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# A Definition and a Formalization of Conservative Adaptation for Knowledge-Intensive Case-Based Reasoning Application to Decision Support in Oncology (A Preliminary Report)

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## Abstract

Case-based reasoning aims at solving a problem by the adaptation of the solution of an already solved problem that has been retrieved in a case base. This paper defines an approach to adaptation called conservative adaptation; it consists in keeping as much as possible from the solution to be adapted, while being consistent with the domain knowledge. This idea can be related to the theory of revision: the revision of an old knowledge base by a new one consists in making a minimal change on the former, while being consistent with the latter. This leads to a formalization of conservative adaptation based on a revision operator in propositional logic. Then, this theory of conservative adaptation is confronted to an application of case-based decision support to oncology: a problem of this application is the description of a patient ill with breast cancer, and a solution, the therapeutic recommendation for this patient. Examples of adaptations that have actually been performed by experts and that can be captured by conservative adaptation are presented. These examples show a way of adapting contraindicated treatment recommendations, treatment recommendations that cannot be applied, and recommendations of ineffective treatments. Finally, several related issues are studied, in particular, the issue of a retrieval process well-suited for conservative adaptation and the issue of case combination following a multiple case retrieval.

**Keywords:** case-based reasoning, knowledge-intensive case-based reasoning, adaptation, conservative adaptation, theory of revision, logical representation of cases, application to oncology

## 1 Introduction

Case-based reasoning (CBR [Riesbeck and Schank, 1989; Kolodner, 1993]) aims at solving a new problem thanks to a set of already solved problems. The new problem is called the *target problem*, denoted by  $\text{tgt}$  in this paper, and the already solved problems are the *source problems*, denoted by  $\text{srce}$ . A case is the representation of a problem-solving episode, that is, at least a problem  $\text{pb}$  and a solution  $\text{Sol}(\text{pb})$  of  $\text{pb}$ . Hence a case is denoted by a pair  $(\text{pb}, \text{Sol}(\text{pb}))$ . A source problem  $\text{srce}$  is a problem that has already been solved in a solution  $\text{Sol}(\text{srce})$ . The pair  $(\text{srce}, \text{Sol}(\text{srce}))$  is a *source case* and the set of source cases is the *case base*. A classical decomposition of the CBR inference points out three steps: retrieval, adaptation and memorization. *Retrieval* selects a source case  $(\text{srce}, \text{Sol}(\text{srce}))$  that is judged similar to  $\text{tgt}$ , according to some similarity criterion. *Adaptation* aims at solving  $\text{tgt}$  thanks to the retrieved case  $(\text{srce}, \text{Sol}(\text{srce}))$ . Thus, a successful adaptation provides a solution  $\text{Sol}(\text{tgt})$  to  $\text{tgt}$ , in general by modification of  $\text{Sol}(\text{srce})$ .

Finally, *memorization* evaluates the utility of storing the new case  $(tgt, Sol(tgt))$  in the case base and stores it when it is useful. Knowledge-intensive approaches of CBR are such that the domain knowledge plays a key role (and not only the case base) [Aamodt, 1990]. This holds for the conservative adaptation as it is shown hereafter.

## 1.1 CBR and Adaptation

In general, it is considered that the CBR inference is based on the following principle:

*Similar problems have similar solutions.* (CBR principle)

This principle has been formalized in [Dubois *et al.*, 1998] by

$$\mathcal{T}(Sol(srce), Sol(tgt)) \geq \mathcal{S}(srce, tgt)$$

(translated with our notations) where  $\mathcal{S}$  and  $\mathcal{T}$  are similarity measures respectively between problems and solutions: the solution  $Sol(tgt)$  is constrained to be similar to  $Sol(srce)$ . There are multiple ways of specifying the adaptation step in accordance to the CBR principle, starting from the so-called *null adaptation*:

$$Sol(tgt) := Sol(srce) \quad (\text{null adaptation})$$

Null adaptation is justified in [Riesbeck and Schank, 1989] by the fact that “people often do very little adaptation”. One limit of null adaptation is that the fact “ $Sol(srce)$  solves  $tgt$ ” may contradict some domain knowledge. In this case, a strategy for adaptation is the following:

$Sol(tgt)$  is obtained by keeping from  $Sol(srce)$  as much as possible features  
while keeping the available knowledge consistent. (conservative adaptation)

Conservative adaptation aims at following the CBR principle in the sense that it tends to make the similarity  $\mathcal{T}(Sol(srce), Sol(tgt))$  maximal.

## 1.2 Overview of the Paper

In section 2, the principle of conservative adaptation is presented with more details. It relates this kind of adaptation with the theory of revision: both of them are based on minimal change.

Section 3 presents the basic principles of the theory of revision. This theory consists in a set of axioms that a revision operator has to satisfy.

Section 4 provides a formalization of conservative adaptation based on a given revision operator.

This work is motivated by an application in oncology: the KASIMIR system, in which a problem represents a class of patients and a solution represents a treatment proposal for these patients. From our study of adaptations actually performed by experts in oncology, several adaptation patterns have emerged [d’Aquin *et al.*, 2006a]. Several of these patterns can be implemented thanks to conservative adaptation; this is what is illustrated in section 5.

The retrieval step of a CBR system aims at selecting a source case to be adapted. In section 6, the issue of retrieval preceding a conservative adaptation is studied.

For some CBR systems, the retrieval consists in choosing several source cases and then, these source cases are “combined”, in order to solve the target problem. Section 7 presents a first study on case combination based on the principle of conservative adaptation.

Section 8 discusses this work and points out relative work.

Finally, section 9 draws some conclusions and points out new directions of work following this study.

## 2 Principle of Conservative Adaptation

Let us consider the following example of conservative adaptation:

**Example 1** *Léon is about to invite Thècle and wants to prepare her an appropriate meal. His target problem can be specified by the characteristics of Thècle about food. Let us assume that Thècle is vegetarian (denoted by the propositional variable  $v$ ) and that she has other characteristics (denoted by  $o$ ) not detailed in this example:  $tgt = v \wedge o$ . From his experience as a host, Léon remembers that he had invited Simone some times ago and he thinks that Simone is very similar to Thècle according to food behavior, except that she is not a vegetarian:  $srce = \neg v \wedge o$ . He had proposed to Simone a meal with salad ( $s$ ), beef ( $b$ ) and a dessert ( $d$ ), and she was satisfied by the two formers but has not eaten the dessert, thus Léon has retained the case  $(srce, Sol(srce))$  with  $Sol(srce) = s \wedge b \wedge \neg d$ . Besides that, Léon has some general knowledge about food: he knows that beef is meat, that meat and tofu are protein foods, and that vegetarians do not eat meat. Thus, his domain knowledge is*

$$DK_{Léon} = b \Rightarrow m \quad \wedge \quad m \Rightarrow p \quad \wedge \quad t \Rightarrow p \quad \wedge \quad v \Rightarrow \neg m \quad (1)$$

where  $b$ ,  $m$ ,  $t$  and  $p$  are the propositional variables for “some beef/meat/tofu/protein food is appreciated by the current guest”. According to conservative adaptation, what meal should be proposed to Thècle?  $Sol(srce)$  itself is not a satisfactory solution of  $tgt$ :  $Sol(srce) \wedge tgt \wedge DK_{Léon}$  is unsatisfiable. However, the features  $s$  and  $\neg d$  can be kept in  $Sol(srce)$  to solve  $tgt$ . Moreover, what conducts to a contradiction is the fact that there is a meat, not in the fact that it is a protein food. Therefore, a solution of  $tgt$  according to conservative adaptation could be  $s \wedge p \wedge \neg d$ . Another one could be to replace beef by tofu:  $s \wedge t \wedge \neg d$ .

As this example illustrates, the adaptation process consists in a shifting from the source context to the target context. If this process is conservative, then this shifting has to operate a minimal change and, in the same time, must be consistent with the definition of the target problem. Both contexts are interpreted in the framework of the “permanent knowledge”, i.e., the knowledge of the CBR system, consisting in the domain knowledge. Therefore, conservative adaptation is based on three kinds of knowledge:

- (KB<sub>1</sub>) The old knowledge that can be altered (but must be altered *minimally*): the knowledge related to the context of the source problem and its solution;
- (KB<sub>2</sub>) The new knowledge, that must not be altered during the process: the knowledge related to the context of the target problem;
- (DK) The knowledge that is permanent (true in any context): the domain knowledge (i.e., the general knowledge of the domain of the CBR system under consideration, e.g., the ontology giving the vocabulary with which the cases are expressed).

The question that is raised is “What is the minimal change on the knowledge base KB<sub>1</sub> that must be done to be consistent with knowledge base KB<sub>2</sub>?” When KB<sub>1</sub> and KB<sub>2</sub> do not contradict, there is no reason to change KB<sub>1</sub> and thus, a conservative adaptation process entails KB<sub>1</sub>, which amounts to a null adaptation.

This principle of minimal change of knowledge can be found in the theory of *revision*: given two knowledge bases  $\psi$  and  $\mu$ , the revision of  $\psi$  by  $\mu$  is a knowledge base  $\psi \circ \mu$  that entails  $\mu$  and makes the *minimal change* on  $\psi$  to make this revision consistent [Alchourrón *et al.*, 1985].

Both  $KB_1$  and  $KB_2$  must be interpreted in consistency with the domain knowledge  $DK$ . Thus, conservative adaptation consists, given a *revision operator*  $\circ$ , in computing  $(DK \wedge KB_1) \circ (DK \wedge KB_2)$  and to infer from this new knowledge base the pieces of information that are relevant to  $So1(\text{tgt})$ .

So, before formalizing conservative adaptation, it is necessary to introduce the notion of revision operator.

### 3 Revision of a Knowledge Base

Revision of a knowledge base has been formalized independently from a particular logic in the so-called AGM theory [Alchourrón *et al.*, 1985]. This theory has been applied, in particular, to propositional logic by [Katsuno and Mendelzon, 1991a] and this is this work that is presented here, since the current paper concentrates on this formalism.

#### 3.1 Preliminaries

The propositional formulas are assumed to be built on  $\mathcal{V}$ , a finite set of propositional variables. An interpretation  $\mathcal{I}$  is a function from  $\mathcal{V}$  to the pair  $\{\text{true}, \text{false}\}$ . If  $a \in \mathcal{V}$ ,  $\mathcal{I}(a)$  is also denoted by  $a^{\mathcal{I}}$ .  $\mathcal{I}$  is extended on the set of formulas in the usual way ( $(f \wedge g)^{\mathcal{I}} = \text{true}$  iff  $f^{\mathcal{I}} = \text{true}$  and  $g^{\mathcal{I}} = \text{true}$ , etc.). A model of a formula  $f$  is an interpretation  $\mathcal{I}$  such that  $f^{\mathcal{I}} = \text{true}$ .  $\text{Mod}(f)$  denotes the set of models of  $f$ .  $f$  is satisfiable means that  $\text{Mod}(f) \neq \emptyset$ .  $f$  entails  $g$  (resp.,  $f$  is equivalent to  $g$ ), denoted by  $f \models g$  (resp.,  $f \equiv g$ ), if  $\text{Mod}(f) \subseteq \text{Mod}(g)$  (resp.,  $\text{Mod}(f) = \text{Mod}(g)$ ), for two formulas  $f$  and  $g$ . Finally,  $g \models_f h$  (resp.,  $g \equiv_f h$ ) means that  $g$  entails  $h$  (resp.,  $g$  is equivalent to  $h$ ) under  $f$ :  $f \wedge g \models h$  (resp.,  $f \wedge g \equiv h$ ).

#### 3.2 Katsuno and Mendelzon's Axioms

Let  $\circ$  be a revision operator.  $\psi \circ \mu$  is a formula expressing the revision of  $\psi$  by  $\mu$ , according to the operator  $\circ$ :  $\psi$  is the “old” knowledge base (that has to be revised),  $\mu$  is the “new” knowledge base (that contains knowledge revising the old one). The axioms that a revision operator on propositional logic has to satisfy are:

- (R1)  $\psi \circ \mu \models \mu$  (the revision operator has to retain all the knowledge of the new knowledge base  $\mu$ );
- (R2) If  $\psi \wedge \mu$  is satisfiable, then  $\psi \circ \mu \equiv \psi \wedge \mu$  (if the new knowledge base does not contradict the old one, then every piece of knowledge of the two bases has to be kept);
- (R3) If  $\mu$  is satisfiable then  $\psi \circ \mu$  is also satisfiable ( $\circ$  does not lead to an unsatisfiable knowledge base, unless the new knowledge is itself unsatisfiable);
- (R4) If  $\psi \equiv \psi'$  and  $\mu \equiv \mu'$  then  $\psi \circ \mu \equiv \psi' \circ \mu'$  (the revision operator follows the principle of irrelevance of syntax);
- (R5)  $(\psi \circ \mu) \wedge \phi \models \psi \circ (\mu \wedge \phi)$ ;
- (R6) If  $(\psi \circ \mu) \wedge \phi$  is satisfiable then  $\psi \circ (\mu \wedge \phi) \models (\psi \circ \mu) \wedge \phi$ .

for  $\psi, \psi', \mu, \mu'$ , and  $\phi$ , five propositional formulas. (R5) and (R6) are less obvious to understand than (R1) to (R4) and are explained in [Katsuno and Mendelzon, 1991a]. They are linked with the idea that a revision operator is supposed to perform a minimal change:  $\psi \circ \mu$  keeps “as much as possible” from  $\psi$  while being consistent with  $\mu$ .

### 3.3 Distance-based Revision Operators and Dalal's Revision Operator

In [Katsuno and Mendelzon, 1991a], a characterization and a survey of revision operators in propositional logic is proposed. This paper highlights a class of revision operators based on distances between interpretations. Let  $\text{dist}$  be such a distance. For  $M_1$  and  $M_2$  two sets of interpretations and  $\mathcal{I}$  an interpretation,

$$\begin{aligned} \text{let } \text{dist}(M_1, \mathcal{I}) &= \min\{\text{dist}(\mathcal{I}, \mathcal{J}) \mid \mathcal{I} \in M_1\} \\ \text{and } \text{dist}(M_1, M_2) &= \min\{\text{dist}(M_1, \mathcal{J}) \mid \mathcal{J} \in M_2\} \\ &= \min\{\text{dist}(\mathcal{I}, \mathcal{J}) \mid \mathcal{I} \in M_1 \text{ and } \mathcal{J} \in M_2\} \end{aligned}$$

Now let  $\psi$  and  $\mu$  be two formulas and  $\Delta = \text{dist}(\text{Mod}(\psi), \text{Mod}(\mu))$ . Then, the revision operator  $\circ_{\text{dist}}$  based on  $\text{dist}$  is defined by

$$\text{Mod}(\psi \circ_{\text{dist}} \mu) = \{\mathcal{J} \mid \mathcal{J} \in \text{Mod}(\mu) \text{ and } \text{dist}(\text{Mod}(\psi), \mathcal{J}) = \Delta\} \quad (2)$$

This equation defines  $\psi \circ_{\text{dist}} \mu$  up to the equivalence between formulas (since we adhere to the principle of irrelevance of syntax, this is sufficient). The proof that axioms (R1) to (R6) hold for  $\circ_{\text{dist}}$  is a rather straightforward application of the definitions above. Note, in particular, that (R2) can be proven thanks to the equivalence  $\text{dist}(\mathcal{I}, \mathcal{J}) = 0$  iff  $\mathcal{I} = \mathcal{J}$ , for two interpretations  $\mathcal{I}$  and  $\mathcal{J}$ .

The intuition of minimal change from  $\psi$  to  $\psi \circ_{\text{dist}} \mu$  is related to the distance  $\text{dist}$  between interpretations:  $\psi \circ_{\text{dist}} \mu$  is the knowledge base whose interpretations are the interpretations of  $\mu$  that are the closest ones to those of  $\psi$ , according to  $\text{dist}$ . Figure 1 may be useful to help the intuition: only the 3 interpretations  $\mathcal{J}$  of  $\mu$  that are the closest ones to interpretations  $\mathcal{I}$  of  $\psi$  are kept:  $\text{dist}(\mathcal{I}_1, \mathcal{J}_1) = \text{dist}(\mathcal{I}_2, \mathcal{J}_2) = \text{dist}(\mathcal{I}_3, \mathcal{J}_3) = \Delta$ , thus  $\text{Mod}(\psi \circ_{\text{dist}} \mu) = \{\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3\}$ .

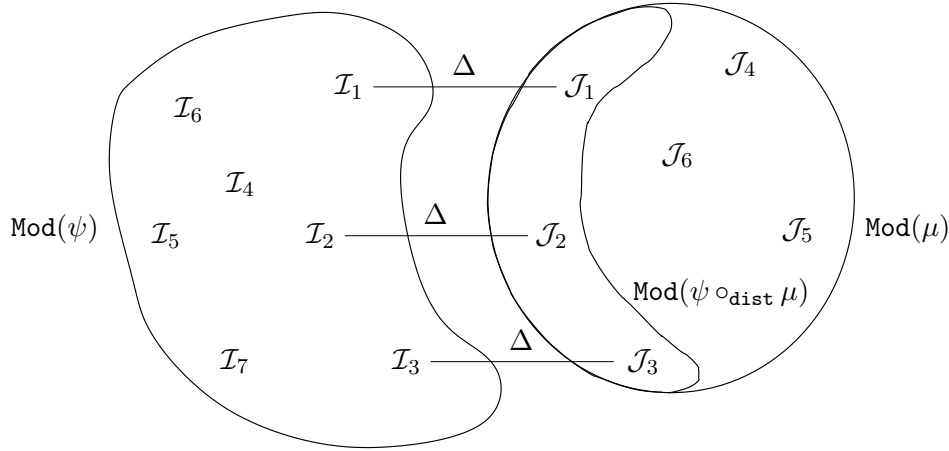


Figure 1: Illustration of  $\circ_{\text{dist}}$  (on this figure,  $\text{dist}$  is the Euclidian distance on the plan).

The Dalal's revision operator  $\circ_D$  [Dalal, 1988] is such a revision operator: it corresponds to the Hamming distance between interpretations defined by:  $\text{dist}(\mathcal{I}, \mathcal{J})$  is the number of propositional variable  $a \in \mathcal{V}$  such that  $a^{\mathcal{I}} \neq a^{\mathcal{J}}$ . This is this operator that has been chosen for the examples of this paper.

## 4 Formalization of Conservative Adaptation

This section presents a formalization of conservative adaptation based on a revision operator in propositional logic, an example using Dalal’s revision operator, and a discussion on the meaning of Katsuno and Mendelzon’s axioms for conservative adaptation.

### 4.1 Conservative Adaptation Process based on a Revision Operator

It is assumed that all the knowledge entities of the CBR system under consideration (problem, solution, domain knowledge) are represented in the formalism of propositional logic. The natural language assertion “pb is the current problem” is translated simply in pb. From this and the informal definition of conservative adaptation presented in section 1.1, it comes that, in order to solve  $\text{tgt}$  by conservative adaptation of  $(\text{srce}, \text{Sol}(\text{srce}))$ , the following knowledge bases are defined:

$$\text{KB}_1 = \text{srce} \wedge \text{Sol}(\text{srce}) \quad \text{KB}_2 = \text{tgt}$$

Let  $\circ$  be a revision operator. The  $\circ$ -conservative adaptation consists in computing  $\text{TSKCA} = (\text{DK} \wedge \text{KB}_1) \circ (\text{DK} \wedge \text{KB}_2)$ , where DK denotes the domain knowledge, and, second, entails from TSKCA pieces of information relevant to solve  $\text{tgt}$  (TSKCA is the target solution knowledge inferred by conservative adaptation).

Figure 1 is also useful to help the intuition of a  $\circ_{\text{dist}}$ -conservative adaptation process, for  $\psi = \text{DK} \wedge \text{srce} \wedge \text{Sol}(\text{srce})$ ,  $\mu = \text{DK} \wedge \text{tgt}$ , and  $\psi \circ_{\text{dist}} \mu = \text{TSKCA}$  (then, all the interpretations of the figure  $\mathcal{I}_1$  to  $\mathcal{I}_7$  and  $\mathcal{J}_1$  to  $\mathcal{J}_6$  are models of DK). Finding a solution  $\text{Sol}(\text{tgt})$  to  $\text{tgt}$  is choosing a subset  $\text{Mod}(\text{DK} \wedge \text{tgt} \wedge \text{Sol}(\text{tgt}))$  of  $\text{Mod}(\mu)$ .  $\text{Mod}(\text{TSKCA})$  is such a subset: the one pointed out by  $\circ_{\text{dist}}$ -conservative adaptation. It is obtained by considering the interpretations  $\mathcal{J} \in \text{Mod}(\mu)$  that are the closest ones to the source context  $\text{Mod}(\psi)$ :  $\mathcal{J}_1$ ,  $\mathcal{J}_2$ , and  $\mathcal{J}_3$ . In other words, the part of the source context that is the closest one to the target context  $\{\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3\}$  is shifted in  $\{\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3\}$  in order to be consistent with this context.

### 4.2 Example

From this principle, the example 1 (section 2) can be treated as follows. The knowledge bases DK,  $\text{KB}_1$ , and  $\text{KB}_2$  are:

$$\text{DK} = \text{DK}_{\text{Léon}} \quad \text{KB}_1 = \neg v \wedge o \wedge s \wedge b \wedge \neg d \quad \text{KB}_2 = v \wedge o$$

With  $\circ_D$ , the Dalal’s revision operator on propositional logic (see section 3), it can be proven that

$$\text{TSKCA} = (\text{DK} \wedge \text{KB}_1) \circ_D (\text{DK} \wedge \text{KB}_2) \equiv_{\text{DK}_{\text{Léon}}} \underbrace{v \wedge o \wedge s}_{(a)} \wedge \underbrace{\neg b \wedge \neg m \wedge p \wedge \neg d}_{(b)}$$

The target problem  $\text{tgt} = v \wedge o = (a)$  is entailed by TSKCA: this is true for any revision operator. Indeed, from axiom (R1),  $\text{TSKCA} \models \text{DK} \wedge \text{KB}_2$ , and  $\text{DK} \wedge \text{KB}_2 \models \text{tgt}$  (since  $\text{KB}_2 = \text{tgt}$ ).

In the example 1, two plausible solutions were proposed:  $\text{Sol}_1(\text{tgt}) = s \wedge p \wedge \neg d$  and  $\text{Sol}_2(\text{tgt}) = s \wedge t \wedge \neg d$ . The former can be entailed from TSKCA:  $(b) \models \text{Sol}_1(\text{tgt})$ . But (b) indicates more precisely that some protein food that is not meat ( $\neg m \wedge p$ ) is appreciated by the guest. This does not involve that the guest appreciates tofu. Now, let  $\text{DK}'_{\text{Léon}}$  be the knowledge of Léon with the additional knowledge that the only available protein food of Léon that is not meat is tofu:  $\text{DK}'_{\text{Léon}} = \text{DK}_{\text{Léon}} \wedge (p \Rightarrow m \vee t)$ . By substituting  $\text{DK}_{\text{Léon}}$  by  $\text{DK}'_{\text{Léon}}$  it comes:

$$\text{TSKCA}' = (\text{DK}'_{\text{Léon}} \wedge \text{KB}_1) \circ_D (\text{DK}'_{\text{Léon}} \wedge \text{KB}_2) \equiv_{\text{DK}'_{\text{Léon}}} \underbrace{v \wedge o \wedge s}_{(a')} \wedge \underbrace{\neg b \wedge \neg m \wedge t \wedge p \wedge \neg d}_{(b')}$$

and  $(b') \models \text{Sol}_2(\text{tgt})$ .

### 4.3 Revision Axioms and Conservative Adaptation

Now, the Katsuno and Mendelzon's axioms (R1) to (R6) can be reconsidered at the light of conservative adaptation.

(R1) applied to conservative adaptation gives  $\text{TSKCA} \models \text{DK} \wedge \text{tgt}$ . If this assertion were violated, this would mean that there exists a model  $\mathcal{I}$  of TSKCA such that  $\mathcal{I} \notin \text{Mod}(\text{DK} \wedge \text{tgt}) = \text{Mod}(\text{DK}) \cap \text{Mod}(\text{tgt})$ . Therefore  $\mathcal{I}$  would contradict

- Either the definition of the target problem (meaning that the conservative adaptation solves *another* target problem!);
- Or the domain knowledge (that has to be preserved by conservative adaptation).

Thus, using a revision operator that satisfies (R1) prevents from these two kinds of contradiction.

Let us assume that  $\text{DK} \wedge \text{KB}_1 \wedge \text{KB}_2$  is satisfiable: in other words  $\text{srce} \wedge \text{Sol}(\text{srce}) \wedge \text{tgt}$  is consistent under the domain knowledge DK. Then, (R2) entails that  $\text{TSKCA} \equiv \text{DK} \wedge \text{KB}_1 \wedge \text{KB}_2$ . Thus,  $\text{TSKCA} \models \text{srce} \wedge \text{Sol}(\text{srce}) \wedge \text{tgt}$ : if  $\text{tgt}$  is consistent with  $\text{srce} \wedge \text{Sol}(\text{srce})$  in DK, then it can be inferred by conservative adaptation that  $\text{Sol}(\text{srce})$  solves  $\text{tgt}$ . This is consistent with the principle of this kind of adaptation:  $\text{Sol}(\text{tgt})$  is obtained by keeping from  $\text{Sol}(\text{srce})$  as much as possible, and if the fact “ $\text{Sol}(\text{srce})$  solves  $\text{tgt}$ ” does not contradict DK, then conservative adaptation amounts to null adaptation.

(R3) gives: if  $\text{DK} \wedge \text{KB}_2$  is satisfiable then TSKCA is satisfiable. The satisfiability of  $\text{DK} \wedge \text{KB}_2 = \text{DK} \wedge \text{tgt}$  means that the specification of the target problem does not contradict the domain knowledge. Thus, (R3) involves that whenever the target problem is specified in accordance with the CBR domain knowledge, conservative adaptation provides a satisfiable result.

(R4) simply means that conservative adaptation follows the principle of irrelevance of syntax.

The conjunction of (R5) and (R6) can be reformulated as follows:

- Either  $(\psi \circ \mu) \wedge \phi$  is unsatisfiable,
- Or  $(\psi \circ \mu) \wedge \phi \equiv \psi \circ (\mu \wedge \phi)$ .

Applied to conservative adaptation, it gives:

- Either  $\text{TSKCA} \wedge \phi$  is unsatisfiable,
- Or  $\text{TSKCA} \wedge \phi \equiv (\text{DK} \wedge \text{KB}_1) \circ (\text{DK} \wedge \text{KB}_2 \wedge \phi)$ .

Let  $\phi$  be a formula representing some additional knowledge about the target problem. If  $\phi$  is consistent with the result of conservative adaptation (TSKCA is satisfiable) then the conjunction of (R5) and (R6) entails that adding  $\phi$  to  $\text{tgt}$  before the conservative adaptation process or after it gives the same result.

## 5 Application: Conservative Adaptation of Breast Cancer Treatments

The KASIMIR project aims at the management of decision protocols in oncology. Such decision protocols have to be adapted for some medical cases. This section shows some examples of adaptations performed by experts (oncologists) and how these examples can be modeled by conservative adaptation.



## 5.1 The KASIMIR Project

A huge research effort has been put on oncology during the last decades. As a consequence, the complexity of decision support in oncology has greatly increased. The KASIMIR project aims at the management of decision knowledge in oncology. A big part of this knowledge is constituted by decision protocols. For example, the protocol for breast cancer treatment is a document indicating how a patient suffering from this disease has to be treated. Therefore, this protocol can be seen as a set of rules  $\text{Pat} \rightarrow \text{Ttt}$ , where  $\text{Pat}$  denotes a class of patients and  $\text{Ttt}$ , a treatment for the patients in  $\text{Pat}$ .

Unfortunately, for about one third of the patients, this protocol cannot be applied, for example because of contraindications (other examples are presented in section 5.3). Indeed, it is practically impossible to list all the specific situations that prevent the application of the protocol: this is an instance of what [McCarthy, 1977] calls the qualification problem. It has been shown that, in these situations, the oncologists often *adapt* the protocol for recommending a treatment to these patients (meaning that they reuse the protocol, but not in a straightforward manner). More precisely, given the description of a target patient,  $\text{tgt}$ , a rule  $\text{Pat} \rightarrow \text{Ttt}$  is selected in the protocol, such that  $\text{Pat}$  is similar to  $\text{tgt}$ , and  $\text{Ttt}$  is adapted to fit the particularities of  $\text{tgt}$ . If the rules  $\text{Pat} \rightarrow \text{Ttt}$  are assimilated to source cases ( $\text{srce}, \text{Sol}(\text{srce}) - \text{srce} = \text{Pat}$  and  $\text{Sol}(\text{srce}) = \text{Ttt}$ ) then this process is an instance of CBR, with the particularity that the source cases are *generalized cases* (as called in [Maximini *et al.*, 2003]), also known as *ossified cases* (in [Riesbeck and Schank, 1989]).

## 5.2 The KASIMIR System

The KASIMIR system aims at assisting physicians in their decision making process. The last version of this system has been implemented as a semantic portal (i.e., a portal of the semantic Web [Fensel *et al.*, 2003]), using as representation language the W3C recommendation OWL DL [Bechhofer *et al.*, 2006], that is equivalent to the expressive description logic *SHOIN*(D) [Baader *et al.*, 2003].

This system performs protocol application: given a protocol written in OWL DL and the description of a patient, it highlights the treatments that the protocol recommends to the patient. It also implements adaptation processes, based on some adaptation knowledge [d'Aquin *et al.*, 2006b]. Current studies aim at acquiring this adaptation knowledge: from experts [d'Aquin *et al.*, 2006a] and semi-automatically [d'Aquin *et al.*, 2007].

Conservative adaptation appears as a promising research direction for adaptation within the KASIMIR system, as next section shows.

## 5.3 Examples

Two examples corresponding to real situations of decision problems of breast cancer treatment are presented below, followed by an explanation in term of conservative adaptation expressed in propositional logic. The first one describes an adaptation in a situation of contraindication; the second one, the adaptation of a therapeutic decision that is not applicable. The third example, that has been invented, is a combination of the first and second examples. The last example is an abstract one, showing the adaptation of an ineffective treatment.

### 5.3.1 Adaptation of a Contraindicated Treatment

**Example 2** *Some hormones of the human body facilitate the development of cells. In particular, oestrogens facilitate the growing of some breast cells, including some cancerous breast cells. A hormonotherapy is a long term treatment that aims at inhibiting the development of hormones (or*

their actions) to lower the chance of having a new tumor developed after the other types of treatment (surgery, chemotherapy and radiotherapy) have been applied. Tamoxifen is a hormonotherapy drug that prevents from the action of oestrogen on breast cells. Unfortunately, tamoxifen is contraindicated for people having a liver disease. The protocol of breast cancer treatment does not take into account this contraindication and the physicians have to substitute tamoxifen by another treatment having the same therapeutic benefit (or a similar therapeutic benefit). For example, they can use anti-aromatases (a drug not contraindicated for people suffering from the liver) instead of tamoxifen or a treatment consisting in the ablation of ovaries (that are organs producing oestrogen).

This example can be formalized as follows. The protocol rules leading to a recommendation of tamoxifen are formalized by  $c_1 \Rightarrow tam$ ,  $c_2 \Rightarrow tam$ ,  $\dots c_n \Rightarrow tam$ . This can be expressed by a single rule  $c \Rightarrow tam$ , where  $c = c_1 \vee c_2 \vee \dots \vee c_n$ . This rule corresponds to a source case ( $srce, Sol(srce)$ ) with  $srce = c$  and  $Sol(srce) = tam$ . Now, let us consider a woman suffering from breast cancer such that (1) the application of the protocol gives tamoxifen and (2) she suffers from a liver disease. This medical case can be formalized by  $\text{tgt} = \gamma \wedge \text{liver-disease}$ , where  $\gamma$  is such that  $\gamma \models_{DK} c$  (see below). The domain knowledge is:

$$\begin{aligned} DK = & \gamma \Rightarrow c \quad \wedge \quad \text{liver-disease} \Rightarrow \neg tam \quad \wedge \quad tam \Rightarrow \text{anti-oestrogen} \quad \wedge \\ & \text{anti-aromatases} \Rightarrow \text{anti-oestrogen} \quad \wedge \quad \text{ovary-ablation} \Rightarrow \text{anti-oestrogen} \end{aligned}$$

$\text{liver-disease} \Rightarrow \neg tam$  represents the contraindication of tamoxifen for people suffering from a liver disease.  $tam \Rightarrow \text{anti-oestrogen}$  (resp.,  $\text{anti-aromatases} \Rightarrow \text{anti-oestrogen}$ ,  $\text{ovary-ablation} \Rightarrow \text{anti-oestrogen}$ ) indicates that if tamoxifen (resp. anti-aromatases, ablation of ovaries) is recommended then an anti-oestrogen treatment is recommended.

The  $\circ_D$ -conservative adaptation leads to:

$$\text{TSKCA} = (DK \wedge c \wedge tam) \circ_D (DK \wedge \gamma \wedge \text{liver-disease}) \equiv_{DK} \gamma \wedge c \wedge \neg tam \wedge \text{anti-oestrogen}$$

If the only anti-oestrogen treatments besides tamoxifen are constituted by anti-aromatases and ablation of ovaries then an additional piece of knowledge can be added to DK:  $\text{anti-oestrogen} \Rightarrow (tam \vee \text{anti-aromatases} \vee \text{ovary-ablation})$ . With this additional knowledge,  $\text{anti-aromatases} \vee \text{ovary-ablation}$  is involved by TSKCA. It can be noticed that this example is very similar to example 1: meat is (in a sense) contraindicated by vegetarians.

### 5.3.2 Adaptation of an Inapplicable Treatment

**Example 3** *The large majority of persons suffering from breast cancer are woman (about 99%). This explains why the protocol of breast cancer treatment has been written for them. When the physicians are confronted to the medical case of a man suffering from this disease, they adapt the protocol. For example, let us consider a man with some characteristics  $c$ , such that, for a woman with these characteristics, the protocol recommends a radical mastectomy (surgery consisting in a breast ablation), a “FEC 100” chemotherapy and an ovary ablation. Both the surgery and the chemotherapy can be applied efficiently to the man, but no ovary ablation (for obvious reasons). The adaptation usually consists in keeping the surgery and the chemotherapy and in substituting the ovary ablation by an anti-oestrogen treatment, such as tamoxifen or anti-aromatases.*

The protocol rule used in this example is the source case ( $srce, Sol(srce)$ ) with  $srce = c \wedge \text{woman}$  and  $Sol(srce) = \text{radical-mastectomy} \wedge \text{FEC-100} \wedge \text{ovary-ablation}$ :  $\text{radical-mastectomy}$  (resp.,  $\text{FEC-100}$ ,  $\text{ovary-ablation}$ ) denotes the persons for which a radical mastectomy (resp., a

FEC 100 chemotherapy, an ovary ablation) is recommended. The target problem is  $\text{tgt} = c \wedge \text{man}$ . The domain knowledge is constituted by the domain knowledge of example 2 (denoted hereafter by  $\text{DK}_{\text{ex. 2}}$ ), the fact that ovary ablation is impossible for men, and the fact that men are not women:

$$\text{DK} = \text{DK}_{\text{ex. 2}} \wedge \text{man} \Rightarrow \neg \text{ovary-ablation} \wedge \neg \text{woman} \vee \neg \text{man}$$

The result of conservative adaptation, TSKCA, is such that:

$$\text{TSKCA} \equiv_{\text{DK}} c \wedge \text{man} \wedge \text{radical-mastectomy} \wedge \text{FEC-100} \wedge \neg \text{ovary-ablation} \wedge \text{anti-oestrogen}$$

If the only available anti-oestrogen therapies are tamoxifen, anti-aromatases, and ovary ablation, then DK can be substituted by  $\text{DK}' = \text{DK} \wedge (\text{anti-oestrogen} \Rightarrow \text{tam} \vee \text{anti-aromatases} \vee \text{ovary-ablation})$ . Then, the  $\circ_{\text{D}}$ -conservative adaptation gives  $\text{TSKCA}'$  such that  $\text{TSKCA}' \equiv \text{TSKCA} \wedge (\text{tam} \vee \text{anti-aromatases})$ .

### 5.3.3 Adaptation of a Treatment with Contraindication and Inapplicability

**Example 4** *This example is a combination of examples 2 and 3. Let us consider a man suffering from breast cancer and from a liver disease and having the same characteristics  $c$  as in example 3. Therefore, if he were a woman, the protocol would recommend a radical mastectomy, a FEC 100 chemotherapy, and an ovary ablation. From conservative adaptation principle, it may be suggested to make this recommendation with the substitution of ovary ablation by another anti-oestrogen treatment, which is not tamoxifen (because of the liver disease), such as anti-aromatases.*

With the same  $(\text{srce}, \text{Sol}(\text{srce}))$  and the same DK as in example 3,  $\text{tgt} = c \wedge \text{man} \wedge \text{liver-disease}$  formalizes the example. Conservative adaptation gives

$$\begin{aligned} \text{TSKCA} \equiv_{\text{DK}} c \wedge \text{man} \wedge \text{liver-disease} \wedge \text{radical-mastectomy} \wedge \text{FEC-100} \wedge \\ \neg \text{tam} \wedge \neg \text{ovary-ablation} \wedge \text{anti-oestrogen} \end{aligned}$$

which is coherent with the example. With the same  $\text{DK}'$  as in example 3, conservative adaptation gives  $\text{TSKCA}'$  such that  $\text{TSKCA}' \models_{\text{DK}'} \text{anti-aromatases}$ .

### 5.3.4 Adaptation of a Treatment that is Ineffective

It may occur that a treatment should not be applied to a patient, because, the specificities of this patient make this treatment non effective: it is neither contraindicated, nor inapplicable, but its application is useless. The idea is then to substitute this treatment by a treatment having a therapeutic benefit similar to what was expected for the patient. For example, when a given drug has been administrated during a long time to a patient, its effect may become lower and lower, and if the dose cannot be augmented any more, then it has to be substituted by another treatment (e.g., another drug). This situation has been met several times during the situations of adaptations that we have met so far. Nevertheless, we have chosen to present an abstract situation, that is much easier to introduce and to understand. Real situations of adaptations of ineffective treatments are often similar to this abstract one.

**Example 5** *Let  $c$ , some patient characteristic, that leads to a desirable effect  $e$ . Let  $\text{tnt}_1$  be a treatment that has been designed for achieving  $e$ . This treatment can be applied on any patient, but it is effective only for patients having the characteristic  $d$ . Now, let  $\text{tnt}_2$  and  $\text{tnt}_3$  be two treatments having the same expected effect  $e$  as  $\text{tnt}_1$ , but which are effective for any patient. It is assumed that the only treatments that may lead to the effect  $e$  are  $\text{tnt}_1$ ,  $\text{tnt}_2$ , and  $\text{tnt}_3$ , and that only one of these*

treatments can be recommended at the same time, for the same patient. This can be modeled by the following domain knowledge:

$$DK = c \Rightarrow e \quad \wedge \quad d \wedge ttt_1 \Rightarrow e \quad \wedge \quad \neg d \wedge ttt_1 \Rightarrow \neg e \quad \wedge \quad ttt_2 \Rightarrow e \quad \wedge \quad ttt_3 \Rightarrow e \quad \wedge \\ e \Rightarrow ttt_1 \vee ttt_2 \vee ttt_3 \quad \wedge \quad \neg ttt_1 \vee \neg ttt_2 \quad \wedge \quad \neg ttt_1 \vee \neg ttt_3 \quad \wedge \quad \neg ttt_2 \vee \neg ttt_3$$

Now, let us consider the protocol rule represented by the source case  $(srce, Sol(srce))$  such that:

$$srce = c \wedge d \quad \quad Sol(srce) = ttt_1$$

( $ttt_1$  has been chosen instead of  $ttt_2$  or  $ttt_3$ , e.g. because it has less undesirable effect). For the target problem  $tgt = c \wedge \neg d$ , since  $ttt_1$  is not applicable,  $ttt_2$  or  $ttt_3$  can be suggested, since they have the same effect  $e$  on  $tgt$  as  $ttt_1$  on  $srce$ .

This adaptation can be realized thanks to a  $\circ_D$ -conservative adaptation:

$$TSKCA \equiv_{DK} = c \wedge \neg d \wedge e \wedge \neg ttt_1 \wedge (ttt_2 \vee ttt_3)$$

## 6 Conservative Adaptation and Adaptation-Guided Retrieval

The philosophy of adaptation-guided retrieval is to relate the retrieval module of a CBR system to the capability of its adaptation module [Smyth and Keane, 1996]. Ideally, retrieval is based on the following preference: a source case is preferred to another one if the adaptation of the former to solve the target problem is “better” than the adaptation of the latter. But, what does “better” mean? The principle of conservative adaptation is to minimize the change. Following this principle, we make the following hypothesis:

**Hypothesis:** A conservative adaptation process is better than another one if the former requires less change than the latter.

Now, the question is “How can changes be compared?” An answer to it can be proposed for the distance-based revision operators  $\circ_{\text{dist}}$  (e.g.,  $\circ_D$ ): given two propositional formulas  $\psi$  and  $\mu$ , the less  $\Delta = \text{dist}(\text{Mod}(\psi), \text{Mod}(\mu))$  is, the less the revision  $\psi \circ_{\text{dist}} \mu$  requires some change. From this and from the definition of conservative adaptation, a criterion useful for preferring a source case  $(srce^1, Sol(srce^1))$  to a source case  $(srce^2, Sol(srce^2))$  for the purpose of retrieval is the following one:

$$\Delta^1 < \Delta^2 \quad \text{with } \Delta^i = \text{dist}(\text{Mod}(DK \wedge srce^i \wedge Sol(srce^i)), \text{Mod}(DK \wedge tgt)) \quad (i \in \{1, 2\})$$

The following example illustrates this idea:

**Example 6** Let  $tgt = c \wedge \text{liver-disease} \wedge \text{allergy-to-FEC}$  (where *allergy-to-FEC* denotes an allergy to the chemotherapy using the FEC drugs) and  $(srce^1, Sol(srce^1))$  and  $(srce^2, Sol(srce^2))$  be two source cases such that  $srce^1 = c_1$ ,  $Sol(srce^1) = \text{FEC-50} \wedge \text{anti-aromatases}$ ,  $srce^2 = c_2$ ,  $Sol(srce^2) = \text{FEC-50} \wedge \text{tam}$ . With  $DK_{ex. 4}$ , the domain knowledge of example 4, let

$$DK = DK_{ex. 4} \quad \wedge \quad c \Rightarrow c_1 \quad \wedge \quad c \Rightarrow c_2 \quad \wedge \quad \text{FEC-50} \Rightarrow \text{FEC} \quad \wedge \\ \text{FEC} \Rightarrow \text{chemotherapy} \quad \wedge \quad \text{CMF} \Rightarrow \text{chemotherapy} \quad \wedge \quad \text{allergy-to-FEC} \Rightarrow \neg \text{FEC}$$

With this domain knowledge,  $\circ_D$ -conservative adaptations with source cases  $(srce^1, Sol(srce^1))$  and  $(srce^2, Sol(srce^2))$  respectively give:

$$TSKCA^1 \equiv_{DK} c \wedge c_1 \wedge c_2 \wedge \text{liver-disease} \wedge \text{allergy-to-FEC} \wedge \neg \text{FEC-50} \wedge \neg \text{FEC} \wedge \text{chemotherapy} \\ \wedge \neg \text{tam} \wedge \text{anti-aromatases} \wedge \text{anti-oestrogen}$$

which requires a change measured by  $\Delta^1 = 3$

$$TSKCA^2 \equiv_{DK} c \wedge c_1 \wedge c_2 \wedge \text{liver-disease} \wedge \text{allergy-to-FEC} \wedge \neg \text{FEC-50} \wedge \neg \text{FEC} \wedge \text{chemotherapy} \\ \wedge \neg \text{tam} \wedge \text{anti-oestrogen}$$

which requires a change measured by  $\Delta^2 = 5$

Since  $\Delta^1 < \Delta^2$ , a retrieval process based on the criterion defined above gives a preference to  $(srce^1, Sol(srce^1))$ , when it is compared to  $(srce^2, Sol(srce^2))$ .

Two research directions follow this first study on “conservative adaptation-guided retrieval”. The first one is theoretical: it may occur that two source cases cannot be distinguished with this criterion. For example, let us consider a target problem  $\text{tgt}$  and two source cases  $(srce^1, Sol(srce^1))$  and  $(srce^2, Sol(srce^2))$  such that  $srce^1 \equiv_{DK} \text{tgt}$ ,  $srce^2 \not\equiv_{DK} \text{tgt}$ , and  $DK \wedge srce^2 \wedge Sol(srce^2) \wedge \text{tgt}$  is satisfiable. For both source cases, conservative adaptation amounts to null adaptation (cf. the application of (R2) to conservative adaptation: see section 4.3) and the preference criterion introduced above cannot distinguish them:  $\Delta^1 = \Delta^2 = 0$ . Nevertheless, it seems natural to prefer  $(srce^1, Sol(srce^1))$  to  $(srce^2, Sol(srce^2))$ : the first source case represents the resolution of the target problem whereas the second source case only represents the resolution of a problem logically compatible with the target problem. Therefore, another preference criterion should be proposed for a more accurate retrieval process. This criterion could be applied to distinguish source cases having the same  $\Delta$ .

The second research direction is practical. Given a source case and a target problem, the computation of  $\Delta$  requires some computing time: it is NP-hard. Indeed, a program that computes  $\Delta = \text{dist}(\text{Mod}(f), \text{Mod}(g))$  can be used to solve the NP-complete problem of satisfiability of a propositional formula:  $f$  is satisfiable iff  $\text{dist}(\text{Mod}(f), \text{Mod}(t)) = 0$ , with  $t$  a tautology (e.g.,  $t = a \vee \neg a$ ). Thus, if the set of propositional variables and the case base are large, computing  $\Delta$  for each source case may become practically impossible. This is a frequent issue for the conception of a CBR system, that is usually solved in two different ways (or a combination of them). The first way is to organize the case base in a hierarchy in order to prune a big part of the case base containing source cases dissimilar to the target problem (see e.g. [Lieber, 2002]). The second way is to find an approximate retrieval criterion that can be computed with a low complexity, in order to define a two-stage retrieval: (1) selection of a small subset of the case base according to this approximate criterion and (2) retrieve in this subset the closest source case according to  $\Delta$  (this two-stage retrieval principle can be found e.g., in [Cunningham *et al.*, 1993]).

## 7 Managing the Combination of Several Source Cases

The presentation of the CBR retrieval step that has been made at the beginning of the paper assumes that only one source case is selected to be adapted. By contrast, many CBR systems use several source cases in order to solve a sole target problem (see e.g. [Smyth, 1996]). This approach is called the *combination* of source cases because it usually consists in selecting parts of these cases and combine these parts to build a solution. The issue studied in this section is how case combination could be performed following the principles of conservative adaptation.

## 7.1 A Sequential Approach to Case Combination

An approach to combination is to consider sequentially the retrieved source cases  $(srce^i, Sol(srce^i))$ . The contribution of the first one,  $(srce^1, Sol(srce^1))$ , consists simply in applying conservative adaptation:

$$TSKCA^1 = (DK \wedge srce^1 \wedge Sol(srce^1)) \circ (DK \wedge tgt)$$

From  $TSKCA^1$ , a solution  $Sol^1(tgt)$  of  $tgt$  is entailed. Let  $tgt^1 = TSKCA^1$ . Intuitively,  $tgt^1$  represents the problem  $tgt$  completed by the constraint that the solution must also be consistent with  $Sol^1(tgt)$ . The contribution of the second source case to solve  $tgt$  consists in applying conservative adaptation of  $tgt^1$ :

$$TSKCA^{1,2} = (DK \wedge srce^2 \wedge Sol(srce^2)) \circ (DK \wedge tgt^1)$$

From  $TSKCA^{1,2}$ , a solution  $Sol^{1,2}(tgt)$  of  $tgt$  is involved, that is more specific than  $Sol^1(tgt)$ :  $Sol^{1,2}(tgt) \models_{DK} Sol^1(tgt)$ . Let  $tgt^{1,2} = TSKCA^{1,2}$ . The third step of this process consists in a conservative adaptation of  $(srce^3, Sol(srce^3))$  to solve  $tgt^{1,2}$  in order to obtain a solution  $Sol^{1,2,3}(tgt)$  of  $tgt$ , etc.

It can be noticed that the way the source cases are ranked plays a role in the problem-solving as the example below shows:

**Example 7** *Let us consider again the example 1 (section 2), with the difference that  $(srce, Sol(srce))$  is denoted by  $(srce^1, Sol(srce^1))$ . Moreover, Léon remembers not only the appreciations of Simone, but also the ones of Sophie who is a vegetarian and who does not share with Thècle the other characteristics  $o$ . Léon had served to Sophie an egg ( $e$ ), pasta ( $\pi$ ), some cheese ( $c$ ), and a dessert ( $d$ ). It seems to Léon that Sophie has enjoyed all the meal. The experience with Simone can be formalized by the source case  $(srce^2, Sol(srce^2))$  such that  $srce^2 = v \wedge \neg o$  and  $Sol(srce^2) = e \wedge \pi \wedge c \wedge d$ . The knowledge base of Léon is the same as in example 1 ( $DK_{ex. 1}$ ), except that the characteristics  $o$  involves that cheese is not appreciated by the guest:  $DK = DK_{ex. 1} \wedge (o \Rightarrow \neg c)$ . If  $(srce^1, Sol(srce^1))$  is considered before  $(srce^2, Sol(srce^2))$ , the sequential combination process gives (with the Dalal's revision operator):*

$$\begin{aligned} TSKCA^1 &= (DK \wedge \neg v \wedge o \wedge s \wedge b \wedge \neg d) \circ_D (DK \wedge v \wedge o) \\ &\equiv_{DK} v \wedge o \wedge s \wedge \neg b \wedge \neg m \wedge p \wedge \neg d \\ TSKCA^{1,2} &= (DK \wedge v \wedge \neg o \wedge e \wedge \pi \wedge c \wedge d) \circ_D TSKCA^1 \\ &\equiv_{DK} v \wedge o \wedge s \wedge \neg b \wedge \neg m \wedge p \wedge \neg d \wedge e \wedge \pi \wedge \neg c \end{aligned}$$

If they are ranked differently, the result is:

$$\begin{aligned} TSKCA^2 &= (DK \wedge v \wedge \neg o \wedge e \wedge \pi \wedge c \wedge d) \circ_D (DK \wedge v \wedge o) \\ &\equiv_{DK} v \wedge o \wedge e \wedge \pi \wedge \neg c \wedge d \\ TSKCA^{2,1} &= (DK \wedge \neg v \wedge o \wedge s \wedge b \wedge \neg d) \circ_D TSKCA^2 \\ &\equiv_{DK} v \wedge o \wedge s \wedge \neg b \wedge \neg m \wedge p \wedge d \wedge e \wedge \pi \wedge \neg c \end{aligned}$$

The fact that  $TSKCA^{1,2} \not\equiv_{DK} TSKCA^{2,1}$  shows that the order between source cases may play a role. If no relevant criterion is found to rank the two source cases, it is possible to consider their disjunction:

$$TSKCA^{1,2} \vee TSKCA^{2,1} \equiv_{DK} v \wedge o \wedge s \wedge \neg b \wedge \neg m \wedge p \wedge e \wedge \pi \wedge \neg c$$

meaning that according to Léon's experience about Simone and Sophie, the following type of food should be appreciated by Thècle: salad, protein food, eggs, pasta and cheese, whereas meat and cheese are not appreciated. For dessert, there are pros (related to the experience with Sophie) and cons (related to the experience with Simone).

This first study on how several source cases can be adapted and combined must be carried on. In particular, it must be studied whether the sequential approach presented above is relevant (i.e., models correctly some of the multiple case combinations performed by experts) and, if so, the question of how the source cases should be ranked for this process must be addressed. This last future work may meet the issue of the retrieval criterions discussed in section 6.

## 7.2 Managing Missing Information about the Target Problem

It may occur that some pieces of information about the target problem are missing. In particular, during the adaptation knowledge acquisition from experts for the KASIMIR system, situations for which some features of the patients were missing have appeared several times [d'Aquin *et al.*, 2006a]. In these observed situations, physicians generally act in a way that can be modeled by the so-called Wald pessimistic criterion [Dubois *et al.*, 2001] (called the minimax strategy in [Wald, 1950]), which consists in taking a decision on the basis of its worst possible consequence. This section shows how this criterion can be integrated in the framework of conservative adaptation. The following example illustrates these ideas:

**Example 8** *Let us consider a patient having a 2 centimeters tumor. Given other features, the protocol recommends a partial mastectomy. Now, the radiography shows some white dots on the image that are rather far away from the tumor and that may be either (a) cancerous cells, or (b) something harmless. Under assumption (a), the tumor region considered for the decision is the union of the observed tumor and of the white dots, which gives a middle-sized tumor for which a radical mastectomy is recommended –decision  $\text{dec}_{(a)}$ . Under assumption (b), a partial mastectomy is recommended –decision  $\text{dec}_{(b)}$ . If no examination before surgery can indicate which of the hypotheses (a) and (b) is correct, the question that is raised is to know whether it is better to do*

$(\text{dec}_{(a)}/b)$  *A radical mastectomy  $\text{dec}_{(a)}$  under assumption (b) –and thus, a larger surgery than necessary– or*

$(\text{dec}_{(b)}/a)$  *A partial mastectomy  $\text{dec}_{(b)}$  under assumption (a) –which would leave cancerous cells in the body of the patient.*

*Moreover, some additional knowledge is available telling that  $(\text{dec}_{(a)}/b)$  is preferred to  $(\text{dec}_{(b)}/a)$ . Therefore, according to the Wald pessimistic criterion, the decision taken is  $\text{dec}_{(a)}$ : the worst possible consequence (w. p. c) of  $\text{dec}_{(a)}$  in the context of the patient is preferred to the w. p. c. of  $\text{dec}_{(b)}$  in this context.*

This example can be formalized as follows. The two source cases are  $(\text{srce}^1, \text{Sol}(\text{srce}^1))$  and  $(\text{srce}^2, \text{Sol}(\text{srce}^2))$ , with  $\text{srce}^1 = c \wedge \text{mst}$ ,  $\text{Sol}(\text{srce}^1) = \text{rm}$ ,  $\text{srce}^2 = c \wedge \text{st}$ , and  $\text{Sol}(\text{srce}^2) = \text{pm}$ , where  $c$  is some patient characteristics,  $\text{mst}$  and  $\text{st}$  stand respectively for middle-sized tumor (between 4 and 7 cm) and small tumor (less than 4 cm),  $\text{rm}$  (resp.,  $\text{pm}$ ) represents the patients for which a radical (resp., partial) mastectomy is recommended, which corresponds to decision  $\text{dec}_{(a)}$  (resp.,  $\text{dec}_{(b)}$ ). The target problem can be formalized as  $\text{tgt} = \gamma \wedge (\text{st} \vee \text{mst})$  with  $\gamma$  such that  $\gamma \models_{\text{DK}} c$ . The patient has either a small or a middle-sized tumor, but

it is not known which one. The assertion “ $(\text{dec}_{(a)}/b)$  is preferred to  $(\text{dec}_{(b)}/a)$ ” can be formalized as  $((a) \vee (b)) \Rightarrow (\text{dec}_{(b)} \Rightarrow \text{dec}_{(a)})$ : when  $(a)$  or  $(b)$  holds, the decision  $\text{dec}_{(a)}$  is preferred to the decision  $\text{dec}_{(b)}$ , meaning that the choice of  $\text{dec}_{(b)}$  is possible only if the choice of  $\text{dec}_{(a)}$  is also made ( $\text{dec}_{(b)} \Rightarrow \text{dec}_{(a)} \equiv \neg(\text{dec}_{(b)} \wedge \neg \text{dec}_{(a)})$ ). In the example, this preference is formalized by  $(st \vee mst) \Rightarrow (pm \Rightarrow rm)$ . Therefore, the domain knowledge is:

$$DK = \gamma \Rightarrow c \quad \wedge \quad (st \vee mst) \Rightarrow (pm \Rightarrow rm) \quad \wedge \quad \neg st \vee \neg mst \quad \wedge \quad \neg pm \vee \neg rm$$

The conjunct  $\neg st \vee \neg mst$  means that a tumor cannot be at the same time small and middle-sized. The conjunct  $\neg pm \vee \neg rm$  means that for a given patient, it is not possible to recommend at the same time a partial mastectomy and a radical one. By applying the sequential combination of cases presented in section 7.1, it comes:

$$TSKCA^{1,2} \equiv_{DK} \gamma \wedge c \wedge mst \wedge rm$$

which is in accordance with the example (in this example, the patient characteristic  $st \vee mst$  has been specialized into  $mst$ , translating the fact that the target patient tumor is considered as middle-sized). It can be noticed that, for this example, the result does not depend on the ranking of the source cases:  $TSKCA^{1,2} \equiv_{DK} TSKCA^{2,1}$ .

## 8 Discussion and Relative Work

This section discusses some research directions that follow this first study on conservative adaptation and presents related researches relevant to get further in these directions.

### 8.1 Learning Domain Knowledge from Failed Conservative Adaptations

The knowledge required for conservative adaptation is the domain knowledge  $DK$  of the CBR system under consideration:  $DK$  is useful to point out the features of the source case that need to be adapted to the context of the target problem. Thus, with insufficient domain knowledge, conservative adaptation may provide an unsatisfying solution to the target problem: this solution contradicts the expert knowledge but does not contradict  $DK$ . In other words, the failed result of conservative adaptation is due to the gap between  $DK$  and the expert knowledge (a gap that cannot be completely filled in practice, due to the qualification problem mentioned in section 5.1). Therefore, from an analysis of the failure, some new domain knowledge can be acquired and added to the current  $DK$ .

Let us consider again the example 1, with the difference that Léon ignores that vegetarians do not eat meat:  $DK$  is obtained by removing from the conjunction defining  $DK_{\text{Léon}}$  (cf. (1)) the implication  $v \Rightarrow \neg m$ . Now,  $DK \wedge \text{srce} \wedge \text{Sol}(\text{srce}) \wedge \text{tgt}$  is satisfiable and thus, conservative adaptation proposes  $\text{Sol}(\text{srce})$  as a solution to  $\text{tgt}$ . Following this proposal, Léon offers a dinner with beef to Thècle who refuses to eat beef and explains him that, since she is a vegetarian, she does not eat any meat. From this explanation, Léon learns  $v \Rightarrow \neg m$  (and proposes a meat-free dinner to Thècle).

Therefore, following the ideas of [Hammond, 1990b], a CBR system may learn new domain knowledge from the explanations that follow failed conservative adaptation, which involves an improvement of its competence. How this can be put in practice is still an open issue.

### 8.2 Towards Extensions of Conservative Adaptation

Conservative adaptation does not capture any kind of adaptation. This section shows two directions for extending it.



### 8.2.1 Towards the Definition of a Less Conservative Adaptation

In some situations, conservative adaptation keeps to much from the source case, as the following example illustrates:

**Example 9** *Let us consider again the example 3 of a man suffering from breast cancer, with the following additional knowledge: the three types of anti-oestrogen treatments (tamoxifen, anti-aromatases, and ovary ablation) are mutually exclusive. This means in particular that if an ovary ablation is recommended, then neither tamoxifen nor anti-aromatases can be recommended. This can be formalized with the following domain knowledge (with  $DK_{ex. 3}$  the domain knowledge of example 3):*

$$DK = DK_{ex. 3} \wedge \neg tam \vee \neg anti-aromatases \wedge \neg tam \vee \neg ovary-ablation \wedge \neg anti-aromatases \vee \neg ovary-ablation$$

With the source case and the target problem of example 3 and this domain knowledge, conservative adaptation gives

$$TSKCA \equiv_{DK} TSKCA_{ex. 3} \wedge \neg tam \wedge \neg anti-aromatases \wedge anti-oestrogen$$

with  $TSKCA_{ex. 3}$ , the TSKCA of example 3.

This may seem contrary to the intuition: an anti-oestrogen treatment is recommended, that is not the ovary ablation (inapplicable to men), but that is neither tamoxifen, nor anti-aromatases, that are the only other anti-oestrogen treatments reified in the domain knowledge! This can be explained by the fact that  $DK \wedge srce \wedge Sol(srce) \models f$  with  $f = \neg tam \wedge \neg anti-aromatases$ , and since  $f$  is consistent with  $DK \wedge tgt$ , then  $f$  is kept by the conservative adaptation process.

This example shows that, due to its conservative nature, this approach to adaptation may keep too much from the source case. Now the question raised is how to define another class of adaptation approaches, similar to conservative adaptation, but which removes some relevant pieces of information on the source case, even when they do not contradict the target problem. This question is another open issue that will deserve some future work.

### 8.2.2 Adaptation Based on Substitution and Repair

In early researches of CBR, several steps were distinguished, in particular the steps of adaptation, explanation, test, and repair [Riesbeck and Schank, 1989]. The result of adaptation, a first solution  $Sol_1(tgt)$  of the target problem  $tgt$ , is tested and, if it does not meet some requirements (of  $tgt$  and of the domain knowledge), this failure is explained and then  $Sol_1(tgt)$  is repaired to meet these requirements. These steps were reformulated into the reuse and revise steps in the [Aamodt and Plaza, 1994] CBR cycle: reuse corresponds to adaptation and revise to test, explanation, and repair. It can be noticed that, to our knowledge, the revise step of the CBR cycle has not been related to the AGM theory of revision: we have found only one paper on CBR using revision techniques [Rial *et al.*, 2001], but not for the purpose of the reasoning process itself, but for the maintenance of the case base and of a rule base when there are some evolutions in time (according to [Katsuno and Mendelzon, 1991b], this is more an update of a knowledge base than a revision of it, but the techniques addressing update and revision share many common features).

Conservative adaptation can be considered from a reuse-and-revise point of view: its reuse step corresponds to a null adaptation –the first solution of  $tgt$  is  $Sol(srce)$ – and its revise step corresponds to the use of a revision operator, in accordance with domain knowledge.

The well-known system CHEF [Hammond, 1990a] is also based on an adaptation-test-explanation-repair scheme. CHEF is a case-based planner whose primary domain is cooking: a problem  $pb$  is a recipe represented by a set of goals and a solution of  $pb$  is a recipe for  $pb$ , i.e., a plan that satisfies its goals. Let us consider the following example in CHEF:

**Example 10** *This example is a simplified version of the “beef and broccoli” example: the target problem is “How to prepare a stir-fry dish including beef and broccoli?” The source case is the “beef and green beans recipe”, that is a stir-fry recipe. A first solution to the target problem is obtained by substituting, in this source recipe, green beans by broccoli. (Actually, in CHEF, some other adjustments are made, in particular, changes in cooking times and add of a step to chop the broccoli.) Then this recipe is tested, thanks to a simulator using some domain knowledge. The simulation leads to the fact that the broccoli is, at the end of the recipe, soggy, which is dissatisfying (broccoli should be crisp). This is explained by the fact that broccoli is cooked in water, this water being produced by cooking the beef, and the beef and the broccoli being cooked together, in the same pan. Finally, this plan is repaired by cooking the broccoli separately from the beef.*

This adaptation can be qualified as a substitution and repair approach. It can be expressed with the help of a revision operator, as explained hereafter. The example is formalized in first order logic (FOL). The source case ( $srce$ ,  $Sol(srce)$ ) corresponding to the beef and beans recipe can be formalized (in a simplified manner) by:

$$\begin{aligned} srce &= \text{cooked}(\text{beef}) \wedge \text{cooked}(\text{beans}) \wedge \text{stir-fry-recipe} \\ Sol(srce) &= \text{cooking-together}(\text{beef}, \text{beans}) \end{aligned}$$

The target problem “How to prepare a beef and broccoli dish” can be formalized by:

$$tgt = \text{cooked}(\text{beef}) \wedge \text{cooked}(\text{broccoli}) \wedge \text{stir-fry-recipe}$$

The substitution step of this adaptation consists in finding a substitution  $\sigma$  such that  $\sigma(srce) \equiv tgt$  and then in proposing  $Sol_1(tgt) = \sigma(Sol(srce))$  as a first solution of  $tgt$ :

$$\begin{aligned} \sigma &= \{\text{beans}/\text{broccoli}\} \\ \sigma(Sol(srce)) &= \text{cooking-together}(\text{beef}, \text{broccoli}) \end{aligned}$$

Note that the retrieval of this recipe from the case base as well as the choice of the substitution is based on some elements of knowledge not considered here (in particular, the fact that broccoli and green beans share the property of being vegetables). The fact that  $\sigma(Sol(srce))$  is *not* a satisfying solution of  $tgt$  is modeled by the fact that  $DK \wedge tgt \wedge \sigma(Sol(srce)) \models \text{failure}$ , where  $DK$  is the following domain knowledge:

$$\begin{aligned} DK &= \forall x \forall y \text{ cooking-together}(x, y) \Rightarrow \text{cooking}(x) \wedge \text{cooking}(y) \quad \wedge \\ &\quad \forall x \forall y \text{ cooking-together}(x, y) \wedge \text{produces-water}(x) \Rightarrow \text{cooked-in-water}(y) \quad \wedge \\ &\quad \text{cooking}(\text{beef}) \Rightarrow \text{produces-water}(\text{beef}) \quad \wedge \\ &\quad \text{cooked-in-water}(\text{broccoli}) \Rightarrow \text{failure} \end{aligned}$$

The repair of  $\sigma(Sol(srce))$  may be performed by considering that it is a solution that has to be altered in order to entail  $\neg \text{failure}$  while being consistent with  $tgt$ . Moreover, if it is assumed that this alteration has to be done with a minimal change, then a revision operator  $\circ$  may be used to compute  $f = (DK \wedge \sigma(Sol(srce))) \circ (DK \wedge tgt \wedge \neg \text{failure})$ . Technically, the use of  $\circ_D$  requires that the example is translated from FOL to propositional logic. This can be done by (1) substituting the universally quantified variables  $x$  and  $y$  by the constants  $\text{beef}$  and  $\text{broccoli}$  and

(2) substituting the FOL atoms by propositional variables, e.g., `cooking(beef)` to `cooking-beef` and `cooking-together(beef, broccoli)` by `cooking-together-beef-broccoli`. Then, the application of the Dalal’s revision operator gives a formula that, translated back to FOL, gives  $f$  such that:

$$\begin{aligned} f &\equiv_{\text{DK}} \text{tgt} \wedge \text{Sol}(\text{tgt}) \\ \text{with } \text{Sol}(\text{tgt}) &\equiv \text{cooking}(\text{beef}) \wedge \text{cooking}(\text{broccoli}) \wedge \\ &\quad \neg \text{cooking-together}(\text{beef}, \text{broccoli}) \end{aligned}$$

This example illustrates how an adaptation by substitution and repair may be performed thanks to a revision operator. The scope of this approach remains to be studied. One of its possible applications lies in the approach to analogical reasoning of [Forbus and Gentner, 1986]. This approach focuses on the establishment of correspondences between the source and the target universes, that can be likened to the substitution  $\sigma$  in the example above. The use of a revision operator may complete this approach when the analogical transfer leads to a contradiction with the knowledge associated with the target universe.

This approach to adaptation differs from conservative adaptation by (1) the need to find a substitution  $\sigma$  and (2) the fact that no problem statement is used on the left side of the revision operator. Both approaches belong to a family of adaptation approaches that may deserve some future researches: the family of revision-based adaptations.

### 8.3 Conservative Adaptation in Taxonomies of Adaptation Approaches

There have been several proposals in the CBR literature of adaptation approach taxonomies. This section aims at situating conservative adaptation in several such taxonomies.

**Riesbeck and Schank’s taxonomy.** In [Riesbeck and Schank, 1989] (pages 44 to 51), some adaptation approaches are presented and discussed. Conservative adaptation can be related to several of them. *Null adaptation* has been presented at the beginning of the current paper (section 1.1) as a starting point for introducing conservative adaptation. *Critic-based adaptation* is an approach that consists, starting from a first solution, in debugging it when necessary (the adaptation performed by CHEF and described in section 8.2.2 is an example of this kind of adaptation). Thus, conservative adaptation may be seen as a combination of null and critic-based adaptations.

*Abstraction and respecialization* approach to adaptation consists in (1) abstracting the solution  $\text{Sol}(\text{srce})$  of  $\text{srce}$  into a solution  $\text{Sol}(A)$  of an abstract problem  $A$ , and (2) in specializing  $\text{Sol}(A)$  in order to solve  $\text{tgt}$ . According to [Bergmann, 1992], this adaptation can be better qualified as a generalization/specialization approach (versus an abstraction/refinement approach), but this distinction is not made in [Riesbeck and Schank, 1989].

The examples of conservative adaptations presented in this paper may be seen as the application of some generalization and specialization adaptations. For instance, in example 3 (section 5.3.2),  $\text{Sol}(\text{srce})$  is generalized by substituting *ovary-ablation* by *anti-oestrogen* and then, whenever it is known that the only available anti-oestrogen treatments besides ovary ablation are tamoxifen and anti-aromatases, *anti-oestrogen* is specialized into  $\text{tam} \vee \text{anti-aromatases}$ .

This behavior of  $\circ_{\text{D}}$ -conservative adaptation can be understood thanks to a definition of distance-based revision operators  $\circ_{\text{dist}}$  (such as  $\circ_{\text{D}}$ ), equivalent to the one given in section 3.3 and inspired from [Dalal, 1988]. This definition is as follows. First, for any real number  $\delta \geq 0$ , let  $G^\delta$  be a function that maps a propositional formula  $\psi$  based on a set of variables  $\mathcal{V}$  to another formula  $G^\delta(\psi)$  on  $\mathcal{V}$ , such that

$$\text{Mod}(G^\delta(\psi)) = \{\mathcal{I} \mid \mathcal{I}: \text{interpretation on } \mathcal{V} \text{ and } \text{dist}(\text{Mod}(\psi), \mathcal{I}) \leq \delta\}$$

$G^\delta$  realizes a generalization:  $\psi \models G^\delta(\psi)$  for any  $\psi$  and any  $\delta$ . Moreover  $G^0(\psi) \equiv \psi$ . Finally, if  $0 \leq \delta \leq \varepsilon$ , then  $G^\delta(\psi) \models G^\varepsilon(\psi)$ . For  $\psi$  and  $\mu$ , two satisfiable formulas on  $\mathcal{V}$ , let  $\Delta$  be the least value  $\delta$  such that  $G^\delta(\psi) \wedge \mu$  is satisfiable.<sup>1</sup> Then,  $\psi \circ_{\text{dist}} \mu$  can be defined by  $\psi \circ_{\text{dist}} \mu = G^\Delta(\psi) \wedge \mu$ . If either  $\psi$  or  $\mu$  is unsatisfiable, then  $\psi \circ_{\text{dist}} \mu = \mu$ . It can be proven easily that this definition of  $\circ_{\text{dist}}$  is equivalent to the one of section 3.3 (as soon as syntax is considered to be irrelevant). Thus  $\psi \circ_{\text{dist}} \mu$  can be interpreted as follows: it is obtained by generalizing  $\psi$  in a minimal way (according to the scale  $(\{G^\delta(\psi)\}_\delta, \models)$ ) in order to be consistent with  $\mu$ , and then, it is specialized by a conjunction with  $\mu$ .

**Kolodner’s taxonomy.** In [Kolodner, 1993], ten general adaptation approaches are introduced. Conservative adaptation appears as an approach rather “orthogonal” to this taxonomy, in the sense that it may be likened to several of them. It has been chosen to focus on two approaches –local search and query memory– that are interesting to compare with conservative adaptation. Local search consists in substituting a solution part by an element similar to it taken in “an auxiliary knowledge structure” (such as an “is-a” hierarchy). Query memory consists in substituting a solution part by an element obtained from querying “either auxiliary knowledge structure or the case [base]”. These two approaches share with conservative adaptation the principle of substituting a solution part by an element of a knowledge base related to this solution part. The main difference between these approaches and conservative adaptation is that for the formers, the knowledge base is considered externally whereas, for the latter, the represented entities are considered modulo the equivalence modulo the domain knowledge ( $\equiv_{\text{DK}}$ ).

**Hanney et al.’s taxonomies.** In [Hanney *et al.*, 1995], adaptation is considered within three taxonomies. The first one classifies CBR systems wrt adaptation according four dimensions: (1) presence/absence of adaptation, (2) simple of multiple case(s) reused, (3) atomic or compound solutions, and (4) existence of interactions between solution parts. In a CBR system using a  $\circ$ -conservative adaptation, (1) adaptation is present (obviously), (2) it uses a single case (though the reuse of several source cases may also be performed with similar principles, as section 7 shows), (3) it uses compound solutions (a solution is represented by a formula that may be non atomic). The interaction between solution parts (cf. dimension (4)) is managed thanks to consistency: if the target problem is consistent with the domain knowledge, then the solution inferred by conservative adaptation is necessarily consistent (cf. axiom (R3)).

The second taxonomy is a taxonomy of tasks performed by the CBR systems (e.g., predict or design). Since conservative adaptation is defined independantly from a specific application, it is difficult to highlight for what kinds of CBR systems it is more useful. However, from the examples related to KASIMIR, it seems that this approach to adaptation may be useful for case-based decision support.

The third taxonomy is the one of the adaptation operators used in adaptation procedures. Four types of such operators are distinguished: (1) target elaboration operators, (2) role substitution operators, (3) subgoal operators, and (4) goal interaction operators. (1) Target elaboration consists in completing the description of  $\text{tgt}$  and/or in re-describing it. It is based on the domain knowledge, thus, if  $\text{tgt}'$  is obtained from  $\text{tgt}$  by elaboration then  $\text{tgt} \equiv_{\text{DK}} \text{tgt}'$ . Therefore,

<sup>1</sup>In fact,  $\Delta = \text{dist}(\text{Mod}(\psi), \text{Mod}(\mu))$  realizes this: (a)  $G^\Delta(\psi) \wedge \mu$  is satisfiable and (b) if  $\delta < \Delta$  then  $G^\delta(\psi) \wedge \mu$  is unsatisfiable. (a) can be proven as follows. Let  $\mathcal{J} \in \text{Mod}(\mu)$  such that  $\Delta = \text{dist}(\text{Mod}(\psi), \mathcal{J})$  (this makes sense since  $\text{Mod}(\psi) \neq \emptyset$ ). Thus,  $\mathcal{J}$  also belongs to  $\text{Mod}(G^\Delta(\psi))$  and so  $\mathcal{J} \in \text{Mod}(G^\Delta(\psi)) \cap \text{Mod}(\mu) = \text{Mod}(G^\Delta(\psi) \wedge \mu)$ , which proves (a). (b) can be proven by contradiction, assuming that there is some  $\delta < \Delta$  such that  $G^\delta(\psi) \wedge \mu$  is satisfiable. If so, let  $\mathcal{J} \in \text{Mod}(G^\delta(\psi) \wedge \mu)$ , thus  $\mathcal{J} \in \text{Mod}(G^\delta(\psi))$  and  $\mathcal{J} \in \text{Mod}(\mu)$ . Therefore  $\Delta = \text{dist}(\text{Mod}(\psi), \text{Mod}(\mu)) \leq \delta$ , which is in contradiction with the assumption  $\delta < \Delta$ . Thus, (b) is also proven.

conservative adaptation gives the same result with  $\text{tgt}$  and with  $\text{tgt}'$  since

$$\text{DK} \wedge \text{srce} \wedge \text{Sol}(\text{srce}) \quad \circ \quad \text{DK} \wedge \text{tgt} \quad \equiv \quad \text{DK} \wedge \text{srce} \wedge \text{Sol}(\text{srce}) \quad \circ \quad \text{DK} \wedge \text{tgt}'$$

(which is a direct consequence of the axiom (R4)). Therefore, the use of an elaboration operator together with conservative adaptation is useless from the viewpoint of the inferred solution (though it might have some consequences on the computing time). (2) A role substitution operator effects substitutions at various levels of granularity. This approach to adaptation is compared to conservative adaptation below, in the section about Wilke and Bergmann's taxonomy. (3) A subgoaling operator aims at decomposing the adaptation task into subtasks while (4) a goal interaction operator handles interactions between solution parts: it detects and repairs bad interactions. It may be considered that conservative adaptation performs a combination of operations of types (3) and (4). The specification of a target problem –the formula  $\text{tgt}$ – can be viewed as a goal specification (the goal is to find a solution consistent with  $\text{tgt}$ ). If  $\text{tgt} \equiv \text{tgt}_1 \wedge \text{tgt}_2$  then  $\text{tgt}_1$  and  $\text{tgt}_2$  are two subgoals of the target problem. Conservative adaptation provides a solution that is consistent with both subgoals. Therefore, this approach to adaptation considers possibly interacting subgoals as a combined use of operators of types (3) and (4) would do. However, if the revision operator is considered as a black box, then the distinction between (3) and (4) operators is not visible.

**Voß's taxonomy.** In [Voß, 1996], the Kolodner's taxonomy is reused. This paper also points out the notion of “shift in grain-size” as a class of operators used during adaptation, with the following subclasses: generalization (or abstraction), specialization (or refinement), focussing (from the whole to a part), and extending (from the part to the whole) operators. Several adaptation processes of CBR systems are studied according to the combined use of these operators. According to this viewpoint, the  $\circ_{\text{dist}}$ -conservative adaptations, as shown below (cf. Riesbeck and Schank's taxonomy), may be seen as combinations of generalization and specialization operators.

**Wilke and Bergmann's taxonomy.** In [Wilke and Bergmann, 1998], adaptation approaches are divided into transformational and generative ones (reusing the distinction between transformational and derivational analogies [Carbonell, 1983; Carbonell, 1986]): a transformational adaptation consists in modifying  $\text{Sol}(\text{srce})$  to build  $\text{Sol}(\text{tgt})$ , whereas a generative adaptation consists in reusing (“replaying”) on  $\text{tgt}$  the reasoning trace  $\text{RT}(\text{srce})$  associated with the source case  $(\text{srce}, \text{Sol}(\text{srce}))$ . Therefore, conservative adaptation is transformational. However, the principle of conservative adaptation might be reused in the following way for generative adaptation: if  $\text{RT}(\text{srce})$  is inapplicable on  $\text{tgt}$  – $\text{DK} \wedge \text{tgt} \wedge \text{RT}(\text{srce})$  is unsatisfiable–  $\text{RT}(\text{srce})$  is modified with a minimal change to infer some knowledge applicable to  $\text{tgt}$ , which could be done by computing

$$\text{DK} \wedge \text{srce} \wedge \text{RT}(\text{srce}) \quad \circ \quad \text{DK} \wedge \text{tgt} \tag{3}$$

(for some given revision operator  $\circ$ ) and by inferring from it a reasoning trace  $\text{RT}(\text{tgt})$  to be applied on  $\text{tgt}$  in order to help its problem-solving. Note that (3) illustrates the idea that generative adaptation corresponds to a transformational adaptation on the reasoning trace.

Among transformational adaptations, Wolfgang Wilke and Ralph Bergmann distinguish null (as in [Riesbeck and Schank, 1989]), substitutional, and structural adaptations. The notions of substitutional adaptation makes sense in a formalism with attribute-value pairs:  $\text{Sol}(\text{srce})$  is adapted in  $\text{Sol}(\text{tgt})$  by substituting values, while keeping the structure of the solution. By contrast, a structural adaptation alters the structure of the solution (e.g., addition or deletion of an attribute-value pair). This distinction between substitutional and structural adaptation is defined at a syntactic level, whereas conservative adaptation is based on a logical representation of cases and thus had to

be defined at a semantic level (it has to respect, in particular, the principle of irrelevance of syntax). However, substitutional and structural adaptations may be combined to revision-based approach to adaptation as it has been illustrated in section 8.2.2: a first solution to  $\text{tgt}$  is generated by a substitutional adaptation, and then, its consistency with the domain knowledge is restored thanks to a revision operator.

**Fuchs and Mille’s taxonomy.** In [Fuchs and Mille, 1999], a general model of adaptation in CBR is presented in a task formalism: starting from the analysis of several CBR systems implementing an adaptation process, they propose a hierarchical decomposition of adaptation in tasks and sub-tasks. The idea is that many (if not all) transformational adaptation procedures implemented in CBR systems may be modelled according to this scheme, considering in general only a subset of these tasks. Conservative adaptation may be seen as a way of instantiating the following subset of tasks:

- Copy solution;
- Select and modify discrepancies (by removing, substituting, and/or adding some pieces of information using the domain knowledge);
- Test consistency.

In fact, in conservative adaptation, this is the revision operator that processes all these tasks: it performs a minimal change that can be seen as a sequence of copy, modification, and test tasks. Moreover, it uses the domain knowledge in order to choose the features to be modified in order to reach consistency.

## 8.4 Conservative Adaptation in Description Logics

The AGM theory of revision is defined independently from a particular logic by a set of postulates. In this paper, we have concentrated on the application of this theory to propositional logic as studied in [Katsuno and Mendelzon, 1991a]. In order to apply conservative adaptation to a CBR system using a knowledge representation formalism, it may be useful to implement a revision operator in this formalism.

In particular, description logics (DLs) constitute a family of formalisms often used for CBR (see, e.g. [Kamp, 1996; González-Calero *et al.*, 1999]). For instance, KASIMIR uses the DL  $\mathcal{SHOIN}(\mathcal{D})$ . A DL is a knowledge representation formalism equivalent to a decidable fragment of FOL. In order to implement conservative adaptation, it is necessary to be able to implement a revision operator on the DL used. Now, the problem is that some DLs, in particular  $\mathcal{SHOIN}(\mathcal{D})$ , are not *AGM-compliant*, meaning that there exists no revision operator satisfying the AGM postulates in these formalisms: in [Flouris *et al.*, 2005], the issue of AGM-compliance for DLs is addressed. This paper also shows that if one of the AGM postulates is relaxed, then, any DL becomes AGM-compliant. Therefore, to apply conservative adaptation to a CBR system where cases and domain knowledge are expressed in a given DL, the following questions can be raised:

- Is this DL AGM-compliant?
- If yes, how could a revision operator be implemented?
- If no:
  - What are the consequences of the relaxation of the AGM theory on conservative adaptation?

- How could an operator satisfying the relaxed version of the AGM theory could be implemented?
- Is it possible to use a revision operator on a less expressive formalism that is AGM-compliant (such as another DL or propositional logic)?

Another issue to be studied about conservative adaptation in DL is how it can be formalized: in the current paper, the principle of conservative adaptation is given for any formalism (cf. section 2), but it is formalized only for propositional logic (cf. section 4). Hereafter, a formalization of conservative adaptation in a given AGM-compliant DL is proposed. For such a DL, let  $\circ$  be a revision operator. It is assumed that  $\text{srce}$ ,  $\text{Sol}(\text{srce})$ , and  $\text{tgt}$  are represented by instances (which correspond to constants in FOL) and are defined by assertions (formulas that characterize the instances and that relate them to other instances):  $\mathcal{A}_{\text{srce}}$ ,  $\mathcal{A}_{\text{Sol}(\text{srce})}$ , and  $\mathcal{A}_{\text{tgt}}$  are the sets of instances respectively characterizing  $\text{srce}$ ,  $\text{Sol}(\text{srce})$ , and  $\text{tgt}$ . As before,  $\text{DK}$  represents the domain knowledge (usually, in the form of a set of (*terminological*) *axioms*).  $\text{TSKCA}$ , the knowledge base from which the characterization of  $\text{Sol}(\text{tgt})$  can be inferred, is defined as in propositional logic:

$$\text{TSKCA} = (\text{DK} \cup \text{KB}_1) \circ (\text{DK} \cup \text{KB}_2)$$

(conjunctions of formulas –assertions and axioms– are not represented in DLs), with  $\text{KB}_1$ , the knowledge related to the source case and  $\text{KB}_2$ , the knowledge related to the target problem. We propose:

$$\begin{aligned} \text{KB}_1 &= \mathcal{A}_{\text{srce}} \cup \mathcal{A}_{\text{Sol}(\text{srce})} \cup \{\text{is-solved-by}(\text{srce}, \text{Sol}(\text{srce}))\} \\ \text{KB}_2 &= \mathcal{A}_{\text{tgt}} \cup \{\text{srce} \doteq \text{tgt}\} \end{aligned}$$

where, for four instances  $\text{pb}$ ,  $\text{sol}$ ,  $a$ , and  $b$ :

- $\text{is-solved-by}(\text{pb}, \text{sol})$  is an assertion meaning that the instance  $\text{sol}$  represents a solution of the problem represented by the instance  $\text{pb}$ .
- $a \doteq b$  is an assertion meaning that  $a$  and  $b$  represent the same individual (i.e., for each model  $\mathcal{I} = (\Delta_{\mathcal{I}}, \cdot^{\mathcal{I}})$  of  $a \doteq b$ ,  $a^{\mathcal{I}} = b^{\mathcal{I}}$ ).

The assertion  $\text{srce} \doteq \text{tgt} \in \text{KB}_2$  is used to relate the source and the target contexts: without it, a priori nothing about the source case can be inferred on the target problem. In other words,  $\text{DK} \cup \text{KB}_1 \cup \mathcal{A}_{\text{tgt}}$  may be satisfiable even if  $\text{srce}$  and  $\text{tgt}$  represent two disjoint problems (e.g.,  $\text{srce}$  represents a medical case of a woman and  $\text{tgt}$  represents a medical case of a man). This was not useful with propositional logic since, in this formalism, statements are about a sole individual (as if each interpretation domain contained only one element).

This formalization of conservative adaptation for DLs remains to be studied deeper and tested, e.g., in the framework of the KASIMIR system.

## 9 Conclusion and Future Work

In case-based reasoning, adaptation is often considered as a difficult task, in comparison to retrieval that is supposed to be simpler to design and to implement. This paper presents an approach to adaptation, called conservative adaptation, that is based on the theory of revision: it consists in keeping as much as possible from the source case while being consistent with the target problem and the domain knowledge. Conservative adaptation is defined, formalized in the framework

of propositional logic, and this formalism can be extended to other knowledge representation formalisms, e.g., the AGM-compliant description logics. Moreover, it is shown through examples that conservative adaptation covers some of the adaptations performed by experts in oncology. This approach to adaptation can be used for knowledge-intensive approaches to CBR, since it requires some domain knowledge. A noticeable feature of conservative adaptation is that the adaptation knowledge is part of the domain knowledge: it is not constituted by, e.g., a set of adaptation rules. This paper has also shown that the AGM theory of revision and the huge amount of researches based on this theory may be of interest for adaptation in CBR: a revision operator should be considered as a tool for designing an adaptation procedure, for conservative adaptation as well as for other approaches to adaptation.

Several theoretical issues about conservative adaptation have been addressed that deserve further investigations. They are listed hereafter. One of them is the design of a retrieval procedure suited to conservative adaptation: in section 6, a retrieval preference criterion is defined, but it raises two difficulties. First, it is sometimes insufficient to distinguish between two source cases. Second, its naive implementation is intractable.

Another issue is the combination of several source cases to solve a sole target problem. A way to do it by considering sequentially these cases is presented in section 7.1 but raises the problem of ranking them. Case combination can also be used to solve a decision problem with missing information, with the help of the Wald pessimistic criterion (cf. section 7.2). Other criterions could be used for such decision problems [Dubois *et al.*, 2001], and their integration in the framework of conservative adaptation constitutes another research direction.

The domain knowledge useful for conservative adaptation may have to be learnt. In section 8.1, a way of learning it based on failed conservative adaptations (leading to solutions that are not correct, according to a domain expert) is examined. It requires further investigation to see how it could be put in practice.

Conservative adaptation covers only a part of the adaptations actually performed by experts. Some other adaptations could be covered thanks to extensions of conservative adaptation, as section 8.2 shows. This section also shows, more generally, that revision operators can be used in various ways as tools for designing and implementing adaptation processes. The study of such extensions is another research direction.

In section 8.3, conservative adaptation is situated among several taxonomies of adaptation approaches in CBR. On the one hand, this section shows that conservative adaptation shares some common features with some adaptation approaches, in particular handling the problems of consistency, extending null adaptation (also called *copy of the source solution*), and, at least for  $\circ_{\text{dist}}$ -conservative adaptation, being equivalent to an adaptation by generalization and specialization. On the other hand, it appears that conservative adaptation may appear as “orthogonal” to these taxonomies, since it is defined at a semantic level, whereas these taxonomies are more oriented by the formalism of case representation.

From a practical viewpoint, future work will consist in the development and the use of a conservative adaptation tool to be integrated within the KASIMIR system. A first tool implementing the Dalal’s revision operator has been implemented, but it can be optimized. As an example, the most complex computation of a revision presented in this paper is the computation of TSKCA<sup>1</sup> in the example 6: it is based on 16 propositional variables and requires about 25 seconds on a current PC. Another practical issue is the integration of conservative adaptation in the KASIMIR system, which raises two problems. The first one is that both the cases and the domain knowledge of KASIMIR are represented in a formalism equivalent to the description logic  $\mathcal{SHOIN}(\mathcal{D})$ . Therefore, as discussed in section 8.4, either a revision operator has to be implemented for a description logic compatible with KASIMIR, or adaptation problems expressed in  $\mathcal{SHOIN}(\mathcal{D})$  are translated



in propositional logic and solved in this formalism.

The second problem of integration is linked with the already existing adaptation module of KASIMIR [d'Aquin *et al.*, 2006b], that is based on adaptation rules (roughly speaking). How conservative adaptation and this rule-based adaptation module can be integrated together in order to provide a unique adaptation module enabling complex adaptation processes (each of them being composed of a conservative adaptation and some rule-based adaptations)? This question should be addressed thanks to earlier work on adaptation composition and decomposition [Lieber, 1999].

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