more precision to be able to identify with no doubt the nature of this abrupt change. A more detailed study of the robustness to the variation of excitability p_T is left for future work, and in the following we will only consider the case of perfect transmission, *i.e.*, with $p_T = 1$.

4.4 “Pacemaker” formation

In order to study what happens in larger groups, we conducted the following experiment. We took a $(X, Y) = (30, 20)$ grid size, each cell initially having a 10% probability to contain an amoeba. The parameters of the model are set to $(p_T, p_E, p_R) = (1, 0.01, 0)$.

Figure 5 shows the evolution of the system for these conditions. We see that at first, randomly chosen cells emit an excitation wave which will attract other non-emitting cells to it. The repetition of the process creates more populated regions of the grid, and by a positive feedback effect, these regions will have a greater probability to emit waves. At the end, one stable compact group is formed. We see that this group emits waves with good regularity (although not perfect). By analogy with the biological term [6], we name this emerging groups *pacemakers*.

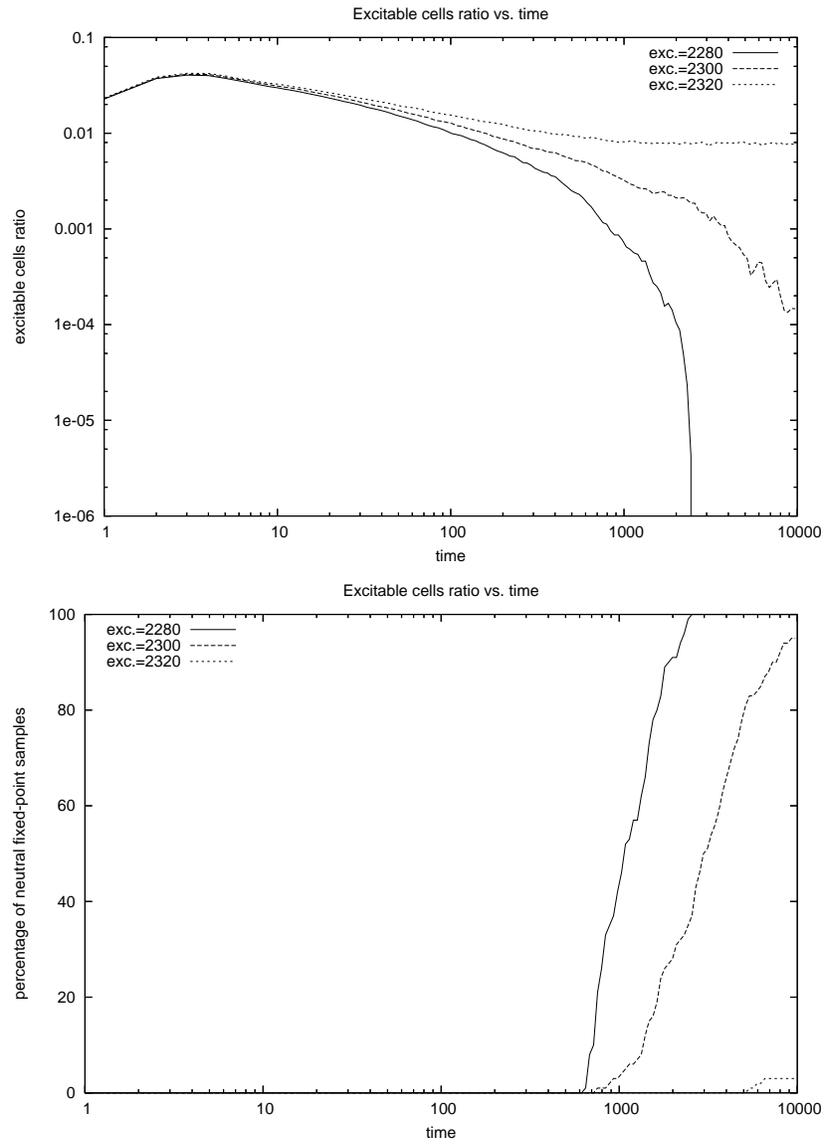


Figure 4: Average evolution of the system of array size is $(X, Y) = (200, 200)$ and 100 samples : (top) Ratio of excited cells as a function of time for different values of excitability. (bottom) Ratio (in %) of runs that have reached the neutral global state as a function of time.

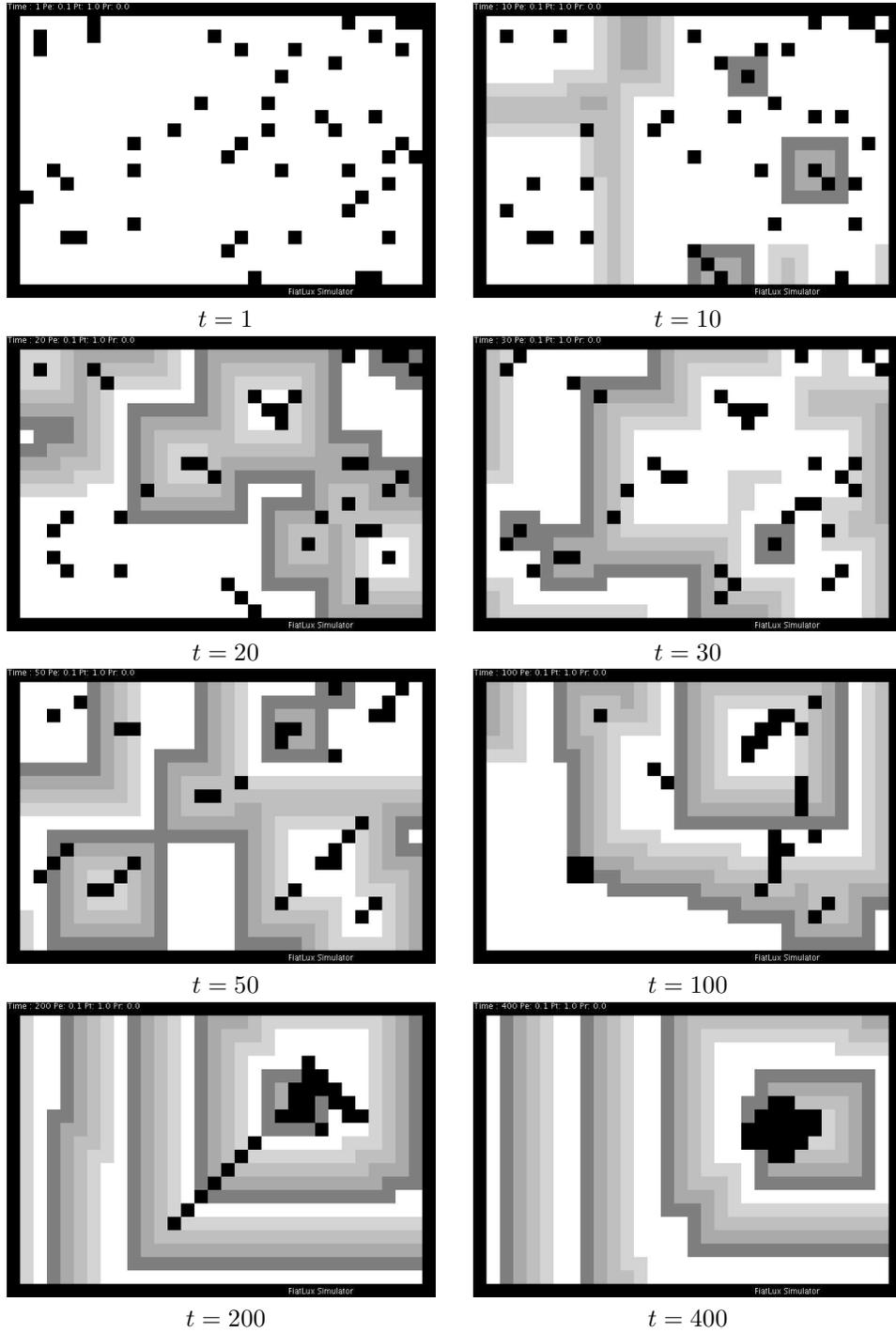


Figure 5: Sequence showing the formation of a “pacemaker” with $(p_T, p_E, p_R) = (1, 0.01, 0)$ and $(X, Y) = (30, 20)$.

5 Large number of agents

This first qualitative exploration of the different behaviour of the model showed us that four type of behaviours were found: static, non-coherent, extinct and, last but not least, self-organising.

In the following we examine how the system behaves for a larger number of amoebae. The grid size is fixed to $(X, Y) = (300, 200)$ and the initial condition is random configuration where all the cells are neutral and where each cell has a 10% probability to contain an amoeba. The average number of amoebae is thus $0.10XY = 6000$. The main question will be to examine how different pacemakers coexist and if the dynamics further evolves.

5.1 Self-organisation with no agitation

As an emitting rate of $p_E = 1$ implies a static system (see Sec. 4.1), we need to lower the value of p_E in order to observe the formation of groups. For example, if we take $p_E = 0.90$, small groups start to appear and these group progressively merge but the dynamics is slow and no major change occurs between $T = 10^4$ and $T = 2 \cdot 10^4$.

However, if we take p_E small enough, it becomes possible to observe groups that progressively merge until they form one single group with all the amoebae in it. For example, for $p_E = 0.1$, it took $T = 1.3 \cdot 10^4$ for the system to merge totally (see Fig. 6). Observing simulations, we notice that some groups are sometimes destroyed and reformed. There are also rare cases where two groups divide into two, one part being attracted by a group the other part by another group. Looking at Fig. 6, one might notice that the streams are orthogonal and follow the four diagonal axis. As mentioned earlier, we believe that this effect is due to the choice of the neighbourhood and could be overcome by choosing a circular neighbourhood [15] or by assigning different weights to different neighbours.

The self-organisation into one single final pacemaker is a result of captures of one pacemaker by another pacemakers, or possibly two. When two pacemakers compete for extending their influence region, we observe that the bigger one has a tendency to win. This can be explained rather easily by noticing that the bigger the size of a group of a pacemaker, the larger the probability to emit a wave. So all happens as if each pacemaker had an “average frequency of emission” that separates two departures of waves. The effect of these regular emission of waves is to extend or shrink influence regions. Indeed, for two neighbouring pacemakers having two different average frequencies of emission, we intuitively perceive that the group that has the highest frequency sees its influence region extend while the other groups sees its influence region shrink.

Once a pacemaker has its influence region disappeared, the cells become under the influence of another pacemaker and they converge towards this pacemaker by forming streams. It is interesting to note that these streams are also observed in *in vitro* experiments on *Dictyostelium D.*

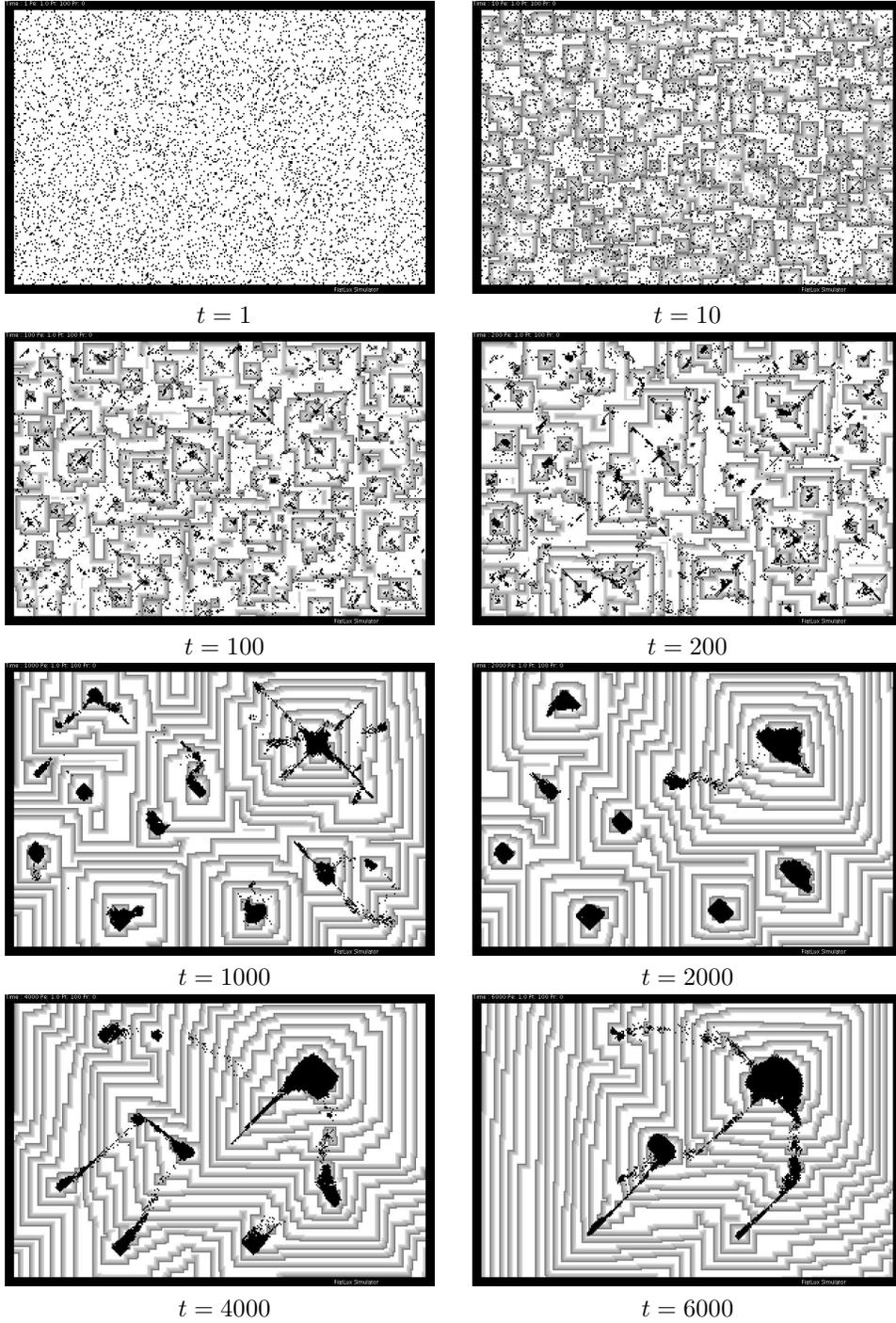


Figure 6: Evolution of the model with perfect transmission rate and no agitation:
 $(p_T, p_E, p_R) = (1, 0.01, 0)$.

Table 1: Synthetic view of the qualitative behaviours observed by simulating the aggregation model

| p_T | p_E | p_R | qualitative behaviour |
|-------|-------|-------|-------------------------|
| 1 | 1 | 0 | static |
| 1 | 0.9 | 0 | self-organising (slow) |
| 1 | 0.1 | 0 | self-organising (quick) |
| 1 | 0.1 | 0.01 | self-organising (quick) |
| 0.9 | 0.1 | 0 | non-coherent |
| <0.23 | 0.1 | 0 | extinct |

5.2 Self-organisation with agitation

In order to examine whether the system is robust to introduction of noise in the moves of the amoebae, we observed the behaviour of the system with $(p_T, p_E, p_R) = (1, 0.01, 0.1)$, *i.e.*, we set the agitation rate to 10%.

Figure 7 shows one evolution of the system for these settings. It took $T = 3 \cdot 10^4$ time steps for the system to group into a compact pattern. Aggregation takes longer than when no noise is present but qualitatively the system behaves the same way. The most noticeable difference is that groups of small size do not appear. This tends to show that there exists a link between the apparition of groups a given size and the parameter p_R . Intuitively, we presume that the noise have a tendency to destroy groups of a small size. Finding a relationship between the minimal group size (if such a property exists) and the quantity of noise is another interesting question that arises from these observations.

5.3 Obstacles on the grid

In Fig. 8, we present the evolution of the system where obstacles (straight lines) are placed randomly on the grid. Obstacles do not allow information nor amoebae to cross.

We notice that the system behaves very well in presence of obstacles. In the parts that are totally disconnected from the rest of the grid, an isolated group is formed. In each covey's part of the grid, we can see that the dynamics of the aggregation is not perturbed by the presence of obstacles. In particular, in some parts of the grid it is possible to observe the amoebae taking narrow spaces to converge to a pacemaker. In this case, the streams formed are thin and they usually split and re-form several times before attaining a pacemaker.

6 Conclusion and perspectives

The model we proposed to solve the Decentralised Grouping Problem is a simple model inspired from the aggregation phenomenon in *Dictyostelium D*. It uses a simplified reaction-diffusion mechanism to propagate information and one principal law to direct the moves of the virtual robots. These laws used three

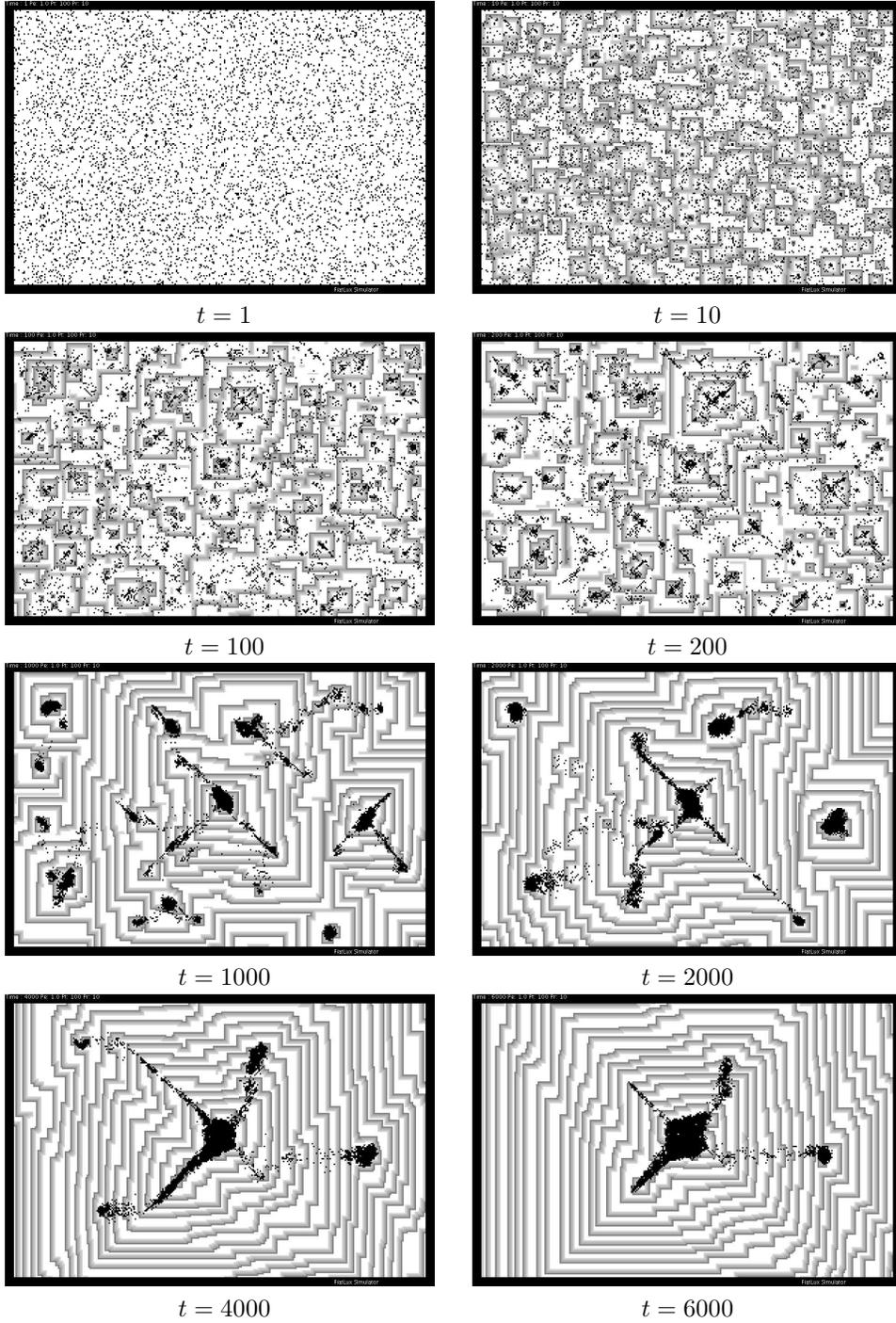


Figure 7: Evolution of the model with perfect transmission rate and small agitation: $(p_T, p_E, p_R) = (1, 0.01, 0.1)$.

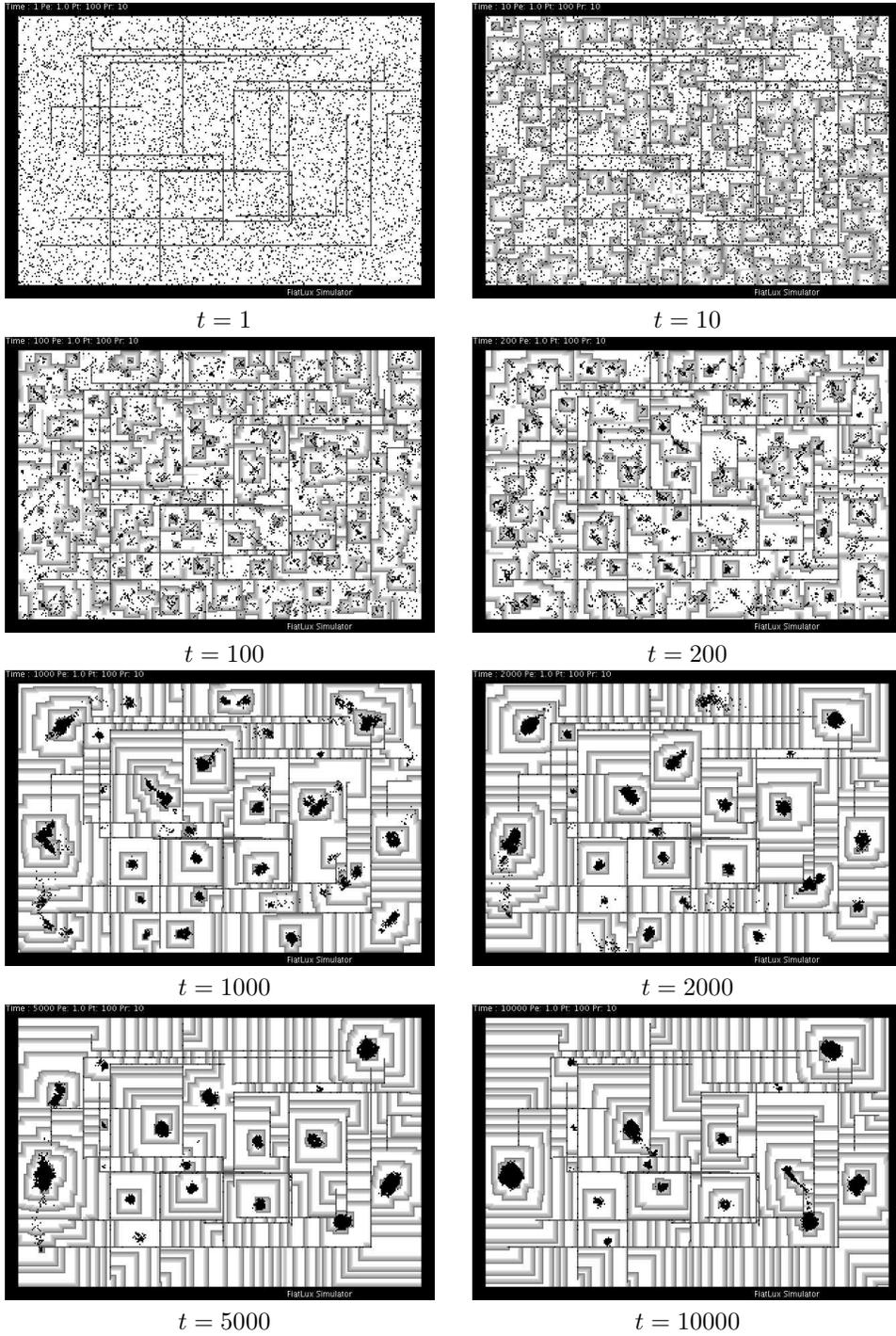


Figure 8: Evolution of the model with agitation and obstacles: $(p_T, p_E, p_R) = (1, 0.01, 0.10)$.

probabilities and the global behaviour of the system was observed according to the values of these three parameters.

The first set of observations suggests that the dynamics of the system is rich, showing different qualitative behaviours among which self-organisation (see 1). The grouping phenomenon, far from trivial, showed a progressive merging of groups, from small groups to large groups. The complexity of this hierarchical dynamics resulted from different emerging behaviours:

1. formation of waves,
2. formation of first “pacemakers”,
3. extension and shrinking of the regions according to their respective size,
4. captures of small groups by little groups.

Among interesting properties observed in the system, it was shown that the grouping could also occur in the presence of obstacles as the virtual amoebae could take advantage of narrow corridors to find their way too an attracting centre.

The development of this work consists in finding an adequate quantitative description of the dynamics of the system. This task will probably require different level of description. For example, we may at first try to gain a better understanding of the wave propagation, in particular to characterise the separation between the extinct and non-coherent regime. In a second step, we might need to study the dynamics of pacemakers, relating their size to their average frequency of emission, and this frequency to the evolution of influence region. The very notions of “pacemaker” or “influence region”, used here informally, also need to be formalised and techniques coming from computer imagery may surely be of great help. Finally, in order to be able to apply these results in real-world situations, it is necessary to study in detail the robustness of the model. Like other bio-inspired models such as virtual ants (*e.g.*, [4]), possible applications may be found in the fields of self-assembling, robots grouping or various problem solving such as data clustering.

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