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1993

Une classe de réseaux de Petri sans conflit pour le contrôle des systèmes de production

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RESUME

Ce papier traite de l'évaluation du comportement et de la gestion des systèmes discrets non cycliques, et en particulier des systèmes de production. Nous introduisons un type de réseaux de Petri appelés CFIO (Conflict-Free nets with Input and Output transitions / Réseaux sans conflit avec transitions d'entrée et de sortie). Nous montrons que les CFIO sont vivants et que, s'ils sont consistants, ils peuvent être maintenus bornés et sont réversibles. Nous développons aussi des règles de réduction qui facilitent le calcul des t-invariants des CFIO. Nous montrons ensuite comment tirer avantage des propriétés qualitatives des CFIO pour la planification et l'ordonnancement. Ces approches sont illustrées par des exemples numériques.

MOTS-CLES

Réseaux de Petri, Systèmes à événements discrets, Systèmes de production, Consistance, Bornes, Réduction des réseaux de Petri, Systèmes sans conflit, t-invariants, Planification.

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A Class of Conflict-Free Petri Nets used for Controlling Manufacturing Systems

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Abstract

This paper is devoted to the behavior evaluation and management of non-cyclic discrete systems, and in particular manufacturing systems. We introduce a special type of Petri nets called CFIOs (Conflict Free nets with Input and Output transitions). It is shown that CFIOs are live, and if consistent, they can be kept bounded and are reversible. We also develop reduction rules which facilitate the computation of the t-invariants of CFIOs. We then show how to take advantage of the qualitative properties of CFIOs to perform planning in manufacturing systems. Numerical examples illustrate these approaches.

Key words: Petri nets, Discrete event systems, Manufacturing systems, Consistency, Boundedness, Reduction of Petri nets, Conflict-free systems, t-invariants, Planning

1 Introduction

Significant changes have been occurring in industrialized countries during the past three decades. To name only a few of them: (i) production is moving towards high quality products and small lot sizes (it is said that the economy of scale has been replaced by the economy of scope), (ii) competitiveness has become a worldwide phenomenon, and (iii) erratic demands are progressively replacing steady demands.

A way to face these trends is to introduce highly automated manufacturing systems with high flexibility and small, or even zero, set-up times. Such systems are highly sophisticated. They require expensive components, are costly to run and maintain and, at the beginning of their life cycle, demand a very complex design process.

The first stage of this process is the so called preliminary design [2], which includes the tasks to be performed in order to specify the manufacturing system, given the parts' specifications.

This stage is also known as the "*paper study*," and ends when the physical implementation of the system starts. Among the tasks to be performed at this level are the establishment of the functional specifications, the modeling and the evaluation of the manufacturing system. Many tools exist to support these tasks. For instance, queueing theory, state-transition, mathematical programming and simulation, are used for the performance evaluation of the system, while entity-relationship approaches and the CIM-OSA[1] related tools are used for the specification of the system.

We claim that Petri nets are the only tool which can support specification as well as functional modeling and performance evaluation (see also [10]). The latter is performed either by using a Petri net based simulation software, or by using the properties of Petri nets, which are particularly rich in some applications. Thus Petri nets can be considered as tools that enable a step towards integration at the preliminary design stage.

Numerous analytical results (see in particular [1], [3], [4], [6], [10] and [11]) are available to support the preliminary design of cyclic manufacturing systems. The reason is that these kinds of manufacturing systems can be modeled using strongly connected event graphs, a special class of Petri nets whose analytical properties are particularly powerful.

However, strongly connected event graphs cannot be employed where non-cyclic manufacturing systems are concerned. The main reasons for this are: (i) we have to be able to introduce and remove parts from the system (and this process cannot be represented by a strongly connected net), and (ii) the system is not *conflict-free* (due to the decision making process).

Non-cyclic manufacturing systems are modeled using basic general Petri nets enhanced by:

i. **Control Places**, which make it possible to take into account external decisions.
ii. **Input transitions**, which can deliver tokens to the system, making it possible to model raw material and semi-finished products coming from outside the manufacturing system.
iii. **Output transitions** which can remove from the system tokens that represent finished or semi-finished products leaving the system.

The way control places are employed to take into account external decisions is explained in [5] and [9] and summarized in section 2.2.

As soon as a decision is applied by means of the control places, some of the transitions are *frozen* (i.e. cannot be fired anymore), which turns the Petri net model into a conflict-free net. This conflict-free net is obtained by removing from the initial Petri net model the frozen transitions, the related arcs, as well as their input and output places, if they are not connected to a non-frozen transition. Such a conflict-free subnet of the initial Petri net model is referred to hereafter as a **CFIO** net, which stands for Conflict Free with Input and Output transitions. A sequence of decisions applied to a non-cyclic manufacturing system can thus be viewed as a sequential activation of conflict-free subnets of the initial Petri net model.

In this paper, we focus on the study of CFIOs. Such a subnet is characterized by input and output transitions, and by the fact that each place has only one output transition.

To make it possible to control the whole manufacturing system, its net should be decomposable into manageable CFIOs.

By **manageability** we mean that:

i. We are in a position to keep the CFIO net bounded, which means that the work-in-process of the related manufacturing system can be held below a given level.

ii. Any marking reachable from the initial marking can be reached from any other marking reachable from the initial marking; this means that the related manufacturing system can always reach any of the states it was designed for.

iii. The ratios of the output transition firings can vary to a large extent, which guarantees the flexibility of the manufacturing system at hand.

The remainder of this paper is organized as follows:

In section 2 we provide the definitions used in the paper. In section 3, we propose some properties of CFIO nets related to manageability. In section 4, we introduce a process to transform a CFIO N into a reduced CFIO N_1 such that N is consistent if and only if N_1 is consistent, and the t-invariants of N can be derived from the t-invariants of N_1. We also propose a powerful algorithm that derives the reduced net and uses it to compute the t-invariants of the initial net and decide if it is consistent or not. An example is presented to illustrate this algorithm. In section 5, we show how to use CFIOs to perform planning in a manufacturing system. The approach is illustrated by a small example. Section 6 presents our conclusions.

2 Definitions

In this section, we first define generic Petri nets and then provide the definition of CFIO nets proposed in this paper (for details see [7]).

2.1 Definitions related to Petri Nets

A Petri net can be viewed from two aspects: static and dynamic. The static aspect of a Petri net is defined by a weighted, bipartite, directed graph which consists of places (represented by circles), transitions (represented by bars), and arcs (represented by arrows). The dynamic aspect is provided by the initial and subsequent markings of the net, as enabled transitions are fired.

Definition 1 A Petri-net is a four-tuple $N = < P, T, F, W >$ where :

* $P = \{p_1, p_2, \ldots, p_n\}$ is a finite set of places,
* $T = \{t_1, t_2, \ldots, t_m\}$ is a finite set of transitions,
* $P \cap T = \emptyset$; i.e. places and transitions are disjoint sets,
* $F \subseteq (P \times T) \cup (T \times P)$ is a set of arcs (flow relations),
* $W : F \to N^+$ is a weight function which assigns a strictly positive value to each arc.

A place $p \in P$ is an input (resp. output) place of $t \in T$ if there exists a directed arc joining p to t (resp. t to p). The set of input places of t is denoted by ${}^\bullet t$, while the set of output places of t is denoted by $t^\bullet$. Similarly, $t \in T$ is an input (resp. output) transition of $p \in P$ if there exists a directed arc joining t to p (resp. p to t). The set of input (resp. output) transitions of p is denoted by ${}^\bullet p$ (resp. $p^\bullet$).

We denote by $^\bullet T$ (resp. $T^\bullet$) the set of all the places of the net which are input (resp. output) places of at least one transition. We also denote by $^\bullet P$ (resp. $P^\bullet$) the set of all the transitions which are input (resp. output) transitions of at least one place.

In the initial models of this paper we consider only arcs with weights equal to unity.

Definition 2 A Marked Petri net is a Petri net $PN =< N, M >$ where:

- $N =< P, T, F, W >$ is a Petri net as defined above,
- $M : P \to \mathbf{N}$ is called a marking of PN.

For $p \in P, M(p)$ is the number of tokens contained in the place p for marking M. In figure 1, we present a Petri-net where tokens are represented by dots.
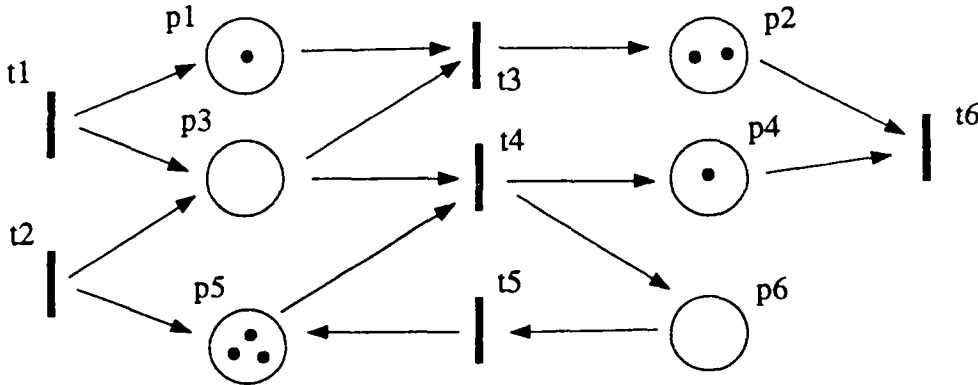

Figure 1: A marked Petri net

The marking represented in figure 1 is $M=<1,2,0,1,3,0>$, which means that $M(p_1) = 1, M(p_2) = 2$ and so on.

Definition 3 A pure Petri net (i.e. such that $^\bullet t \cap t^\bullet = \emptyset, \forall t \in T$) can be represented by an incidence matrix $C = [c_{i,j}], i = 1, 2 \ldots, n; j = 1, 2, \ldots, m$, where:

$$c_{i,j} = \begin{cases} -1, & \text{if there is an arc from place i to transition j} \\ +1, & \text{if there is an arc from transition j to place i} \\ 0, & \text{otherwise} \end{cases}$$

As an example, the following incidence matrix corresponds to the Petri net presented in figure 1.

$$C = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

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In a marked Petri net, a transition $t \in T$ is said to be *enabled* if every $p \in {}^{\bullet}t$ contains at least one token. *Firing* an enabled transition consists of removing one token from each of the input places and adding one token to each output place (i.e. to every place $p \in t^{\bullet}$).

For instance, in the Petri net given in figure 1, t_6 is enabled. Firing t_6 consists only of removing one token from p_2 and p_4; because $t_6^{\bullet} = \emptyset$, no place receives a token.

We can also associate a *firing time* to each transition. Such a time represents the time between the beginning and the end of a transition firing. Since transitions model operations in a manufacturing system, firing times represent manufacturing times. Tokens are reserved in the input places during the firing process. They are removed from the input places, and the output places are marked, at the end of the firing process.

Notations

- By firing a sequence of transitions σ, we can reach a marking M from an initial marking M_0. We say that M is *reachable* from M_0, and that σ is a *firing sequence* starting at M_0 and leading to M. We denote:

$$M \in R(M_0)$$

and

$$M_0 \xrightarrow{\sigma} M$$

- A basic relationship in Petri net theory is that:

$$M = M_0 + C\alpha_\sigma \tag{1}$$

where $\alpha_\sigma = [\alpha_\sigma^1, \alpha_\sigma^2, \ldots, \alpha_\sigma^m]$ and α_σ^i is the number of times transition t_i is fired in σ. α_σ is called the *counting vector*.

- An *elementary circuit* is a directed path which starts from a node (place or transition) of the Petri net and ends at the same node, and is such that no node of the path is included more than once within itself. For instance, $<p_5, t_4, p_6, t_5, p_5>$ is an elementary circuit of the net of figure 1.

A Petri net marking M may be sometimes referred to hereafter as the state of the system, which reflects the state of the manufacturing system modeled by the net.

In manufacturing systems, tokens can be used for representing parts, resources, or other information, while places represent buffers, and transitions represent operations.

2.2 Control Places

Krogh et al. [5] were the first to propose an extension of Petri nets taking into account external decisions which can influence the behavior of the system. This is achieved by introducing control places in the nets (represented by double concentric circle). A control place p is such that:

1. ${}^{\bullet}p = \emptyset$: A control place has no input transitions.
2. $M(p) \leq 1$: There is at most one token in each control place.

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3. The marking of a control place is decided from outside the system. Consequently, if the output transition of a control place fires, (which means that there is one token in the control place), then the token is *not* removed from the control place. The usual firing mechanism holds for the regular places.
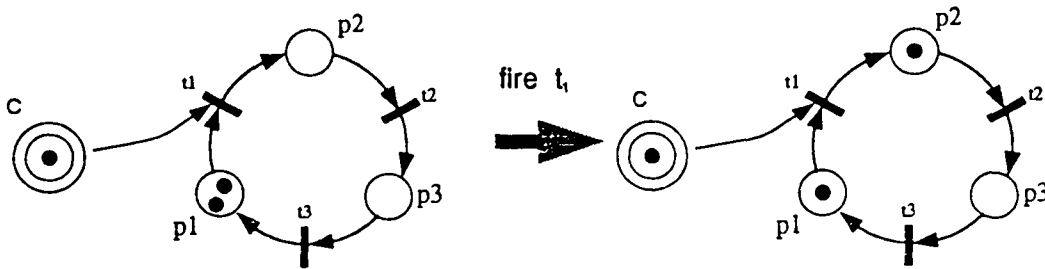
Figure 2 illustrates these rules.

Figure 2: Behavior of a control place

2.3 Definition of a CFIO Net

A **CFIO net** is a subclass of Petri nets, with the following three properties:

1. It is *conflict-free:*

$$^\bullet t_1 \cap {}^\bullet t_2 = \emptyset, \quad \forall t_1, t_2 \epsilon T$$

In other words, each place has only one output transition,

2. Has neither input nor output places (also called source/sink places):

$$^\bullet T = T^\bullet$$

3. Has at least one input and at least one output transition:

$$T - P^\bullet \neq \emptyset$$
$$T - {}^\bullet P \neq \emptyset$$

The net represented in figure 3 is a CFIO net, where t_1 and t_2 are the input transitions, and t_8 and t_9 are the output transitions.
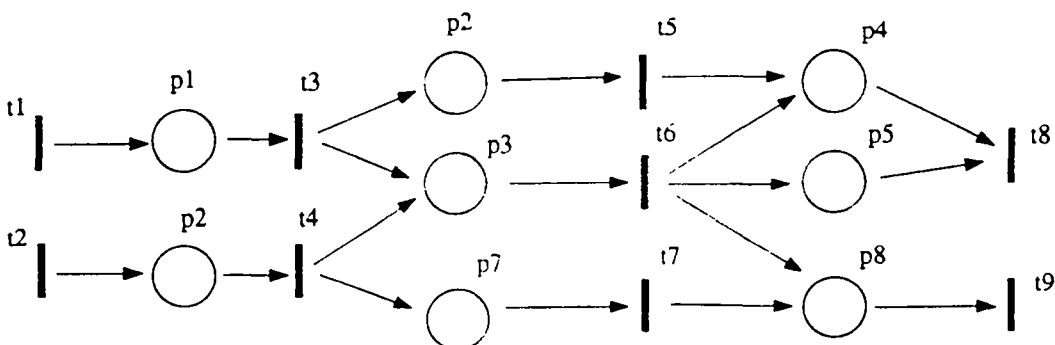

Figure 3: A CFIO net

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3 Properties of CFIO nets

In the following section we consider liveness, boundedness and consistency of CFIOs, and we will show their relationship to manageability.

Note that a CFIO net is structurally conflict-free. From the definition of CFIOs follows that they are structurally live. We show that CFIOs are not structurally bounded, but they can be kept bounded under special conditions. We will see that reversibility is a consequence of consistency in this particular case, and that the net is reversible for any initial marking such that every elementary circuit has at least one token. The relationship between boundedness and consistency is also established.

3.1 Liveness

A petri net is said to be structurally live if there exists at least one marking for which the marked net is live.

A Petri net is structurally live if there exists a vector x of positive integers such that $Cx \geq 0$ (for details on structural properties see [14], [15]).

Result 1: A CFIO net is structurally live.

Proof: A CFIO net has neither input nor output places. Thus for every place $p \in P, {}^{\bullet}p \neq \emptyset$ and $p^{\bullet} \neq \emptyset$. Also a CFIO is a *conflict-free* net, thus the set $p^{\bullet}$ has only one element.

Consider C the incidence matrix of a CFIO. Each row has exactly one (-1) element, and at least one $(+1)$ element. Thus a vector x whose elements are all equal to $(+1)$ is such that $Cx \geq 0$

Q.E.D.

3.2 Boundedness

A Petri net is said to be structurally bounded if there exists an integer $k > 0$ such that, for any initial marking M_0 and for any $M \in R(M_0), M(p) \leq k, \forall p \in P$.

A Petri net is structurally bounded if and only if there exists a vector y of positive integers such that $y^T C \leq 0$.

Result 2: A CFIO net is not structurally bounded.

Proof: A CFIO net has at least one input transition, thus at least one column of the incidence matrix contains only non-negative values, and at least one of these values is 1. If $[a]$ is this column, then $\not\exists y > 0$ such that $y^T[a] \leq 0$. Thus, $\not\exists y > 0$ such that $y^T C \leq 0$.

Q.E.D.

Result 3: A CFIO net can be kept bounded by controlling the firings of the transitions belonging to $T - P^{\bullet}$ (i.e. the input transitions) if and only if the Petri net obtained by removing those transitions, is bounded.

Proof:

(a) The condition is sufficient: Let $A = [A_1 A_2]$ be the incidence matrix of the CFIO net where A_1 is composed with the columns of $T - P^\bullet$. Thus the columns of A_1 do not contain (-1) values.

Boundedness of the net defined by A_2 means that :

$$\exists K_1 = [k_1, k_1, \ldots, k_1]^T , \quad k_1 \in \mathbf{N}^+ \text{ s.t. } M_0 + A_2 \alpha_{\sigma_2} \leq K_1$$

for any initial marking M_0 and for any feasible sequence σ_2 of transition firings related to $P^\bullet$, α_{σ_2} being the counting vector.

But, for any $K = [k, k, \ldots, k]^T$, $k > k_1$, there exists a feasible sequence σ_1 of transition firings related to transitions $T - P^\bullet$ such that:

$$A_1 \alpha_{\sigma_1} \leq K - K1$$

Thus, $\exists K_1$ and σ_1 such that:

$$M_0 + [A_1 A_2] \begin{bmatrix} \alpha_{\sigma_1} \\ \alpha_{\sigma_2} \end{bmatrix} \leq K$$

and the CFIO net can be kept bounded for a given σ_1 for any initial marking M_0.

(b) The condition is necessary: If A_2 as defined above, is not bounded, then there does not exist $K_1 > 0$ such that:

$$M_0 + A_2 \alpha_{\sigma_2} \leq K_1$$

for at least one feasible sequence σ_2 related to an initial marking M_0.

Also, because $A_1 \alpha_{\sigma_1} \geq 0$, there does not exist $K > 0$ such that:

$$M_0 + A_1 \alpha_{\sigma_1} + A_2 \alpha_{\sigma_2} = M_0 + A \alpha_\sigma < K$$

and thus the CFIO net cannot be kept bounded.

<div align="right">QED</div>

Note: The Petri net obtained by removing the input transitions in a *consistent* CFIO net is bounded, because removing the input transitions limits the number of transition firings and, as a consequence, the number of tokens in the system.

Thus, the following corollary can be derived from result 3:

Corollary 1: A Consistent CFIO net can be kept bounded by controlling its input transitions.

3.3 Consistency

A CFIO net is consistent if there exists a marking M_0 and a transition firing sequence σ were every transition occurs at least once in σ, such that $M_0 \xrightarrow{\sigma} M_0$.

We know that consistency holds if and only if there exists a vector x of positive integers such that $Cx = 0$.

Result 4: If a CFIO net is consistent, then for any initial marking M_0 such that each elementary circuit contains at least one token, and for every $M \in R(M_0)$, we have $M_0 \in R(M)$ (i.e. the net is reversible for every initial marking M_0 such that all elementary circuits are marked).

Proof: The proof consists of three parts:

a. Defining a partial order on the set of transitions. We partition the set of transitions into ordered *minimal* disjoint sets $T_1, T_2, \ldots, T_v$ such that:

$$T_j = \left\{ t \mid {}^\bullet t \subseteq \bigcup_{i \leq j} T_i^\bullet \right\}$$

These subsets can be obtained by the following algorithm:

1. Initially: $E = \emptyset$, $i = 1$
2. Until $E = T$ do:

    a. In case:

- $\exists t \in T$ s.t. ${}^\bullet({}^\bullet t) \subseteq E$ then $T_i = \{t\}$
- $\nexists t \in T$ s.t. ${}^\bullet({}^\bullet t) \subseteq E$ then choose any $t \notin E$ such that ${}^\bullet({}^\bullet t) \cup E \neq \emptyset$ and

        - Determine the strongly connected subnet S containing t
        - $T_i = \{s \mid {}^\bullet s \in S\}$

    b. $E = E \cup T_i$
    c. $i = i + 1$

Finally, we obtain an ordered sequence of v subsets which also constitutes a partition of T.

- $\bigcup_{i=0}^{v} T_i = T$
- $T_i \cap T_j = \emptyset, \forall i, j \quad i \neq j$

Note that the above algorithm defines an order O_R on the set $\{T_1, T_2, \ldots, T_v\}$: it is the order by which the subnets were defined, i.e. $T_i < T_j$ if and only if $i < j$. Thus, because some T_i may contain more than one transition, O_R is a partial order on the set T of transitions.

Note also that, if $t \in T_i$ does not belong to a circuit, then t is the only element of T_i. Otherwise, all the transitions of T_i are the transitions of a strongly connected subgraph.

b. Constructing a complete firing sequence. Assume that the CFIO net is consistent. Then there exists an integer vector $x, x > 0, x = [x_1, x_2, \ldots, x_m]^T$ such that $Cx = 0$. Let y_i be the vector whose components are the components of x which correspond to the transitions of T_i. Note that if T_i is the set of transitions belonging to a strongly connected subgraph, the components of y_i are equal.

We will construct a sequence $\bar{\sigma}$ as a concatenation of sequences $\bar{\sigma}_i$, $\bar{\sigma}_i$ being a sequence of transitions belonging to T_i such that the number of times each transition appears in $\bar{\sigma}_i$ corresponds to the respective component of y_i. We denote

$$\bar{\sigma} = \bar{\sigma}_1 \circ \bar{\sigma}_2 \circ \cdots \circ \bar{\sigma}_v$$

where ∘ denotes the concatenation.

- $\tilde{\sigma}_1$ is a sequence which contains only one input transition. Thus $\tilde{\sigma}_1$ is firable for any M_0.
- Assume that we have constructed the sequence $\tilde{\sigma}_1 \circ \tilde{\sigma}_2 \circ \cdots \circ \tilde{\sigma}_{i-1}$ which is firable.
  We will show that there exists $\tilde{\sigma}_i$ such that the sequence $\tilde{\sigma}_1 \circ \tilde{\sigma}_2 \circ \cdots \circ \tilde{\sigma}_i$ is firable.

  i. If T_i is a singleton set $\{t\}$ then $\tilde{\sigma}_i$ is a sequence containing y_i times the transition t since $^\bullet(^\bullet t) \subseteq \bigcup_{j=1}^{i-1} T_j$.

  ii. Otherwise T_i is a set of transitions belonging to a strongly connected subset S, in which each elementary circuit contains at least one token.

     Assuming that the firing of $t \in S$ is not blocked by $M(p), p \in {}^\bullet t, p \notin S$, then there exists a firable sequence σ where each $t \in S$ occurs the same number of times, such that $M_S \xrightarrow{\sigma} M_S$, for all markings M_S of S where every elementary circuit in S contains at least one token.

     In our net, for every $p \in {}^\bullet t, t \in S, p \notin S$, we observe that $^\bullet p \subseteq \bigcup_{j=1}^{i-1} T_j$. Transition t is not blocked by p because after firing the sequence $\tilde{\sigma}_1 \circ \tilde{\sigma}_2 \circ \cdots \circ \tilde{\sigma}_{i-1}, M(p) \geq y_i$ Thus for $\tilde{\sigma}_i = \sigma$ the sequence $\tilde{\sigma}_1 \circ \tilde{\sigma}_2 \circ \cdots \circ \tilde{\sigma}_i$ is firable.

Then $\tilde{\sigma} = \tilde{\sigma}_1 \circ \tilde{\sigma}_2 \circ \cdots \circ \tilde{\sigma}_v$ is such that $M_0 \xrightarrow{\tilde{\sigma}} M_0$, since M_0 is a marking where every elementary circuit contains at least one token.

Furthermore, the same result holds when replacing x by any vector kx where k is any positive integer. As a consequence we can always suppose that $\alpha_{\tilde{\sigma}} \geq \alpha_{\sigma}$ for *any* sequence σ.

c. **Proving that** $\forall M \in R(M_0), M_0 \in R(M)$

Let σ_M be such that $M_0 \xrightarrow{\sigma_M} M$. σ_M is a subsequence of $\tilde{\sigma}$ which may be made by non-consecutive elements of $\tilde{\sigma}$. Since in a *conflict-free* net, firing a transition does not disable other transitions, it is still possible to fire transitions of $\tilde{\sigma}$ that do not belong in σ_M, after firing σ_M. Those transitions are fired in their order in $\tilde{\sigma}$. Thus, the final marking of the CFIO is M_0.

Q.E.D.

The following corollary is derived from result 4:

Corollary 2: For every initial marking M_0 such that all elementary circuits contain at least one token each, $M_1 \in R(M_0)$ and $M_2 \in R(M_0)$, then $M_1 \in R(M_2)$ and $M_2 \in R(M_1)$.

This corollary means that, in a consistent CFIO net, for any initial marking M_0 with at least one token in every elementary circuit, any marking reachable from M_0 can be reached from any other marking reachable from M_0. Result 4 shows that the consistency of a CFIO net implies item *(ii)* of manageability as described in the introduction.

4 Reduction of CFIO nets

Consistency is a key property of all manageable CFIO nets. Thus, we should be able to verify this property even for large-scale nets in a reasonable amount of time. We also need to compute all the minimal t-invariants of the CFIO net as fast as possible. This will allow the control of the CFIO net according to external criteria. In this subsection, we propose two results

which show that it is always possible to transform a CFIO net N, into a reduced CFIO net N^1 which is consistent if and only if N is consistent. N^1 has only input and output transitions which correspond to the input and output transitions of N.

4.1 Circuit Reduction

Result 5: Let N be a CFIO net.

Case 1: If, for at least one elementary circuit $\gamma \subseteq N$, there exists a place $p \in \gamma$ which has more than one input transition, then N is not consistent.

Case 2: Otherwise, if each elementary circuit contains at least one token, then the net N^1 is consistent if and only if N is consistent. N^1 is derived from N by replacing each elementary circuit γ by a transition t_γ such that any input (resp. output) place of a transition of γ which is not included in γ becomes an input (resp. output) place of t_γ.

Proof of Case 1: Assume that, for an elementary circuit $\gamma \subseteq N$, there exists a place $p \in \gamma$ which has more than one input transition. Assume also that N is consistent.

According to the definition of consistency, there exists a marking M_0 and a transition firing sequence σ where every transition occurs at least once in σ, such that $M_0 \xrightarrow{\sigma} M_0$.

Since N is conflict-free, $\forall p \in \gamma \Rightarrow p^\bullet \in \gamma$. This implies that the total number of tokens in γ cannot be decreased. Now if $\exists p \in \gamma$ such that $^\bullet p - \gamma \neq \emptyset$ then firing a transition $t \in ^\bullet p - \gamma$ will increase the number of tokens in γ. Since σ contains t at least once, firing σ will increase the number of tokens in γ. Thus M_0 cannot be recovered and N is not consistent.

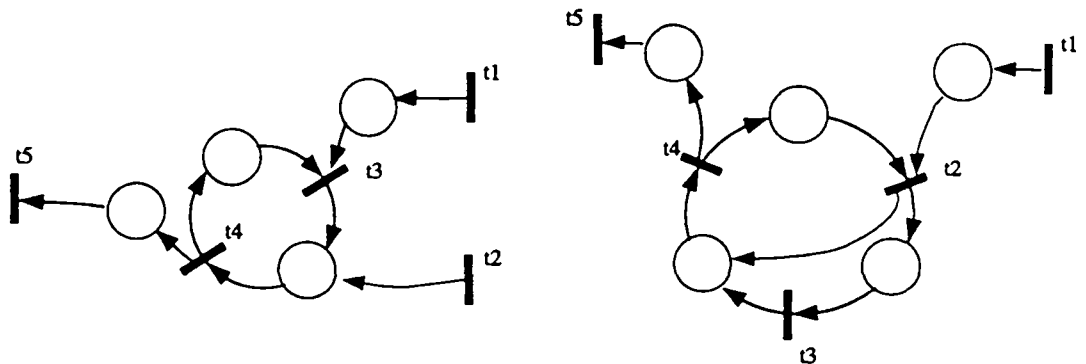In Figure 4, we give two examples of such inconsistent CFIO nets.

Figure 4: Two inconsistent CFIO nets

Proof of Case 2: Let us now assume that each elementary circuit contains at least one token and that, in any elementary circuit, a place has only one input transition. We want to prove the reduction rule presented in figure 5.
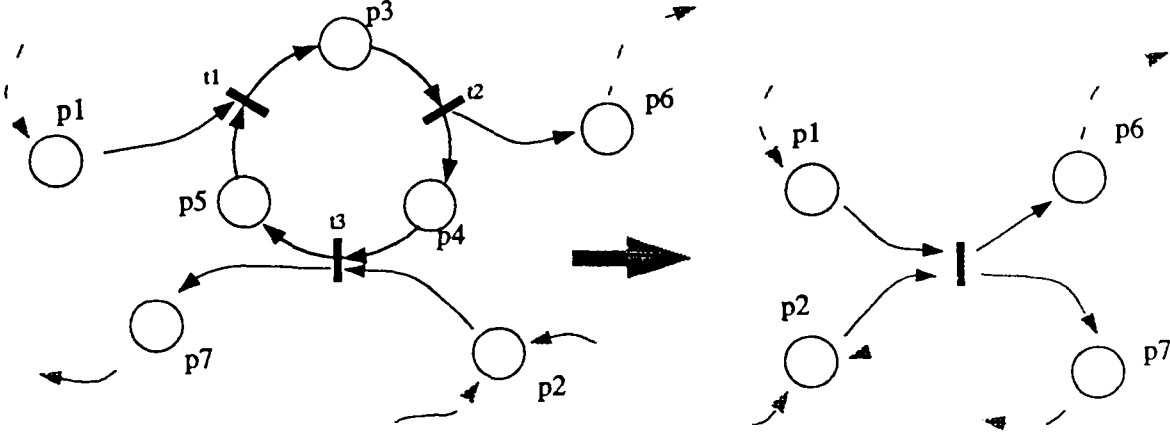

Figure 5: First reduction rule

In the remainder of the proof, C and C^1 are the incidence matrices of N and N^1 respectively. The proof consists of two parts:

(a) If N is consistent then N^1 is consistent.

(b) If N is not consistent then N^1 is not consistent.

(a) *Assume first that N is consistent:*

According to the definition of the consistency, $\exists x \in \mathbf{N}^m, x > 0$ such that $Cx = 0$. Without loss of generality, we can assume that N contains only one elementary circuit $< p_{i_1}, t_{j_1}, p_{i_2}, t_{j_2}, ..., p_{i_s}, t_{j_s} >$. We assume that the rows (resp. the columns) of C are ordered according to the indices of the places (resp. the transitions).

C^1 is derived from C as follows:

α. We first derive C^a from C by setting:

$$c_{i,j}^a = \begin{cases} c(i,j) & \text{for } j \neq j_1 \\ \sum_{k=1}^s c(i,j_k) & \text{for } j = j_1 \end{cases}$$

β. We then derive C^b from C^a by removing the columns of transitions $j_2, j_3, ..., j_s$.

γ. We finally obtain C^1 by removing the rows of places $i_1, i_2, ..., i_s$ of C^b.

Furthermore, each row $i \in \{i_1, ...i_s\}$ in C is such that:

$$c_{i,j} = \begin{cases} -1, & \text{for exactly one } j \in \{j_1, ..., j_s\}, \text{ say } j_i^- \\ +1, & \text{for exactly one } j \in \{j_1, ..., j_s\}, \text{ say } j_i^+ \\ 0, & \text{otherwise} \end{cases}$$

and $Cx = 0$ implies:

$$\sum_{j=1}^m c_{i,j} x_j = 0 \text{ for } i = 1, 2, ..., n$$

$$\Longrightarrow c_{i,j_i^-} x_{j_i^-} + c_{i,j_i^+} x_{j_i^+} = 0$$

$$\Longrightarrow c_{i,j_i^-} x_{j_i^-} + c_{i,j_i^+} x_{j_i^+} = 0 \text{ for } i \in \{i_1, i_2, \ldots, i_s\}$$

$$\Longrightarrow x_{j_i^-} = x_{j_i^+}$$

This being true for any $i \in \{i_1, \ldots i_s\}$, it leads to: $x_{j_1} = x_{j_2} = \cdots = x_{j_s}$

Let us now build x^1 as follows:

(i) if $j \notin \{j_1, j_2, \ldots j_s\}$,

$x_k^1 = x_j$, if the kth column of C^1 has been derived from the jth column of C.

(ii) $x_v^1 = x_{j_1}$, where v is the rank in C^1 of the column obtained by adding the columns $j_1, j_2, \ldots, j_s$ of C.

Based on the process followed to build C^1 and the definition of x^1, $Cx = 0$, we obtain $C^1 x^1 = 0$, and N^1 is consistent.

Q.E.D.

(b) *Assume that N is not consistent:*

If N^1 would be consistent, then:

$$\exists x^1 \in \mathbf{N}^{m_1}, x^1 > 0 \text{ s.t. } C^1 x^1 = 0$$

where m_1 is the number of transitions of N^1.

It would then be possible to expand x^1 to obtain $x \in \mathbf{N}^m$ by setting:

1. $x_j = x_k^1$

   if the k-th column in C^1 is the j-th column in C.

2. $x_{j_1} = x_{j_2} = \ldots = x_{j_s} = x_k^1$

   if transition t_k replaces the elementary circuit $\gamma \subseteq N$ containing $t_{j_1}, t_{j_2}, \ldots, t_{j_s}$.

Using the same arguments as in (a), we can see that $x > 0$ and $Cx = 0$.

Q.E.D

4.2 Path algorithm

From now on, we assume that the CFIO net at hand does not contain any elementary circuits.

For each transition t, we denote by n_t the number of times t has been fired. Assuming that initially there is no token in the CFIO net (i.e. the initial marking is zero), the following inequalities hold:

$$n_t \le \sum_{u \in \,^\bullet p} n_u, \quad \forall p \in \,^\bullet t \tag{2}$$

Assume that we apply relation (2) repetitively, starting from an output transition t_j. We finally obtain a set of inequalities where only the numbers of input and output transition firings are involved:

$$n_{t_j} \leq \sum_{k \in E} \beta_k^s n_{t_k} \qquad \text{for } s = 1, 2, \ldots, H_j \text{ and } j \in \text{F} \qquad (3)$$

where:

- $E = \{k | t_k \text{ is an input transition}\}$
- $F = \{j | t_j \text{ is an output transition}\}$
- H_j refers to the number of different inequalities obtained starting from output transition t_j.
- β_k^s are positive integers.

Inequalities (3) provide the maximal number of times an output transition can fire, knowing the number of times the input transitions have been fired, assuming that the initial marking is zero. Furthermore:

(i) The CFIO net is consistent only if it is possible to remove from the CFIO net all the tokens which have been introduced by firing the input transitions, assuming that each input transition has been fired at least once.

According to the process which leads to inequalities (3), a necessary and sufficient condition for the CFIO net to be consistent is then to find $n_{t_k} > 0$ for every $k \in E$ and $n_{t_j} > 0$ for every $j \in F$ such that inequalities (3) turn into equalities.

Similarly, the CFIO net has a t-invariant if there exists $n = [n_{k_1}, n_{k_2}, \ldots, n_{k_L}], n \not\geq 0$, $\{k_1, k_2, \ldots, k_L\} = E \cup F$, such that the inequalities (3) turn into equalities. Note that if a Petri net is consistent, it has at least one t-invariant, but the converse is not true.

(ii) It is possible, starting from inequalities (3), to construct a reduced CFIO net with the following properties:

- input transitions are those of the initial net,
- output transitions are also those of the initial net,
- it does not contain any other transition other than the input and output transitions

The incidence matrix C^1 of this reduced net is obtained starting from the inequalities derived from (3), i.e.:

$$-n_{t_j} + \sum_{k \in E} \beta_k^s n_{t_k} = 0 \qquad (3 - \text{bis})$$

Each inequality provides one row of C^1.
In the row related to relation (3–bis) (i.e. the sth relation related to t_j) :

- the element of C^1 corresponding to output transition t_j is (-1),
- for every $k \in E$, the element of C^1 corresponding to input transition t_k is β_k^s,
- the other elements of the row are 0.

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This reduced CFIO net is hereafter denoted by N^1, while the initial CFIO net is denoted by N.

Like N, N^1 is consistent if inequalities (3) turn into equalities for strictly positive values. N^1 has a t-invariant if, like N, inequalities (3) turn into equalities for values which are positive or zero, with at least one of them being positive.

Assume that we know x^1 ($x^1 > 0$ or $x^1 \not> 0$) such that $C^1 x^1 = 0$. Firing the input transitions of N as many times as indicated in x^1 for the corresponding input transition of N^1, and then firing the enabled transitions until none remains enabled anymore, we produce the following results :

(i) the vector x whose components represent the number of times each transition fired is such that $Cx = 0$

(ii) the components of x and x^1 corresponding to the input and output transitions are the same.

The following result summarizes the above remarks:

Result 6: Given a CFIO net N, it is always possible, by repeatedly applying inequalities (2), to upper bound the number of times each output transition can fire by a set of positive linear combinations of the number of times each input transition has been fired, assuming that the initial marking of N is a zero-vector (see inequalities (3)). From these upper bounds, it is possible to derive a reduced net N^1 as defined previously. Based on the fact that N and N^1 are CFIO nets, N is consistent if and only if N^1 is consistent.

Furthermore:

(i) A vector $x > 0$ s.t. $Cx = 0$ is derived in an unique way from a vector $x^1 > 0$ s.t. $C^1 x^1 = 0$, and vice versa, where C (resp. C^1) is the incidence matrix of N (resp. N^1).

(ii) A vector $x \not> 0$ s.t. $Cx = 0$ is derived in a unique way from a vector $x^1 \not> 0$ s.t. $C^1 x^1 = 0$, and vice versa. Vector x (resp. x^1) is a t-invariant of N (resp. C^1).
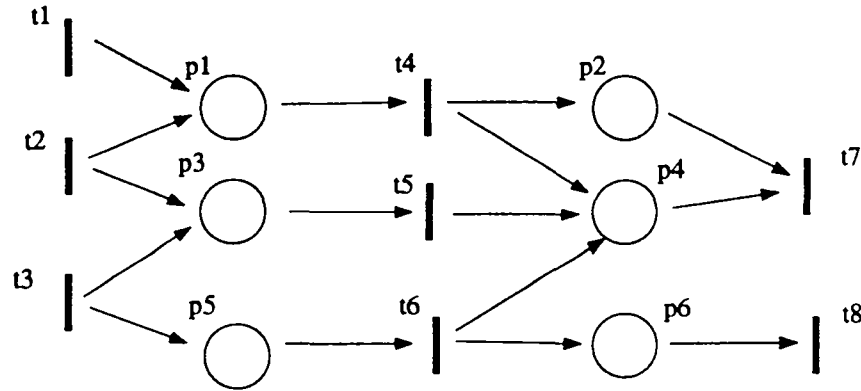

Figure 6: Initial net N

Let us present an example. Consider the CFIO net given in figure 6.

First we apply relation (2) to the output transition t_7 and we obtain:

$$n_7 \leq n_4 \qquad (4-1)$$

$$n_7 \leq n_4 + n_5 + n_6 \qquad (4-2)$$

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Considering t_4, t_5 and t_6, we obtain, respectively:

$$n_4 \leq n_1 + n_2 \qquad (4-3)$$

$$n_5 \leq n_2 + n_3 \qquad (4-4)$$

$$n_6 \leq n_3 \qquad (4-5)$$

Combining (4-3) with (4-1), and (4-3) through (4-5) with (4-2), we obtain, respectively:

$$n_7 \leq n_1 + n_2 \qquad (5-1)$$

$$n_7 \leq n_1 + 2n_2 + 2n_3 \qquad (5-2)$$

Similarly, starting from t_8, we obtain:

$$n_8 \leq n_3 \qquad (5-3)$$

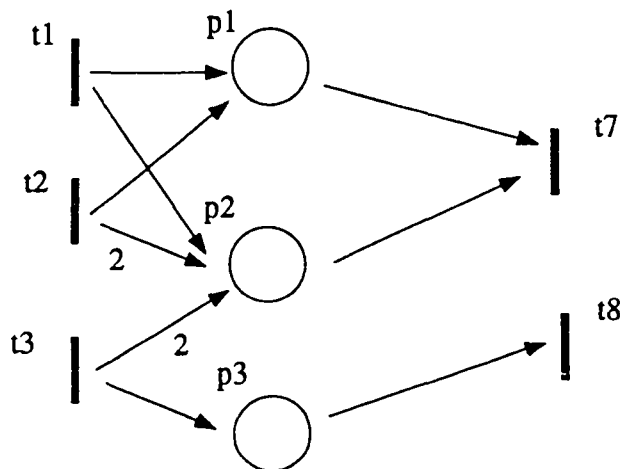Inequalities (5-1) to (5-3) comply with the inequalities (3).

Figure 7: Reduced net N^1

The related reduced net N^1 is given in figure 7.
In this example:

$$
C = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{array}
\begin{array}{cccccccc}
t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_8 \\
\left[\begin{array}{cccccccc}
1 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\
0 & 1 & 1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & -1 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1
\end{array}\right]
\end{array}
$$

and

$$
C^1 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array}
\begin{array}{ccccc}
t_1 & t_2 & t_3 & t_7 & t_8 \\
\left[\begin{array}{ccccc}
1 & 1 & 0 & -1 & 0 \\
1 & 2 & 2 & -1 & 0 \\
0 & 0 & 1 & 0 & -1
\end{array}\right]
\end{array}
$$

N^1 is not consistent, because there is no solution of $C^1 x^1 = 0$ such that $x^1 > 0$. Thus, N is not consistent. However, vector $x^1 = [1, 0, 0, 1, 0]$ is such that $C^1 x^1 = 0 : x^1$ is a t-invariant, which means that if we fire t_1 once starting from a zero-marking, then we come back to a zero marking by firing t_7.

We see that:

$n_1 = n_7 = 1$ and $n_2 = n_3 = n_8 = 0$ is a solution which turns inequalities (4) into equalities.

We finally can see that, by firing t_1 in N, we can fire t_4 and t_7. Thus $x = [1, 0, 0, 1, 0, 0, 1, 0]$ is a t-invariant of N.

Note that:

(i) the number of places in the reduced net is equal to the number of inequalities (3),
(ii) the weight of the arcs is not always 1 in the reduced net; for instance, weights of (t_2, p_2) and (t_3, p_2) are both equal to 2 in this example.

4.3 Computation of the minimal t-invariants

The minimal t-invariants are those which cannot be expressed as a linear combination of other t-invariants. From the point of view of linear programming, the t-invariants of C^1 are the extremal solutions of $C^1 x^1 = 0, x^1 \ngtr 0$. As soon as a minimal t-invariant x^1 of N^1 is known, it is easy to reconstitute the corresponding t-invariant x of N as shown in the previous section.

The minimal t-invariants of N^1 are obtained by computing all the extremal solutions of $C^1 x^1 = 0$, using the classical linear programming approaches [12], [13], [14]. This computation is usually very fast, due to the limited size of C^1. If, for each component, there exists at least one t-invariant in which this component is strictly positive, then N^1, and thus N, is consistent. Furthermore, the minimal t-invariants of N are derived from the t-invariants of N^1 as shown in the previous sub-section (i.e. by firing as many transitions as possible starting from the firing of the input transitions of N as required by the minimal t-invariants of N^1).

4.4 Software Implementation

A software which analyzes a CFIO net is available. It detects whether the net is consistent, and provides all its minimal t-invariants.

The different steps of the algorithm are summarized as follows:

1. Check and remove the circuits as shown in result 5. If, in at least one of the circuits, a place has more than one input transition, the net is inconsistent. A message is displayed and the computation stops.
2. Find inequalities (3) starting from the incidence matrix of the initial net N. This is done by applying repetitively relations (2) starting from the output transitions.
3. Find all the independent sets of values (positive or null) which turn inequalities (3) into equalities. This is done by using a classical linear programming approach to find all the

extremal solutions of $C^1 x^1 = 0$, where C^1 is the incidence matrix of the reduced net N^1 derived from inequalities (3).

4. Decide if N is consistent, by verifying if each of the components of a transition vector has positive value in at least one minimal t-invariant.

5. Construct the minimal t-invariants of N as shown in section 4.3.

Note that the reduced net appears only through its incidence matrix, which is derived from inequalities (3).

5 Planning of Manufacturing Systems

Planning of a manufacturing system consists of the determination of the list of operations to be performed during some consecutive periods of time. In terms of Petri net models, we have to provide the list of transition firings which occur during these periods. Hereafter, we show how to use CFIOs in order to cope with the computational burden. As described in the following section, this trade-off is achieved at the expense of flexibility. Using CFIOs as the basis to compute the planning guarantees that the qualitative properties which are essential for managing manufacturing systems are preserved.

5.1 Decomposition of a Petri Net

Let us consider a Petri Net N with input and output transitions which models the arrival and the departure of parts from the system.

A subset NC of N is a *CFIO of N* if:

1. NC is a connected CFIO net.

2. The input (resp. output) transitions of NC are input (resp. output) transitions of N.

3. Every node (place or transition) of NC is a node of N.

4. For every transition t of NC, the set of input (resp. output) places of t in NC is equal to the set of input (resp. output) places of t in N.

Definition 4 We call a Petri net N with input and output transitions, a **decomposable Petri Net**, if there exist consistent CFIOs of N, $NC_1, NC_2, \ldots, NC_r$ such that:

$$N = \bigcup_{i=1}^{r} NC_i$$

Result 7: A decomposable Petri Net such that each elementary circuit contains at least one token:

(i) is consistent

(ii) can be kept bounded by controlling its input and output transitions

(iii) is live

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Proof:

(i) Let C be the incidence matrix of the decomposable Petri Nets N and $NC_1, NC_2, \ldots, NC_r$ a set of consistent CFIOs such that $N = \bigcup_{i=1}^{r} NC_i$.

The consistency of NC_i, $i = 1, 2, \ldots, r$ implies that there exists a vector $x^i \in \mathbb{N}^m$ such that $Cx^i = 0$ and:

$$x_k^i \begin{cases} > 0, & \text{if } t_k \in NC_i \\ = 0, & \text{otherwise} \end{cases}$$

Let us consider $x^* \in \mathbb{N}^m$ such that

$$x_k^* = \sum_{i=1}^{r} x_k^i \quad \text{for } k = 1, 2, \ldots, m \tag{6}$$

then

$$Cx^* = C \sum_{i=1}^{r} x^i = \sum_{i=1}^{r} Cx^i = 0 \tag{7}$$

Furthermore

$$x_k^* = \sum_{i=1}^{r} x_k^i > 0 \quad \text{for } k = 1, 2, \ldots, m \tag{8}$$

because every transition t_k, $k = 1, 2, \ldots, m$ belongs at least to one of the CFIOs, and thus one of the x_k^i, $k = 1, 2, \ldots, m$ is strictly positive for at least one $i \in \{1, 2, \ldots, r\}$.

From equations (7) and (8) we conclude that N is consistent.

(ii) Given that $NC_1, NC_2, \ldots, NC_r$ are *consistent* subnets, we consider a vector x such that:

$$x = \sum_{i=1}^{r} n_i x^i = [x_1, x_2, \ldots, x_m] \tag{9}$$

where x^i are the vectors introduced in *(i)* above, and n_i are non-negative integers. Then if transition t_k is fired x_k times, $k = 1, 2, \ldots, m$ (which is always possible) the resulting marking is the initial marking.

Thus, N can be kept bounded if the transitions are fired according to (9).

(iii) Liveness results from the fact that N is the union of CFIO nets.

<div align="right">Q.E.D.</div>

In the following sections, we only consider Petri nets which are decomposable. According to numerous previous experiences, and the very nature of manufacturing processes, it appears that the Petri net model of a manufacturing system is decomposable.

5.2 Manageability of a manufacturing system

As far as manufacturing systems are concerned, the goal is to satisfy a variable demand (i.e. firing the output transitions a given number of times) while optimizing some secondary criteria, such as work-in-process, well-balanced utilization of the machines, etc.

We assume that the Petri net model N of the manufacturing system at hand is decomposable. Let $NC_1, NC_2, \ldots, NC_r$ be some consistent CFIOs such that $N = \bigcup_{i=1}^{r} NC_i$. For each NC_i, there exists at least one integer vector $x^i > 0$ as defined in result 7. We select a vector x^i whose elements have a highest common factor equal to 1. Usually x^i is not unique, but we decide to select only one x^i for each NC_i.

Activating NC_i consists in firing the transitions of NC_i as many times as required by the value of the corresponding component in x^i. Several firing orders are usually available. In fact, any enabled transition is firable at any time unless the number of times this transition has already been fired is equal to the corresponding component of x^i.

Note:

(i) Let $M \in R(M_0)$ such that M is reachable by sequential activation of the CFIOs of N. Then, if M_0 is such that any elementary circuit contains at least one token, it is possible to reach M_0 from M by sequential activations of the same CFIOs. This is because these CFIOs are reversible for the subset of M_0 corresponding to the CFIOs (see result 4).

(ii) If the CFIOs corresponding to N are all the CFIOs built from the minimal t-invariants, then N is reversible for any marking M_0, such that each elementary circuit contains at least one token.

According to result 7, it is clear that, if we control the system by sequential activation of the CFIO nets of the Petri Net model, we are sure that the system will retain its expected qualitative properties, namely boundedness, reversibility and liveness.

However, we have to emphasize that choosing only one x^i vector for each NC_i reduces the number of possible ways NC_i can be activated, and thus the number of possible ways the whole system can be controlled. In fact, this approach is a trade-off between the flexibility of the control and the computational burden. A maximal flexibility of the control would be obtained by using the minimal CFIOs of the Petri Net model, i.e., those CFIOs whose transitions correspond to the *minimal* t-invariants.

The number of times each CFIO has to be activated to satisfy the demand may also depend on the secondary criteria introduced previously, since there are usually several ways to meet the demand.

Finally, note that the number of times each CFIO has to be activated provides the number of times each transition has to be fired to meet the demand in a way which retains the expected qualitative properties. The result of this step is the production plan. Defining the order is the scheduling activity. The order of these firings may depend on other constraints, like resource constraints. A planning activity based on CFIOs is illustrated in the following example.

5.3 Example

We consider two product types whose manufacturing processes are represented in figure 8. In each box we show the machines on which the corresponding operation can be performed. The manufacturing times are given in parentheses.

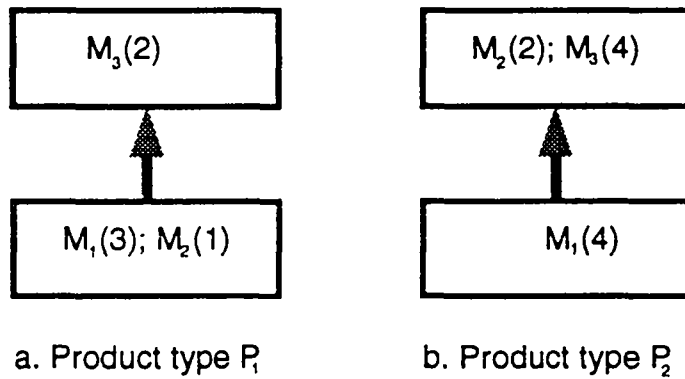a. Product type P_1 b. Product type P_2

Figure 8: Two manufacturing processes

The Petri net model of these processes are given in figure 9.
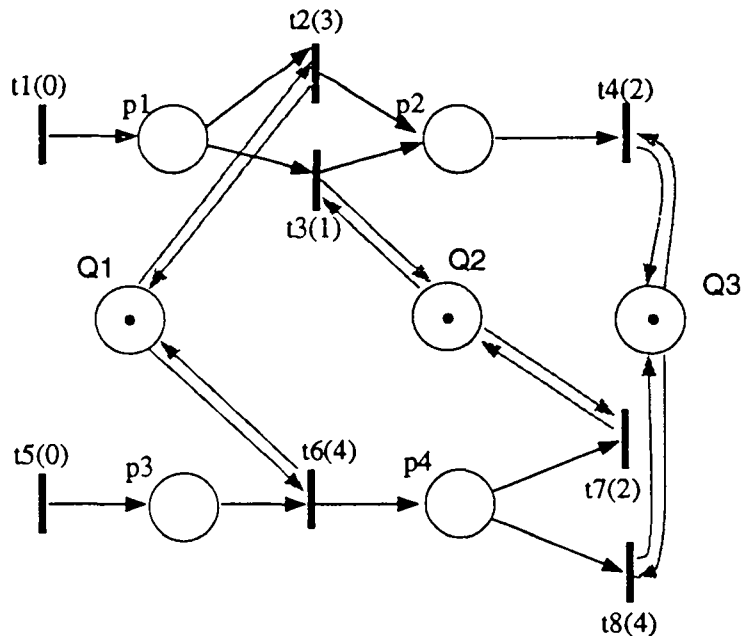
Figure 9: Petri net model

The unique token in places Q_1, Q_2, Q_3 are used to ensure that one machine cannot be used to manufacture more than one product at the same time.

In this example, we construct the following consistent CFIOs: (we do not consider the self-loops)

$$NC_1 = <t_1, p_1, t_2, p_2, t_4>$$
$$NC_2 = <t_1, p_1, t_3, p_2, t_4>$$
$$NC_3 = <t_5, p_3, t_6, p_4, t_7>$$
$$NC_4 = <t_5, p_3, t_6, p_4, t_8>$$

and the corresponding t-invariants are:

$$H_1 = <1, 1, 0, 1, 0, 0, 0, 0>$$
$$H_2 = <1, 0, 1, 1, 0, 0, 0, 0>$$
$$H_3 = <0, 0, 0, 0, 1, 1, 1, 0>$$
$$H_4 = <0, 0, 0, 0, 1, 1, 0, 1>$$

Assume that the demand is given for 4 consecutive periods as shown in table 1. The costs are presented in table 2.

Table 1 : Demand

Period / Product	1	2	3	4
1	2	15	7	3
2	6	1	4	17

Table 2 : Costs

Product	Backlogging Cost	Inventory Cost
1	20	4
2	30	2

Assume that the length of an elementary period is 40. We denote by $a_{i,k}$ the number of times the ith CFIO has to be activated during the kth period.

The capacity constraints are as follows:

$$
\begin{aligned}
4a_{1,k} & & + & 4a_{3,k} & + & 4a_{4,k} & \leq 40 \\
& a_{2,k} & + & 2a_{3,k} & & & \leq 40 \\
2a_{1,k} & + 2a_{2,k} & + & 4a_{4,k} & & & \leq 40
\end{aligned}
\tag{10}
$$

The left hand side of each inequality represents the utilization time of the related machine.

For this example the criterion consists in minimizing the sum of the inventory and the backlogging costs. The criterion to be minimized can be written:

$$
\mathcal{F} = \sum_{r=1}^{4} \left\{ 4 \left[\sum_{k=1}^{r} (a_{1,k} + a_{2,k} - d_k^1) \right]^+ + 20 \left[\sum_{k=1}^{r} (d_k^1 - a_{1,k} - a_{2,k}) \right]^+ + \right.
$$

$$
\left. + 2 \left[\sum_{k=1}^{r} (a_{3,k} + a_{4,k} - d_k^2) \right]^+ + 30 \left[\sum_{k=1}^{r} (d_k^2 - a_{3,k} - a_{4,k}) \right]^+ \right\}
$$

where d_k^i is the demand for product i during period k.

By setting

$$
y_r^1 \geq \sum_{k=1}^{r} (a_{1,k} + a_{2,k} - d_k^1)
$$

$$
z_r^1 \geq \sum_{k=1}^{r} (d_k^1 - a_{1,k} - a_{2,k})
$$

$$
y_r^2 \geq \sum_{k=1}^{r} (a_{3,k} + a_{4,k} - d_k^2)
$$

$$
z_r^1 \geq \sum_{k=1}^{r} (d_k^2 - a_{3,k} - a_{4,k})
$$

(11)

The problem to solve becomes:

$$
\text{Minimize}: \quad \sum_{r=1}^{4} (4y_r^1 + 20z_r^1 + 2y_r^2 + 30z_r^2)
$$

subject to (10), (11) and

$$
a_{i,k} \geq 0 \quad i = 1,2,3,4; \quad k = 1,2,3,4
$$
$$
y_r^s \geq 0, \; z_r^s \geq 0 \quad r = 1,2,3,4; \quad s = 1,2
$$

The result is the number of times each transition has to be fired during each elementary period. (See table 3)

Table 3 : Planning Results: transition firings

Transition Period	t1	t2	t3	t4	t5	t6	t7	t8
1	2	2	0	2	6	6	6	0
2	15	10	5	15	2	2	2	0
3	7	0	7	7	10	10	10	0
4	3	0	3	3	10	10	10	0

From table 3 we derive the production during each elementary period. (See table 4)

Table 4 : Production quantities

Product Periods	P1	P2
1	2	6
2	15	2
3	7	10
4	3	10

The subsequent step would be to define the starting time of each transition firing: this constitutes the scheduling problem, which is NP-hard. This problem is beyond the scope of this paper. Nevertheless, it must be noted that the results of planning cannot always be implemented in a schedule. In that case, the solution is to recompute a new planning with more stringent capacity constraints.

6 Conclusion

In this paper, we introduced the conflict-free nets with input and output transitions, called CFIO nets. It has been shown that CFIOs, if manageable, exhibit qualitative properties which are essential for manufacturing systems planning. Furthermore, CFIOs facilitate the trade-off between computational burden and decision choices. In this approach, the qualitative properties are built in. Thus, the decision-making system, which is in charge of making optimal decisions, can ignore the qualitative aspects of the system behavior. The next step of our research is to study further the Petri Net model structure to develop new scheduling approaches.

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