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PERFORMANCE ANALYSIS OF A FLEXIBLE MANUFACTURING CELL FEEDING SEVERAL MACHINES WITH FINITE BUFFER CAPACITY

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Performance Analysis of a Flexible Manufacturing Cell Feeding Several Machines with Finite Buffer Capacity

Analyse des Performances d'une Cellule de Production Flexible
Approvisionnement Plusieurs Machines Avec Stocks Tampons de Capacités Finies

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Abstract

We study the relationship between buffer capacity and productivity in a system consisting of a flexible cell feeding several dedicated machines. The processing times at both the flexible cell and the dedicated machines are random variables. Each dedicated machine has a finite local buffer to hold parts that are waiting to be processed and the flexible cell is required to produce parts according to specified production ratios. We develop a simple analytic estimate for machine utilization in steady state for this system. Such an analytic estimate may be useful in the planning stages of this kind of manufacturing system both in terms of understanding the tradeoff between increasing buffer capacity and utilization and in estimating the appropriate size and distribution of buffers in the system. Comparison with simulation for the case of exponential processing times indicate that the estimate works reasonably well.

Résumé

Nous étudions la relation entre les capacités des stocks tampons et la productivité dans un système composé d'une cellule flexible qui approvisionne plusieurs machines dédiées, chacune à un type de produit. Les temps de fabrication sur la cellule et les machines sont aléatoires. Chaque machine a son propre stock tampon et la cellule fabrique les différents types de produits suivant des ratios déterminés. Nous proposons une approximation simple du taux d'utilisation des machines en régime permanent pour ce système. Cette approche peut être utile au niveau de la conception de tels systèmes pour comprendre la relation entre le taux d'utilisation des machines et la taille des stocks tampons et pour définir cette dernière. Des comparaisons effectuées avec la simulation dans le cas de distributions exponentielles négatives des temps de fabrication indiquent que nos estimations sont acceptables pour une première évaluation.

Key words: Performance analysis, Random processing times, Design of manufacturing systems, Finite buffers

Mots clés: Analyse de performances, Temps opératoires aléatoires, Conception de systèmes de production, Stocks tampons de capacités finies

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1 Introduction

The problem we consider arises in the following type of manufacturing system. A flexible cell produces parts for a set of N dedicated machines. The flexible cell has an infinite supply of raw material at its disposition and it is capable of producing N different part types, one part type for each dedicated machine. The time needed to produce a type i part at the flexible cell and the time to process a type i part at machine i are stochastic. A set of production ratios, specifying the proportion of parts of each type to produce over an infinite horizon, are assumed given. It is the job of the flexible cell to supply parts to the dedicated machines while respecting the production ratios. Each dedicated machine has a finite local buffer to hold parts that are waiting to be processed at that dedicated machine. Because of these finite buffers, the flexible cell can be blocked when a buffer is full. The general question we wish to investigate is, given a scheduling policy at the flexible cell that will respect the production ratios - i.e. a method by which parts are selected for processing at the flexible cell - and given a set of buffer capacities, what will be the utilization of each of the downstream dedicated machines, where utilization of a machine is defined to be the fraction of time a machine is busy.

Exact analysis of this type of system, even under fairly simple assumptions, has been unsuccessful mainly because of the presence of finite capacity buffers. An exact Markovian model can be stated in the case of exponential processing times but the resulting steady state flow equations have not been solved analytically because of the size of the system. Numerical solution, of course, is possible but the basic problem with the Markov chain approach is the dimension of the state space, which has size $\approx \prod_{i=1}^N B_i$, where B_i is the size of buffer i . If one tries alternative analytic approaches, the basic problem that becomes apparent as soon as one begins to analyze the system is the non-linear interaction of blocking probabilities that occurs when more than one of the dedicated machines can be blocked. Because exact analysis does not seem feasible, we have concentrated on developing a simple analytic approximation which estimates the steady state utilization of the dedicated machines as a function of the system parameters. Our goal is to develop a relatively simple approximate model that models this system and

which can be used potentially in the early planning stages of such a manufacturing system.

There exists a wide literature on the modelling and analysis of manufacturing systems with finite buffers. Many of these papers have focused on the simplest of these problems, the case of two machines in tandem separated by a finite buffer. For example, see Gershwin and Berman (1981), Gershwin and Schick (1983), Buzacott and Hanifin (1978), Coillard and Proth (1983), Ignall and Silver (1977), Proth (1981), Forestier (1980), and the references therein. Generally these papers present models capable of predicting the effect of finite buffers on various performance measures, notably, steady state measures of system throughput and mean buffer content. Various approximations have been proposed for the case of several (>3) machines in tandem, each separated by a finite capacity buffer. For example, see Gershwin (1987), Dallery, David and Xie (1987), and Terracol and David (1987).

By contrast, the specific problem studied here of a flexible cell feeding several distinct machines has not been extensively studied. Seidmann and Schweitzer (1984) use a Markov Decision Process Model to find the optimal part selection policy at the flexible cell where the objective is to minimize the expected shortage penalty per unit time incurred by the downstream machines when they are starved of parts. Tenenbaum and Seidmann (1986) expanded on this approach treating the more general problem of several flexible cells feeding dedicated machines. The objective again was to find an optimal dynamic part selection policy using a dynamic programming model of the system. In both papers, the authors assume that production times are exponentially distributed. The emphasis in these papers is to find an optimal scheduling policy where there are no constraints on the production ratios on the dedicated machines.

As far as we are aware, there has been no work done on the problem of analyzing an unreliable system where the production ratios are stated as constraints on the control of the system. In contrast to the work done in [11] and [13], we are not interested in finding an optimal control policy but in characterizing the tradeoff between the expense of increasing buffer capacity (in terms of both cost associated with work in process, and cost

associated with supplying additional pallets) and the resulting increase in system productivity, given a set of production ratios.

The approach we take for developing an approximation for mean utilization is briefly described as follows. First, we observe that the mean utilization of each dedicated machine can be expressed in terms of the marginal steady state probabilities of buffers being full. Since we do not know these marginal probabilities, we estimate them in terms of what we call the isolated full buffer probabilities. The isolated full buffer probability of a given machine buffer, i , is defined as the probability that buffer i is full in a modified system in which all system parameters are identical except that all other buffers except i are assumed to have infinite capacity. We are then left with the problem of finding these so-called isolated full buffer probabilities. However, finding each of these probabilities (one for each buffer) is equivalent to finding the probability of full buffers in a $G/G/1/K$ system. Hence, we have reduced the original problem to a simpler problem, although we are still left with the difficulty of evaluating or estimating the probability of full buffers in a $G/G/1/K$ system. In the numerical examples in Section 4, we assume that the processing times are exponential and we are thus able to use (as a reasonable approximation) the well known results for the $M/M/1/K$ system. For other processing time distributions, we would have to find appropriate estimates for the full buffer probabilities. However, several studies indicate that the assumption of exponential processing times may not lead to significant loss in accuracy (see, e.g., Suri (1983) and Yao and Buzacott (1985)).

The remainder of the paper is organized as follows. Section 2 defines the problem and states necessary assumptions. Section 3 describes the approximate model. Section 4 presents a series of examples designed to test the approximation method and to illustrate various interesting aspects of the utilization - buffer capacity tradeoff. Section 5 is a conclusion.

2 System Description and Assumptions

The system under consideration consists of a flexible manufacturing cell that produces parts for N dedicated machines (Fig. 1). Each dedicated machine requires parts specifically processed for it by the flexible cell. We define:

$N \equiv$ number of dedicated machines supplied with parts by the flexible cell =

number of part types produced by flexible cell

$F(i) \equiv$ distribution function for processing times for type i parts at the flexible cell

$G(i) \equiv$ distribution function for processing times for parts at machine i

$1/\lambda_i \equiv$ mean processing time of a type i part at the flexible cell

$1/\mu_i \equiv$ mean processing time of a type i part at dedicated machine i

$B_i \equiv$ buffer capacity of dedicated machine i (including one space for the part currently in process)

$f_i \equiv$ fraction or ratio of parts of type i to be produced in steady state

The f_i , $1 \leq i \leq N$, must satisfy

$$\sum_{i=1}^N f_i = 1, \quad f_i > 0$$

We assume that the processing times are mutually independent random variables.

$x_i(t) \equiv$ number of parts in buffer i at time t (including part in service)

$\mathbf{x}(t) \equiv (x_1(t), x_2(t), \dots, x_N(t))$

We assume that the stochastic process $\mathbf{x}(t)$, which depends on the schedule used at the flexible cell, is ergodic and hence approaches a discrete steady state distribution as $t \rightarrow \infty$, which we denote by $P(\mathbf{x})$. We are mainly interested in the marginal stationary probabilities $P_i(\mathbf{x}) \equiv \text{prob}(\mathbf{x} \text{ parts in buffer } i \text{ in steady state}), 1 \leq i \leq N$. We define:

Mean Utilization of Machine $i \equiv 1 - P_i(0)$

In order to develop the approximation we need to make the following additional assumption regarding the schedules used at the flexible cell. No matter what scheduling policy is followed at the flexible cell in order to meet the long-term production ratios, the flexible cell must cease production whenever any of the downstream buffers is full and wait until the full buffers are reduced before resuming production.

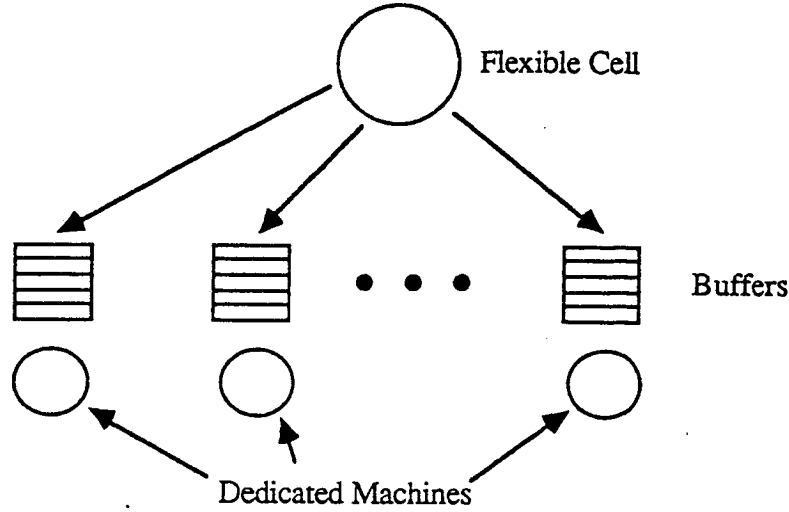


Fig. 1: Schematic Drawing of System

3 Analytic Approximation

In order to develop our analytic approximation, we need to define the following quantities:

$$\lambda_{\max}(i) = \frac{f_i}{\sum_{j=1}^N \frac{f_j}{\lambda_j}} \quad 1 \leq i \leq N \quad (3.1)$$

$\lambda_{\max}(i)$ represents the mean rate at which parts would arrive to dedicated machine i if the system had infinite capacity buffers (i.e. if there were no possibility of blocking).

Further, we define the "isolated probabilities"

$P_i^0(x) \equiv$ probability of x parts in buffer i in steady state for an equivalent system in which all buffers except the i^{th} buffer have infinite capacity.

In particular, we call $P_i^0(B_i)$ the "isolated full buffer probability of machine i ".

We first express a set of N analytically exact equations based on the observation that in steady state the rate at which parts enter the buffer of machine i must equal the rate at which parts leave machine i . We can express this mathematically as:

$$\mu_i(1 - P_i(0)) = \lambda_{\max}(i) \left(1 - \sum_{j=1}^N P_j(B_j)\right) \quad 1 \leq i \leq N \quad (3.2)$$

These equations can be justified as follows. Machine i works at rate μ_i when pieces are present and pieces are present with probability $(1-P_i(0))$. On the other hand, parts arrive at buffer i at mean rate $\lambda_{\max}(i)$ when the flexible cell is working and the flexible cell is working (i.e. not blocked) with probability $1 - \sum_{j=1}^N P_j(B_j)$. The reason we sum these probabilities is because of the fact that since we do not allow the flexible cell to work if any buffers are full, in steady state the probability that more than one buffer will be blocked at the same instant is zero.

Before continuing with the development of our approximation, we observe that the above set of relations allow us to define the following system wide performance measure which we use to measure system performance:

$$\text{Fraction of Max. Utilization} \equiv 1 - \sum_{j=1}^N P_j(B_j) \quad (3.3)$$

This quantity can be interpreted as the fraction of maximal utilization that is achieved by having the given set of buffers where we define

$$\text{Maximal Utilization} \equiv \min(1, \lambda_{\max}(i)/\mu_i)$$

The maximal utilization is the utilization that would be achieved in steady state if all the buffers were infinite (and hence, blocking were eliminated). In other words, the performance measure (3.3) characterizes the loss in potential system productivity due to the introduction of finite buffer capacity. It is a convenient performance measure since it is identical for each dedicated machine.

We now develop a set of approximate expressions for the probabilities $P_i(B_i)$ in terms of the independent full buffer probabilities $P_i^0(B_i)$, $1 \leq i \leq N$. These expressions are based on the following observation. The probability that the buffer of machine i will be full is certainly less than or equal to the probability that the buffer of machine i will be full in a system in which all the other buffers are infinite, i.e.

$$P_i(B_i) \leq P_i^0(B_i) \quad (3.4)$$

The amount by which the former quantity is less than the latter is approximately given by the probability that none of the other buffers are full. Thus, we use the approximation

$$P_i(B_i) \approx P_i^0(B_i) \left(1 - \sum_{j=1; j \neq i}^N P_j(N_j)\right) \quad 1 \leq i \leq N \quad (3.5)$$

This is a set of N linear equations in the unknowns $P_i(B_i)$ which can be written in standard form as:

$$\begin{bmatrix} 1 & P_1^0(B_1) & \dots & P_1^0(B_1) \\ P_2^0(B_2) & 1 & P_2^0(B_2) & \dots & P_2^0(B_2) \\ \vdots & & & & \vdots \\ P_N^0(B_N) & P_N^0(B_N) & \dots & & 1 \end{bmatrix} \begin{bmatrix} P_1(B_1) \\ P_2(B_2) \\ \vdots \\ P_N(B_N) \end{bmatrix} = \begin{bmatrix} P_1^0(B_1) \\ P_2^0(B_2) \\ \vdots \\ P_N^0(B_N) \end{bmatrix} \quad (3.6)$$

In the above equations, we have assumed that the isolated full buffer probabilities, $P_i^0(B_i)$, are known. In general, these probabilities will depend on both the schedule used at the flexible cell as well as the production time probability distributions $F(i)$ and $G(i)$. To find these probabilities, we view queue i in a system in which all other buffers are infinite as an isolated single server, finite buffer system with service time distribution $G(i)$. The interarrival time distribution to queue i , however, is not simple to characterize, since it is, in general, a complicated function of the distributions $F(i)$, $1 \leq i \leq N$ and the part selection policy used at the flexible cell. In fact, the interarrival time process may not even constitute a renewal process. However, depending on the nature of the schedule at the flexible cell and the processing time distributions $F(i)$, $1 \leq i \leq N$, we may at least be able to estimate the interarrival time distribution. If this is the case, we may use the results for $G/G/1/K$ systems to find estimates for the isolated full buffer probabilities.

4 Examples

In this section, we examine some simple examples designed to test the accuracy of the approximation proposed above and to examine the buffer capacity - utilization tradeoff. For simplicity, we assume that processing times are independent and exponentially distributed. As a part selection policy at the flexible cell, we use the following randomized schedule: at each decision epoch, we choose part type i to process

next with probability f_i . It is clear that this schedule will respect the production ratios, f_i , in steady state.

We must first find estimates for the isolated full buffer probabilities $P_i^0(B_i)$, $1 \leq i \leq N$. Consider the modified system in which all buffers except buffer i are infinite. Even in this simple case, the interarrival time process is not easy to find. For example, in the case of two dedicated machines, we can express the random variable representing the interarrival time to Buffer 1 as follows:

$Y \equiv$ a random variable representing the processing time at Buffer 1

$X_i, i \geq 1, \equiv$ a sequence of i.i.d. random variables (exponentially distributed)

representing the processing times of type 2 parts at the flexible cell

$A \equiv$ a random variable representing the interarrival time of parts to buffer 1

We have that

$$A = Y + \sum_{n=1}^{\infty} \left(\sum_{i=1}^n X_i \right) (1 - f_1)^n f_1$$

Thus, the interarrival time distribution (i.e. the distribution of A) in this simple case is the convolution of an infinite set of exponential distributions. Rather than calculate the exact interarrival distribution for the general case, we estimate it by an exponential distribution with rate equal to the known mean arrival rate $\lambda_{\max}(i)$. Since we have assumed that the arrival process is Poisson and that the processing time at machine i is exponentially distributed, we can use the well known results for M/M/1/K systems (cf. for example Kleinrock (1975)) to estimate the isolated blocking probabilities:

$$P_i^0(B_i) \approx \frac{\left(1 - \lambda_{\max}(i)/\mu_i\right)}{1 - \left(\lambda_{\max}(i)/\mu_i\right)^{B_i+1}} \left(\lambda_{\max}(i)/\mu_i\right)^{B_i}$$

Having estimated the probabilities $P_i^0(B_i)$, $1 \leq i \leq N$, we can then solve the set of N linear equations given in (3.6) for the probabilities $P_i(B_i)$, which in turn will yield the approximation for the performance measure defined in (3.3). Note that in deriving our estimate, we have made two approximations. First, we approximate the arrival processes in the isolated systems by Poisson processes. Second, we use the set of approximate linear equations (3.6) to estimate the true full buffer probabilities. The following

examples illustrate the accuracy of the estimate despite these two fairly rough approximations. The examples indicate that, as expected, as the complexity of the system grows, the higher the error of the estimate becomes. One question that arises is which of the two approximations is responsible for the errors in the estimates. To answer this question, we examined a two machine model and solved the exact Markov chain to get the probability of full buffers. The results here indicate that making the approximation of Poisson arrivals seems to have very little effect on the estimates. This is not surprising since utilization concerns the first moment only of the arrival process, which is known, and no higher moments. Hence, the approximation that seems largely responsible for the error in the estimates is the linear relationship stated in equations (3.6), although this remains a hypothesis.

Example No. 1: Two machines

In this example, we examine the case in which the flexible cell supplies parts to two machines. The system parameters are given in Table 1. Figure 2 plots the fraction of maximal utilization as a function of the buffer capacity of queue 1 while all other parameters are held fixed. The figure reveals the usual increasing convex relationship between utilization and buffer capacity. Table 4 lists comparisons between simulation results and theoretical estimates for various buffer configurations. For this example, the error of our estimate usually falls between 0 and 3%.

Example No. 2: Three machines

In this example, we examine the case in which the flexible cell supplies parts to three machines. The system parameters are given in Table 2. Table 5 lists comparisons between simulation results and theoretical estimates for various buffer configurations. Among other things, this example shows that the distribution of the buffers, and not only the total number of slots available, is important in optimizing the system's productivity. The errors encountered in this example were all less than 5%.

Example No. 3: Four machines

In this example, we examine the case in which the flexible cell supplies parts to four machines. The system parameters are given in Table 3. Table 6 lists comparisons

between simulation results and theoretical estimates for various buffer configurations. Here, we observe significant errors in the theoretical estimate for certain buffer configurations although it is interesting to note that the estimate is still very good for cases in which the buffers capacities are low or very high. The largest errors occur for buffer distributions of moderate size for reasons which are unclear.

5 Conclusion

We have developed an analytical estimate for machine utilization as a function of buffer capacity. Comparisons with simulation for the simple case of exponential processing times and a randomized part selection policy indicate that the estimate is reasonably robust. Directions for further research include: 1) a refinement of the approximation - in particular of the linear relation given in (3.5); 2) bounds on the error of the estimate; 3) development of methods for estimating the isolated full buffer probabilities in the case of processing time distributions which are not exponential 4) comparison with simulation in systems with other processing time distributions.

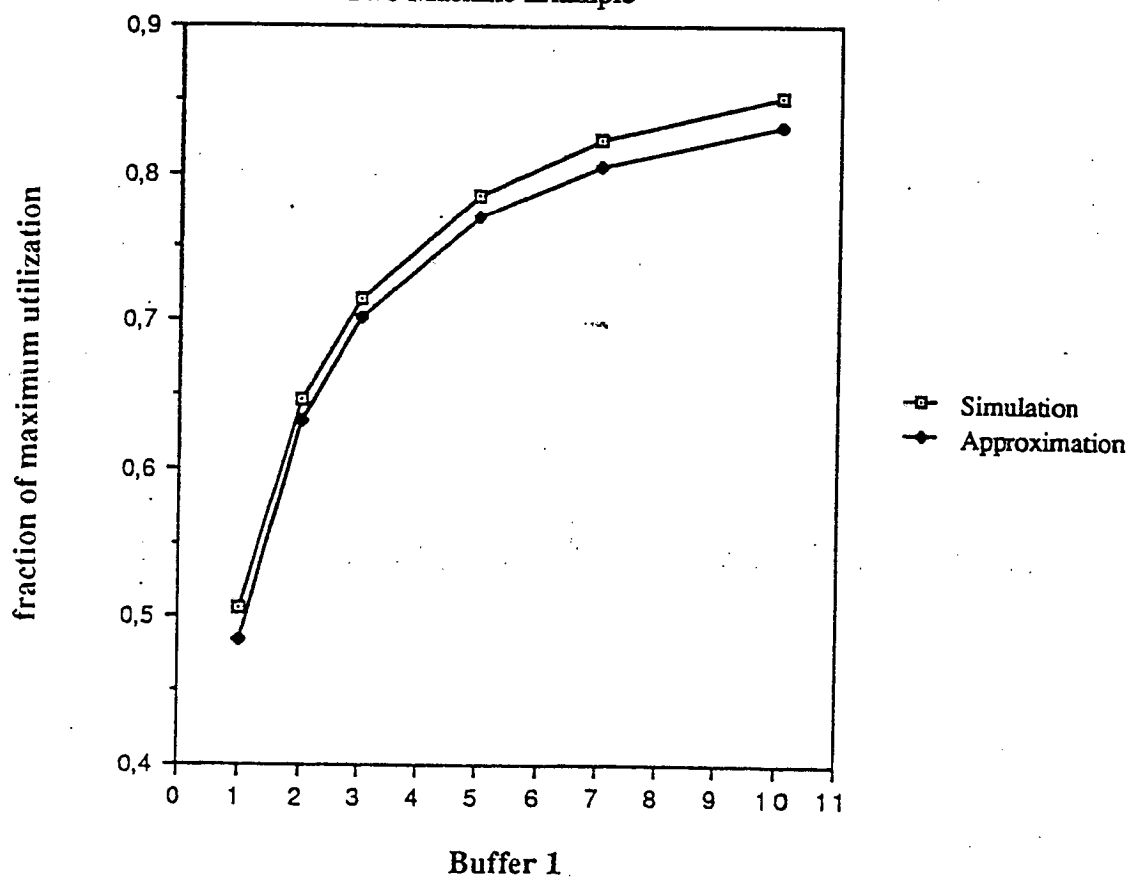
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Fig. 2: Utilization vs. Buffer Capacity

Two Machine Example



Buffer 2 = 3; See table 1 for system parameters

Table 1: Parameters for Two Machine Example

Part #	Flexible Cell Production time distribution	1/mean	Machine Production time distribution	1/mean	Prod. Ratio	Max. Util.
1	exponential	10	exponential	6,5	0,75	0,923077
2	exponential	5	exponential	3	0,25	0,666667

Table 2: Parameters for Three Machine Example

Part #	Flexible Cell Production time distribution	1/mean	Machine Production time distribution	1/mean	Prod. Ratio	Max. Util.
1	exponential	10	exponential	4,5	0,5	0,91954
2	exponential	8	exponential	2,75	0,3	0,902821
3	exponential	6	exponential	1,8	0,2	0,91954

Table 3: Parameters for Four Machine Example

Part #	Flexible Cell Production time distribution	1/mean	Machine Production time distribution	1/mean	Prod. Ratio	Max. Util.
1	exponential	10	exponential	4	0,4545455	0,967742
2	exponential	8	exponential	2,6	0,2727273	0,8933
3	exponential	8	exponential	1,8	0,1818182	0,860215
4	exponential	6	exponential	1	0,0909091	0,774194

Table 4: Results for 2 machine example

Buffer Configuration (B1,B2)	Frac. of Max. Util.		% Error
	Theoretical Est.	Simulation	
1,1	0,3861	0,3854	0,18%
5,5	0,8286	0,8271	0,18%
10,10	0,9358	0,9236	1,32%
10,1	0,5782	0,5937	-2,61%
1,10	0,5184	0,5189	-0,10%
1,3	0,4846	0,5058	-4,19%
3,10	0,7756	0,7574	2,40%
5,10	0,8604	0,8435	2,00%
20,10	0,9753	0,9668	0,88%
3,3	0,7024	0,7145	-1,69%
5,3	0,7712	0,7854	-1,81%
10,3	0,8312	0,851	-2,33%

Table 5: Results for 3 machine example

Buffer Configuration (B1,B2,B3)	Frac. of Max. Util.		% Error
	Theoretical Est.	Simulation	
10,1,1	0,3468	0,35306	-1,77%
7,7,7	0,7696	0,8097	-4,95%
10,10,1	0,4913	0,5164	-4,86%
1,1,1	0,2672	0,2672	0,00%
20,20,20	0,9511	0,9542	-0,32%

Table 6: Results for 4 machine example

Buffer Configuration (B1,B2,B3)	Frac. of Max. Util.		% Error
	Theoretical Est.	Simulation	
1,1,10,10	0,3425	0,3462	-1,07%
15,15,15,15	0,9123	0,9345	-2,38%
1,1,1,1	0,2224	0,2223	0,04%
5,5,5,5	0,6522	0,7179	-9,15%
7,5,5,3	0,6306	0,6984	-9,71%

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