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Central configurations of four gravitational masses with an axis of symmetry (Abstract)

Daniel Lazard

This talk describes a common work with J.-C. Faugère.

A central configuration of n masses under gravitational potential is a configuration such all accelerations are oriented toward the center of masses and proportional to the distance to this center. It follows that the plane central configurations are those that remain homothetic to themselves, with convenient initial velocities. For three masses, these configurations are well known as Lagrange positions.

A well known conjecture is that the number of central configurations of n bodies is always finite when the masses are fixed.

In this talk, we describe the number of central configurations of four masses in the plane, which have an axis of symmetry such symmetric masses are equal. In the case where none of the masses are on the axis (trapeze configuration), it is rather easy to show that for any value of the two masses, there is exactly one central configuration.

In the case of two masses on the axis and two symmetric masses normalized to 1 (this is the only other case if the bodies are not aligned), some reductions allow to model the problem by the system of equations

$$\begin{aligned}
&(b-d)^2 - 2(b+d) + 1 + f = 0,\\
&m(B-1) - (D-F)(d-b+1) = 0,\\
&n(D-1) - (B-F)(b-d+1) = 0,\\
&b^3B^2 = 1, \qquad d^3D^2 = 1, \qquad f^3F^2 = 1.
\end{aligned}$$

where m and n are the remaining masses, the lower case other variables are squares of distances and the upper case variables are the cubes of the inverses of the same distances. Thus all variables should be positive. This system seems rather simple; however it is not having a Bezout number of 1000 and 102 complex solutions for m and n fixed.

The number of central configurations is 1, 3 or 5 depending on the values of the masses m and n. This number is determined by the position of the point (m, n) relatively to a plane curve in the space of the masses. The implicit equation of this curve has degree 424, more than 50,000 monomials and integer coefficients of more 200 decimal digits.

For proving this result, we have designed a general algorithm for discussing zero-dimensional problems depending on few parameters. In this particular case, it needs algorithms for Gröbner basis computation, for elimination, for drawing plane curves, for the certification of a drawing (proving that no branch is omitted) and for fast real solving zero-dimensional systems of equations.

The size of the curve which is a part of the solution shows that this problem is near from the limit of the present technology, and we guess that the only software which is presently able to do such a computation is the last version of FGb/RealSolving, which is developed in our team.