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Progressive Polygon Foliage Simplification[#]

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Abstract

A leaf polygon decimation method, Progressive Leaves Union (PLU), is presented to gradually diminish the number of sparse polygons while approximately keeping the spatial occupation and color distribution of the foliage. In each step of decimation, a new leaf is constructed to represent other two leaves close in position and similar in shape. Many measurements of similarity are used to choose the best leaf pair to be united. All steps of simplification are recorded in preprocessing, and appropriate simplified models are chosen for different viewing positions and for different resolutions in visualization. Experiments have shown that PLU keeps visual effect of original foliage, and when combined with branch polygon simplification, is efficient for multi-resolution representation and view-dependent plant community visualization.

Keywords: polygon decimation, multi-resolution, plant visualization

1 Introduction

Plant is necessity in our living and working environment. Many industries are plant centered, such as agronomy, forestry and environmental defense. And many researches are plant based also, such as landscape image synthesis, computer simulation, theoretical biology, botany, ecology, horticulture, plant-environment interaction modeling. Different applications of plants in all these domains have particular interests in common in plant architectures [1], [2], [3].

Plant architecture modeling and growth simulation have been widely investigated, so that many successful modeling systems appear, such as AMAP, L-system, Xfrog, etc. AMAP is based on the theory of finite automata for plant life cycles [1], [4]. L-system, based on the thought of fractal patterns in living things' growth, uses syntactic methods to model plant growth [5], [6]. Xfrog is an easy tool to generate complex plant structures [3]. However, all these models are formed by such a great number of polygons that real-time visualization is not possible. Plant community visualization has attracted more and more people working in realistic image synthesis and virtual reality, in which efficient plant model simplification becomes a new focus [7], [8].

Recently, the application of ecophysilogic and morphogenetic modeling of GreenLab (AMAP) has begun to be used in the production in agronomy and forestry in INRIA [4], LIAMA [9] and CIRAD [4], [9]; therefore plant community visualization becomes important. The work in this paper is part of the application in forestry.

2 Related Work

Plant community contains a great amount of data in detailed foliage and branching information, independent, small in size, highly repetitive, convergent in distribution, and close to terrain. This leads to high rendering time and aliasing artifacts. Many methods have been developed to deal with these problems, mainly in five aspects, polygon decimation [10], [11], image based rendering [12], [13], [14], volume [13] and texel [15], [16] rendering, point based rendering [17], [18], and silhouette estimation [19], [20].

There are many achievements in mesh simplification [10], [11], so that it becomes an efficient method obtained in realistic and interactive display. But its object is continuous mesh, in which the connectivity relation of polygons is kept in processing. It is useful for solid parts of plants, such as branches and fruits. In some work of AMAP range products, polygon simplification is considered for branches when the plant is far from the viewer, where small branches are not displayed. The ellipsoidal skeleton is a generic model with efficient vertex clustering. Multi-level representation of a tree with ellipsoidal skeleton is impressive [19].

For foliage, the leaf polygons are discontinuously distributed in the space since leafstalks are always omitted. Foliage simplification is first considered in Foliage Simplification Algorithm (FSA) [7]: two leaves disappear to create a new one. Leaves obtained preserve an area similar to that of the collapsed leaves, and visual effect of foliage is kept to some

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degree after simplification with FSA. But FSA has the following drawbacks: (1) its construction of new leaf from Hausdorff distance does not keep the spatial expansion, so that the new leaf is sometimes smaller than one of leaves decimated, therefore in this case the area is not kept; (2) its choice of leaf pair for decimation is confined to the leaves of less than two *lnumber* difference, where *lnumber* means the number of leaves that have been collapsed to create this leaf, so that the chosen leaves pair are often far away and the new leaf is narrow and long; (3) its simplification degree is only the number of leaves to be kept after simplification and it need be specified by the user; so it is hard to chose this degree for the same tree in different positions. It is not clear how to chose this degree to generate the image of many trees growing together simplified by FSA (Figure 9 of [7] and Figure 10 of [8]) and why are so many burrs in the simplified foliage.

We provide another foliage simplification method, Progressive Leaves Union (PLU), to improve these shortages. Like FSA, the shape of leaves to be decimated is still confined to quadrilaterals in PLU, and 2 ideas of FSA are kept: using a new leaf to represent two small leaves and using Hausdorff distance to measure the leaf similarity. As innovation, all the three drawbacks above are overcome with concrete considerations: (1) using leaf diameter to represent its spatial occupation; (2) *lnumber* difference, called as union age difference in PLU, is not confined to any number; but is used as one of many measurements of similarity; (3) the simplification degree is calculated and dependent on three elements: distance of the tree to the eye, the viewing angle and the size of image to be synthesized, therefore PLU is view-dependent. And in addition, when using PLU, each simplified model of foliage keeps visual effect of the original model.

3 Leaves Union

Basic ideas

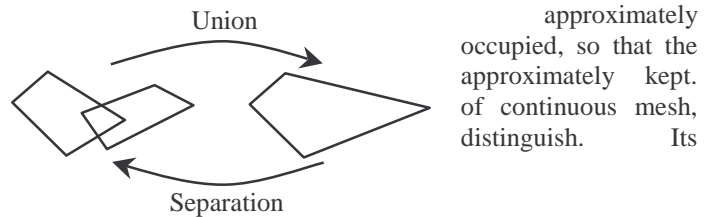
The characteristic of plants is that they have a highly repeated geometry. Each model is made up of two parts: sparse part and continuous part. The geometry of sparse part is isolated polygons, which are similar to each other in position and in shape, including leaves and flowers. The geometry of continuous part, mainly branches, is a special mesh, cylinder-like in shape.

When displaying a tree, especially a far tree, the size of a leaf is always smaller than a pixel. Therefore, foliage community display is both time consuming and causes aliasing effect. The purpose of PLU algorithm is to ameliorate time spending and aliasing effect in polygon tree display. The basic idea of PLU is to repeatedly make two leaves be united to a new one, so the connection relation of foliage polygons is changed. Therefore foliage simplification is different from traditional mesh simplification. It is a process of sparse polygon simplification.

In this paper, we focus our attention on developing efficient foliage simplification methods, so branch geometry simplification is only slightly introduced. It is carried out through the approximation of prism to cylinders with permitted error, so that the final is a mixture of polygon model and line model.

Leaves union

Leaves union means that a new leaf is generated to represent the space two or more leaves have shape and color of the overall plant are This process is rather different from edge collapse so the name LEAVES UNION is used to inverse process is leaves separation (Fig.1).



Attributes of leaves for union

The *areas* of leaves represent the distribution of the maximum of all the distances of different leaf X , $D(X) = \max\{d(x, y); x, y \in X\}$, represents the length it has occupied in space. Union *age* represents the times it is regenerated; it is also the number of original leaves decimated to generate it. The union age of the leaves in original plant model is set one as the beginning age.

Fig.1 Leaf union and separation

foliage. Leaf *diameter*,
vertices of the same

4 Similarity of Leaf Polygons

Distance and diameter represent the spatial expansion a point set has occupied. Normal and area contribute to foliage color distribution. Union age is proportional to leaf size. Therefore these five items are elements for similarity two leaves.

Normal Similarity

Normal similarity measures co-planarity of the two leaves for union

$$S_1(X,Y)=1-\left|\left\langle N(X),N(Y) \right\rangle\right| \quad (1)$$

Where $N(X)$ and $N(Y)$ represent the normal of leaf X and Y , and $\langle *,* \rangle$ is the inner product of two vectors. $S_1(X,Y)$ is between 0 and 1. The more co-planar the two leaves X and Y , the smaller $S_1(X,Y)$ is. The absolute value is needed because the leaves are double sided, and the inverse direction of leaf polygon is also its direction.

Positional Similarity

We use Hausdorff distance of two leaf vertex sets, the minimum of distances of all pairs of vertices of each leaf as positional similarity, which describes the degree of coincidence of two leaves.

$$S_2(X,Y)=\min\{d^2(p_1,p_2); p_1 \in X, p_2 \in Y\} / D_{Tree} \quad (2)$$

where D_{Tree} is the diameter of the whole tree. $S_2(X,Y)$ is much smaller than 1 at the beginning of leaves union. The less the value of (1) is, the closer the two leaves are.

Area Similarity

The pair of two leaves of close area values is more preferential to be united than the pair of ones of different area.

$$S_3(X,Y)=[A(X)-A(Y)]^2/[A(X)^2+A(Y)^2] \quad (3)$$

Where $A(X)$ is the area of leaf X . $S_3(X,Y) \in (0,1)$.

Diameter Similarity

Diameter similarity represents shape deformation. This is a special measurement on the similarity before and after union.

$$S_4(X,Y)=1-[D(X)+D(Y)]/[2*D(X,Y)] \quad (4)$$

Two leaves of the same shape in different positions have different diameter changes for union. $S_4(X,Y) \in (0,1)$.

Union Age Similarity

Union ages represent union order when a new leaf is generated. It still makes contribution to further union.

$$S_5(X,Y)=[G(X)+G(Y)+|G(X)-G(Y)|]/N_{Tree} \quad (5)$$

Where $G(X)$ is the union age of leaf X and N_{Tree} is the number of all leaves of the tree.

Diameter Penalty

Smaller leaves should be united earlier than the bigger ones. Leaves with similar size should make relatively smaller errors after union.

$$S_6(X,Y)=D(X)+D(Y) \quad (6)$$

where $D_{Foliage}$ is the diameter of the set of all leaves.

5 Progressive Leaves Union Algorithm

In PLU, we use weighted average similarity to show a comprehensive priority for leaves union.

Finding the best similarity

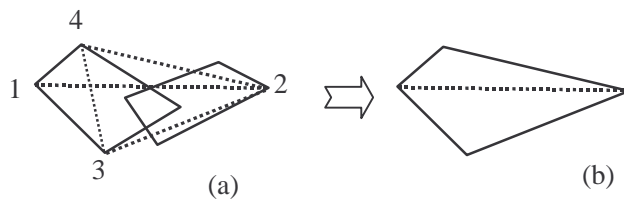
Integral similarity is constructed as a weighted mean value of all these six aspects of similarity:

$$S(X,Y)=k_1S_1(X,Y)+k_2S_2(X,Y)+k_3S_3(X,Y)+k_4S_4(X,Y)+k_5S_5(X,Y)+k_6S_6(X,Y) \quad (7)$$

Where k_i ($k_i > 0$) are weights and $k_1+k_2+k_3+k_4+k_5+k_6=1$. The smaller $S(X,Y)$ is, the more similar X and Y are. Different values of k_i have different influences on visual effect (Fig. 4).

Construction of a New Leaf

(1) Treating all eight vertices of leaves as equal; (2) Finding the different vertices from the eight two ends are chosen as vertices of point 1 and point 2 in Fig.1 (a). distance would be the diameter of Finding a vertex from other six



the two simplified longest distance of vertices, and these the new leaf, see The longest the new leaf. (3) which has the

Fig.2 New polygon construction

biggest sum of distances from this vertex to the two chosen in the former step, see point 3 in Fig.1 (a); (4) Finding a vertex from the other five of biggest sum of distances from this vertex to the three one chosen in the former two steps, see point 4 in Fig.1 (a); (5) Translating point 3 and point 4 so that their center coincides with that of point 1 and point 2, therefore the new quadrilateral is planar. (6) Keeping the original normal vectors at point 1 and point 2 from the two leaves before union, and choosing those at point 3 and point 4 the normal vector of the new leaf.

Union Process Records

The computation of all these similarities is time consuming, so the whole work of repeated leaves union is carried out in pre-processing and the whole information is recorded in file. We use two arrays to record the information.

Geometry and the serial number of all polygons decimated and produced in each step are sufficient for overall simplification process. *Polygon array* $\{P[i]; 1 \leq i < 2n\}$ is used to record all the polygons ever appeared, where $\{P[i]; 1 \leq i \leq n\}$ is the list of polygons of the original model, and $\{P[n+i]; 1 \leq i < n\}$ is the list of union polygons produced in simplification step i . *New polygon Info Array* $\{N[i] = (A[i], B[i], C[i], D[i]); 1 \leq i < n\}$ is used to record the information of newly generated leaves, where $A[i]$ and $B[i]$ are the serial number of polygons decimated in simplification step i ; $C[i]$ is the serial number of polygon generated in simplification step i ; and $D[i]$ is the diameter of $P[n+i]$.

Polygon Reorganization

All the leaves union process forms a binary tree FBT (Foliage Binary Tree). In FBT, all the leaves are the leaves of the plant, and all the branches are newly generated leaves. From the construction method of the new leaf, it is not hard to prove that the diameter of the child in FBT is no greater than that of his father. FBT is a diameter monotonous binary tree, which is a useful property for FBT traverse.

Overall PLU process

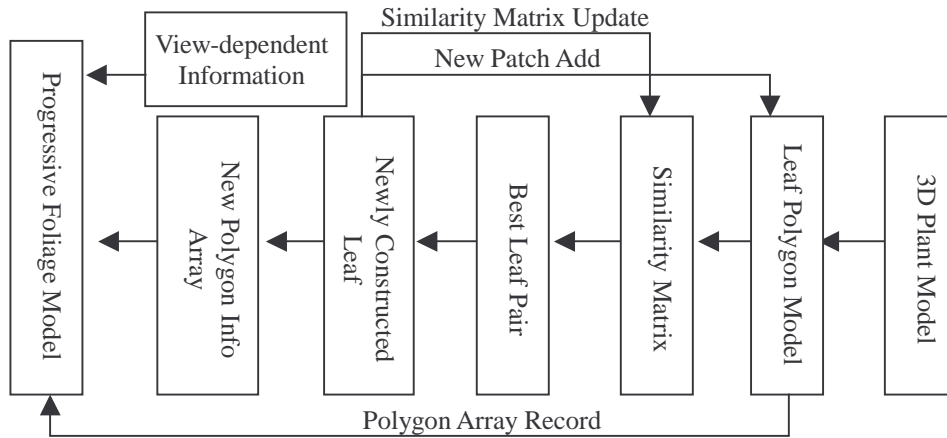


Fig.3 Overall Simplification Process

6 Experiment

Model choice

Four models with quadrilateral leaves or flowers have been generated by commercial AMAP software. A 4 years old holly tree model is used to show the functions of coefficients, an 8 years old holly tree model to show the progressive process, a 10 year old lilac model is used to show view dependence, a 14 year old holly model is used for community visualization. Flowers of lilac tree are also quadrilaterals in shape, so they are simplified in the same way as leaves.

We only use polygon models by AMAP software, so this algorithm is not confined to AMAP format.

Coefficients

To test the functions of all the six aspects of similarity, we set only one coefficient as the main coefficient with value 0.95 and all others as 0.01 in each of six tests. We chose a simple model, a 4 years old holly tree with 355 leaves, 1.85 meters high and in distance from the viewer, and its foliage is simplified to %40. We obtain the following pictures, all

355×799 pixels, where the first (left) is the original foliage model.

It can be seen in Fig. 4 that: (1) When normal similarity coefficient k_1 is the main, foliage distribution is not properly kept; (2) When area similarity coefficient k_3 is the main, there is many deformation in foliage; (3) When diameter

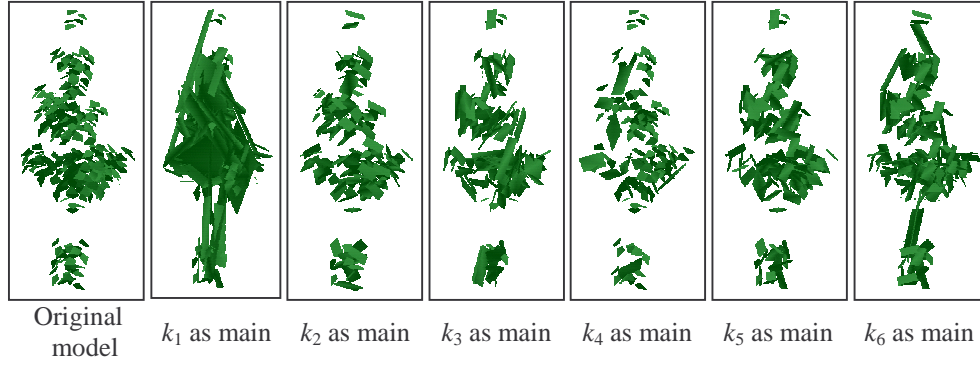


Fig. 4 Functions of coefficients

similarity coefficient k_4 is the main, some big leaves are generated in foliage silhouette; (4) When positional similarity coefficient k_2 , or union age similarity coefficient k_5 , or diameter penalty coefficient k_6 is the main, original shape is kept;

In all models in this paper, we set coefficients k_1 , k_3 and k_4 with small value and k_2 , k_5 and k_6 with great value, see Table 1.

Table 1 Values set for coefficients

Coefficient	k_1	k_2	k_3	k_4	k_5	k_6
Values	0.0375	0.3125	0.0375	0.0375	0.2875	0.2875

Simplification degree

We need choose a right measurement for the simplification degree, i.e., estimation of the error between before and after union. It is very hard to measure this error, so at present only leaf diameter is used. The final result of display is an image, so the degree is image size dependent also. Therefore, the diameters of all the leaves after simplification need be no greater than a special value, product of a constant and permitted pixel error. This constant is called as *detail constant*, which can be chosen by user to determine detail level. The whole process of simplification is to traverse FBT to get all the leaves with appropriate size. When a father leaf is chosen, all generations of his children leaves are discarded.

Visual effect

We chose the 8 years old holly tree to show the simplification effect, which is 2.96 meters high and it is 7.41 meters away from the viewer. In order to show an easy comparison of simplification, we increase permitted pixel error to make all images the same size for different simplification degrees. The size of all these images is 370×543 pixels. The detail constant is set as 30. See Fig. 5 for visual effect, especially foliage silhouette. See the number of polygons after simplification in Table 2.

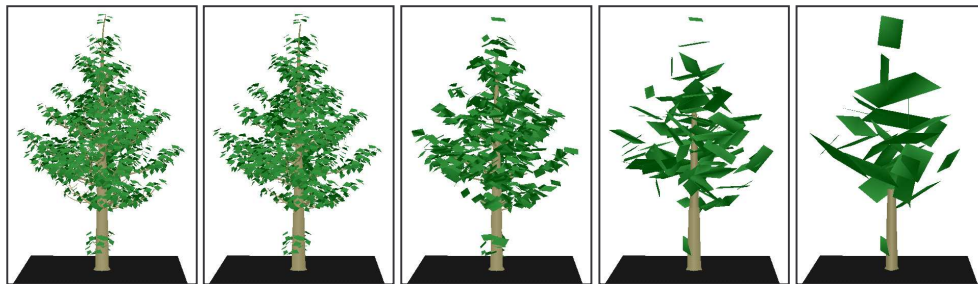


Fig.5 Progressive simplification of a holly tree

Table 2 Simplification for different resolution

Permitted Pixel Err	Leaf Polygon Number	Percentage After simplification	Branch Polygon Number
0.50	1785	%100.0	10291
1.50	1659	%92.9	3069
3.00	590	%33.1	828
7.00	96	%5.4	195
13.00	27	%1.5	78

Information Inherited In Simplification

It is easily deduced from the construction of the new leaf, that, in each simplified model, at least half of all vertex coordinates and corresponding normal vectors are copied from original foliage model, which represent foliage silhouette and shading distribution, see Fig. 5.

Effect Comparison

In order to compare the visual effect of simplified model with original one, the same tree is positioned in different places of three different distances: tree pair Lilac 1 and Lilac 2, pair Lilac 3 and Lilac 4, and pair Lilac 5 and Lilac 6 have the same distance respectively. They are all displayed in the same environment (Fig. 5, 1262×705 pixels). Lilac 1, Lilac 3, and Lilac 5 are view-dependently simplified and Lilac 2, Lilac 4, and Lilac 6 use original foliage model. It can be seen that each pair of tree have similar visual effect and Lilac 2 looks alias since its leaves are too small in the image.

Table 3 Data of view-dependant simplification

Model name	Model type used	Leaf Polygon Number	Leaf Polygon Percentage	Branch Polygon	Distance
Lilac 1	Simplified model	3065	%46.1	2249	78.43
Lilac 2	Original model	6644	%100.0	2249	78.43
Lilac 3	Simplified model	4239	%63.8	3025	58.52
Lilac 4	Original model	6644	%100.0	3025	58.52
Lilac 5	Simplified model	5632	%84.8	4036	38.56
Lilac 6	Original model	6644	%100.0	4036	38.56

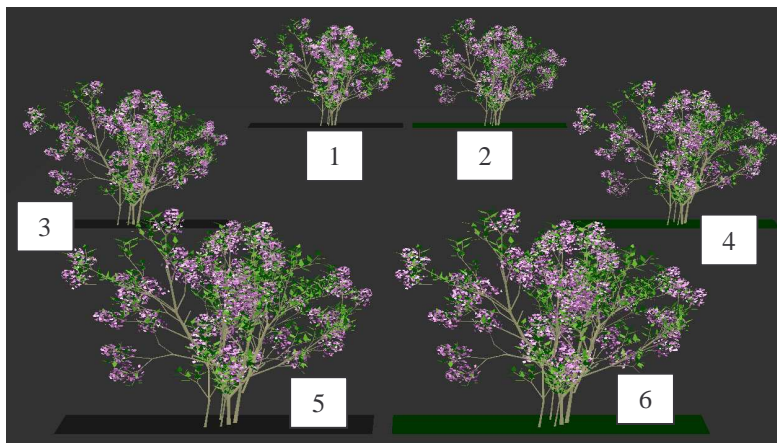


Fig.6 View-dependant simplification

Performance

Table 4 Performance In Pre-processing

Model names and sparse group(s)	8 years old holly leaf	10 years old lilac				14 years old holly leaf
		Flower	Leaf 1	Leaf 2	Leaf 3	
Polygon Number	1785	2268	1072	2604	700	7016
Time spent (seconds)	1284	2460	260	3794	70	75730

It can be seen that, in pre-processing, when leaf number increases, time spent increase much faster.

This test for numerical calculation and image synthesis is taken in Silicon Graphics Fuel™ Workstation with

R14000ATM processor (1500 MHZ), 1024 MB memory, and VProTM Graphics hardware V12 (128MB graphics memory).

7 Application: View-dependent Visualization

In forest visualization and outdoor scene navigation, there are a large number of leaves and flowers to be displayed; this method can be used to decrease the polygon number of tree models, so that the display speed can be accelerated.

We chose a complex model, a 14 years old holly tree with 7016 leaves, 5.17 meters high. It is positioned in 18 places with different distance to viewer from 74.11 meters to 370.04 meters, where the foliage of the closest tree is so close that it is not simplified and the farthest tree only has 1055 leaves. We obtain the following pictures, Fig.6, with 1457×512 pixels. The number of all the polygons in this picture is 67,548, much smaller that that of its original model, 157,642 quadrilaterals. Therefore it is faster than real-time when navigating in this environment.

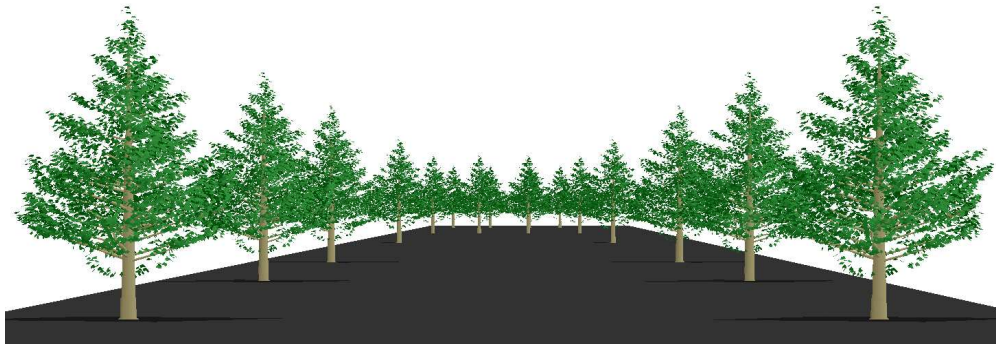


Fig.7 View-dependent Forestry Visualization

This method can also be a useful reference to create level of detail plant models and to realize view-dependant visualization.

8 Conclusion and Further Work

From the above description and experiment, it is evident that PLU method in this paper has four advantages for foliage simplification: (1) Keeping closed silhouette; (2) Keeping closest leaf density distribution or the average of leaf areas in all directions; (3) The simplification process is view-dependent; (4) Aliasing is ameliorated.

This paper still has the following disadvantages for further improvement:

- General shape of leaves and flowers: only quadrilateral leaves and flowers are treated in this paper. For other shapes, methods should be developed.
- Topological structure maintenance: the process of leaf union should keep the topological structure of the tree. In each step, the union should be confined to all the leaves belonging to the same level of branch, including the union of a leaf cluster and that of a compound leaf. The destination of simplification should be keeping the outer silhouette surface of the tree.
- Silhouette computation: silhouette is an important visual effect of a tree, so special computation is required for silhouette computation, so that the leaves in silhouette will be latest simplified. Ellipsoidal skeleton can be a useful choice.
- Transparency: union of overlapped leaves to a new big one will change the transparent effect, so that the simplified tree will be too dark, therefore a consideration of transparency changes is needed. Especially when leaf area increases after union, its transparency needs decreasing.
- Efficient similarity computation: computation of similarities and that for the new produced polygon is very time consuming since it includes many calculations of distance from one leaf to all others, and the time spent increases much faster than the number of leaves in consideration. The high-level data structure of ellipsoidal skeleton and its partition of cloud points sets can be used for separating a big leaf group to many smaller ones and saving preprocessing time.
- Geomorphing: Construction of continuously transferring intermediate models between different resolution representations for navigation in a continuously changing environment to avoid hopping.
- Leaf simplification errors: methods need be developed to estimate errors of the difference between before and after

leaves union in each step of simplification, so that error control would be more rational.

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