



HAL
open science

Rough Sets based Proofs Visualisation

Laurent Vigneron, Anita Wasilewska

► **To cite this version:**

Laurent Vigneron, Anita Wasilewska. Rough Sets based Proofs Visualisation. 18th International Conference of the North American Fuzzy Information Processing Society - NAFIPS'99, invited session on Granular Computing and Rough Sets, 1999, New York, USA, pp.805-808. inria-00098752

HAL Id: inria-00098752

<https://inria.hal.science/inria-00098752>

Submitted on 26 Sep 2006

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Rough Sets based Proofs Visualisation

Laurent Vigneron

LORIA – Université Nancy 2

Campus Scientifique, B.P. 239

54506 Vandœuvre-lès-Nancy Cedex, France

vigneron@loria.fr

Anita Wasilewska

Department of Computer Science

State University of New York

Stony Brook, NY 11794-4400, USA

anita@cs.sunysb.edu

Abstract

We present here an approach we used for proving important properties of clopen topological spaces. We combine powerful theorem provers techniques (and implementations) with a graphical technique based on a graphical representation of a rough set, called Rough Diagrams. Rough Diagrams are a generalization of a classical notion of Venn Diagrams for algebra of sets to clopen topological spaces. We use them as a powerful automated technique of constructing counter-models of properties the prover has a hard time proving and the user might suspect of being false. It means we propose to add a visual tool to a prover that after some fixed number of prover deductions would start constructing a visual counter-model for a property the prover is trying to prove. A prover with the visual tool is called a visual prover. The visual prover has a completeness property: for any rough set equality we can construct its proof or its counter-model.

1. Introduction

The principle of rough sets [2] is to consider that some elements of the universe are equivalent in the context of the knowledge about them. This theory provides a complementary model to fuzzy [3] or evidence [5] models for dealing with imprecise, noisy, or incomplete information. It is also a departure point for studies in machine learning, knowledge discovery and lately in database mining [13]. It has also been applied to various domains such as medical diagnosis, information retrieval, control algorithms acquisition, and market analysis.

We have extended this notion of rough equality to generalized rough sets [10], i.e. any topological space with a clopen property.

A *topological space* is defined by a function I over

a set X , called the *interior* of A , such that the following properties are satisfied, for any $A, B \in X$: $I(A \cap B) = IA \cap IB$, $IA \subset A$, $IIA = IA$, and $IX = X$. For every $A \subset X$, the set $-I-A$, where $-$ is the complement function over X , is called the *closure* of A and denoted by CA .

A topological space satisfies the *Clopen sets property* if, for any $A \subset X$, $CIA = IA$ and $ICA = CA$.

This relation between rough sets and topological spaces has been described by Wiweger [12]: rough sets represent any approximation topological space, i.e. any triple (X, I, R) , where (X, I) is a topological space, R is an equivalence relation, and I is induced by R .

Topological spaces and clopen topological spaces being represented respectively by topological Boolean algebras and rough Boolean algebras [6, 1, 4, 9], we have used a theorem prover for studying these abstract algebras.

This study was motivated by the lack of information concerning these algebras. This is due to the fact that they are very difficult to study “by hand”. Running an automatic tool brings this advantage to perform a fastidious and repetitive work.

The prover used is called `daTac`, and has been developed by one of the authors [7]. Its principle is to apply deductions in first-order logic with equality. Its specificity is to apply deductions with a built-in theory. This theory is composed with associative-commutative properties of operators (such as union and intersection for Boolean algebras), some very costly properties when explicitly used as axioms.

We have mainly used the prover as a generator of properties. The principle is to give `daTac` the specification of an algebra and to ask the prover to do some deductions. Then it is arbitrarily stopped after some time, and we get the set of newly generated properties.

For topological and rough Boolean algebras, we got

hundreds of new properties [11]. From these properties, we have proved some patterns, and also compared algebras, isolating for instance some formulas of rough Boolean algebras that are not properties of topological Boolean algebras.

We have also studied the rough equality [10]. We have shown that there exists an algebra such that the topological equality is a congruence with respect to the operators of that algebra. This has been done by defining new union and intersection operators.

2. Limits of the Prover

We have used the prover `daTac` for generating hundreds of properties. This has been obtained after hours (or days, sometimes) of automatic work, impossible for a human being. But a major problem is to exploit this amount of new knowledge. Some properties may be quite large, with several variables. They may be difficult to understand. What we have done is to infer some patterns from a subset of properties, and sometimes we had got the intuition of some additional properties. Using the prover for trying to confirm an idea is a natural step, but this is not always successful. Since there are infinitely many properties, the user cannot be sure that the prover will be able to prove something in a reasonable delay. This may take several days, or the prover may even stop by lack of memory.

For instance, we have inferred the following patterns, for all n and m :

$$\begin{aligned}
 I(Ca_1 \cup \dots \cup Ca_n \cup Ib_1 \cup \dots \cup Ib_m) \\
 &= Ca_1 \cup \dots \cup Ca_n \cup Ib_1 \cup \dots \cup Ib_m \\
 C(Ca_1 \cap \dots \cap Ca_n \cap Ib_1 \cap \dots \cap Ib_m) \\
 &= Ca_1 \cap \dots \cap Ca_n \cap Ib_1 \cap \dots \cap Ib_m
 \end{aligned}$$

This intuition has been due to the fact that the prover has proved some of the first elements of these patterns. For a formal proof of the whole pattern, we have had to study how the first elements have had been generated, and we have found a generalization of these proofs. For this step, the prover could not help.

Then, from these patterns and from our try to define new union and intersection operators for proving topological equality can be a congruence, we have had the intuition of the following properties:

$$\begin{aligned}
 I(a \cup Ib) &= Ia \cup Ib & C(a \cup Ib) &= Ca \cup Cb \\
 C(a \cap Ib) &= Ca \cap Ib & I(a \cap Ib) &= Ia \cap Ib
 \end{aligned}$$

But the prover has been unable to prove even one of them, and we have had no idea of how to do it. All our tries have been unsuccessful, and we have doubt

of the existence of these properties.

In general, this happens very often that we do have an intuition for a property, but not for the way to prove it. And sometimes we have the intuition that a property is false, but we are unable to find a counter-example.

In this case of rough sets, we have used a graphical representation for being able to study some particular cases (see Figure 1).

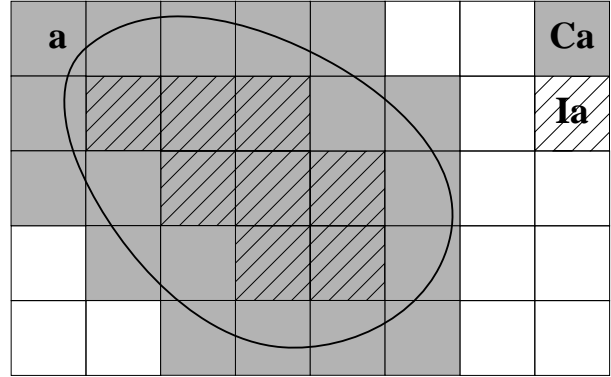


Figure 1. Rough Diagrams

3. Rough Diagrams

Rough Diagrams are a generalization of a classical notion of Venn Diagrams for algebra of sets to clopen topological spaces.

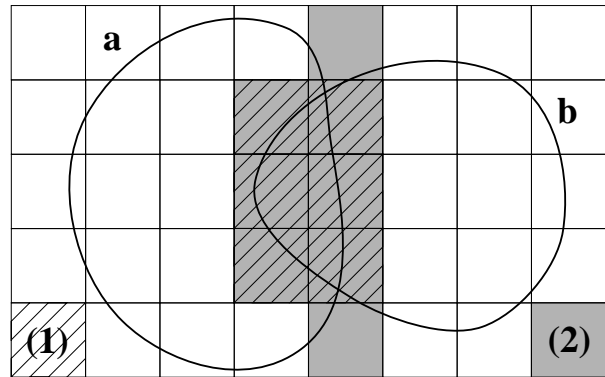


Figure 2. (1): $C(a \cap b)$, (2): $Ca \cap Cb$

A graphical representation has the advantage to be much more easy to understand than an algebraic formula. For instance, there are two important properties that are not satisfied by rough Boolean algebras: $C(a \cap b)$ is not equal to $Ca \cap Cb$, and $I(a \cup b)$ is not

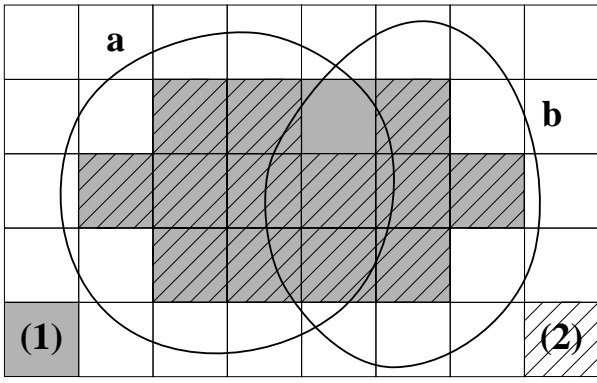


Figure 3. (1): $I(a \cup b)$, (2): $Ia \cup Ib$

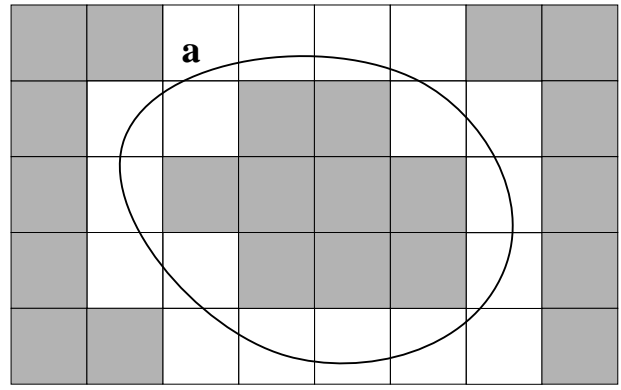


Figure 5. $I(a \sqcup \neg a) (\neq I1 = 1)$

equal to $Ia \cup Ib$. One can build counter-examples of these properties, as illustrated in Figures 2 and 3.

Finding counter-examples by drawing sets is much more easy than doing calculi with algebraic formulas. Such graphical calculi can also be used for testing some cases of a property that looks to be satisfied, as shown in Figure 4 for $I(a \cup Ib) = Ia \cup Ib$.

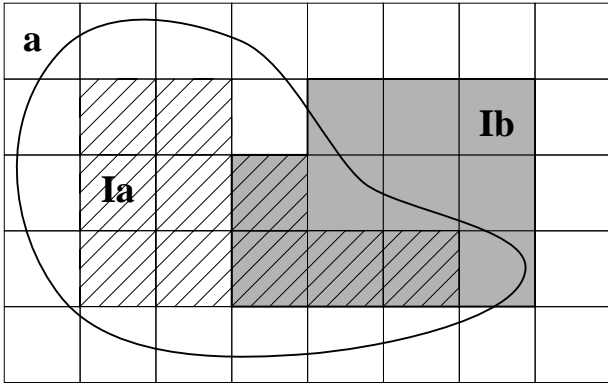


Figure 4. $I(a \cup Ib) = Ia \cup Ib$

We have used rough diagrams for guiding our search for some properties, when the prover unable to do it. We have also used them for defining new union and intersection operators [8] for defining a new algebra, and proving topological equality is a congruence. Without this graphical representation, we would never have succeeded to do it.

For example, this has been easy to show that, in the congruent algebra, $a \sqcup \neg a$ is not equivalent to 1 , where \sqcup is our new union operator; $I(a \sqcup \neg a)$ and $C(a \sqcup \neg a)$ would have to be equal to $I1$ and $C1$, respectively. A counter-example is show in Figure 5.

4. Towards A Visual Prover

From our experiments with Rough Diagrams, we wonder if this could be possible to define a method for building counter-examples, graphically. Such a method can be based on a database of the most current cases, for two or three sets. Looking for a counter-example would consist in trying each case, and trying new cases resulting of the combination of the initial ones.

An even more interesting result would be to define a general method, able to deal with any kind of formula, whatever the number of sets is. This should be possible by decomposing the formula for considering simple cases, and then combining these cases for considering back the whole formula.

We are currently working on this subject, and the first results are very encouraging. Our purpose is to define a visual tool, either automatic or semi-automatic. This tool would be very useful for a lot of researchers working on rough Boolean algebras, or modal logics since the S5 modal logic is a special case of these algebras.

Besides this use of the visual tool, we propose to combine it with a prover, to get a visual prover. Our study of clopen topological spaces has been done by the prover and by Rough Diagrams. We have worked on Rough Diagrams when the prover was unable to prove some properties. And we have used the prover for getting a formal proof of some properties pictured after some drawings. These two aspects are complementary. Offering the opportunity to combine them into an automatic tool is our challenge. This would lead to a complete tool, able to look for a formal proof or to find counter-examples.

References

- [1] J. C. C. McKinsey and A. Tarski. The Algebra of Topology. *Annales of Mathematics*, 45:141–191, 1944.
- [2] Z. Pawlak. *Rough Sets*. Theory and Decision Library. Kluwer Academic Publishers, 1991.
- [3] Z. Pawlak. Rough Sets – A New Approach to Vagueness. In L. Zadeh and J. Kacprzyk, editors, *Fuzzy Logic for the Management of Uncertainty*, pages 105–118. John Wiley & Sons, Inc., 1992.
- [4] H. Rasiowa. *An Algebraic Approach to Non-Classical Logics*, volume 78 of *Studies in Logic and the Foundations of Mathematics*. North-Holland, 1974.
- [5] A. Skowron and J. W. Grzymala-Busse. From Rough Set Theory to Evidence Theory. In M. Fedrizzi and J. Kacprzyk, editors, *Advances in the Dempster Shafer Theory of Evidence*, pages 193–236. John Wiley & Sons, Inc., 1994.
- [6] M. H. Stone. Boolean algebras and their relation to topology. *Proceedings of National Academy of Sciences*, 20:197–202, 1934.
- [7] L. Vigneron. Automated Deduction Techniques for Studying Rough Algebras. *Fundamenta Informaticae*, 33(1):85–103, 1998.
- [8] L. Vigneron and A. Wasilewska. Rough Diagrams. In *Proceedings of the 6th Workshop on Rough Sets, Data Mining and Granular Computing (RSDM-GrC'98) at the 4th Joint Conference on Information Sciences*, Research Triangle Park (North Carolina), October 1998.
- [9] A. Wasilewska. Topological Rough Algebras. In T. Y. Lin and N. Cercone, editors, *Rough Sets and Data Mining: Analysis of Imprecise Data*, pages 411–425. Kluwer Academic Publishers, 1996.
- [10] A. Wasilewska and L. Vigneron. On Generalized Rough Sets. In *Proceedings of 5th Workshop on Rough Sets and Soft Computing (RSSC'97), 3d Joint Conf. on Information Sciences, Research Triangle Park (NC)*, 1997.
- [11] A. Wasilewska and L. Vigneron. Rough Algebras and Automated Deduction. In L. Polkowski and A. Skowron, editors, *Rough Sets in Knowledge Discovery 1*, pages 261–275. Springer-Verlag, July 1998.
- [12] A. Wiweger. On Topological Rough Sets. *Bulletin of Polish Academia of Science (Mathematics)*, 37:51–62, 1988.
- [13] W. Ziarko. Rough Sets, Fuzzy Sets and Knowledge Discovery. In *Proceedings of the International Workshop on Rough Sets and Knowledge Discovery (RSKD'93)*, pages 12–15, Alberta (Canada), October 1993. Springer-Verlag.