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► **To cite this version:**

Laurent George, Paul Mühlethaler, Nicolas Rivierre. Optimality and non-preemptive real-time scheduling revisited. [Research Report] RR-2516, INRIA. 1995. inria-00074162

**HAL Id: inria-00074162**

**<https://inria.hal.science/inria-00074162>**

Submitted on 24 May 2006

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INSTITUT NATIONAL DE RECHERCHJE EN INFORMATIQUE ET EN AUTOMATIQUE

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scheduling revisited*

Laurent George,  
Paul Muhlethaler,  
Nicolas Rivierre

**N° 2516**

Avril 1995

PROGRAMME 1

Architectures parallèles,  
bases de données,  
réseaux et systèmes distribués

*R*apport  
*de recherche*





# Optimality and non-preemptive real-time scheduling revisited

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Projet Reflex

Rapport de recherche n°2516- avril 1995

27 pages

**Abstract:** In this paper, we investigate the non-preemptive scheduling problem as it arises with single processor systems. We extend some previously published results concerning preemptive and non-preemptive scheduling over a single processor. We examine non-idling and idling scheduling issues. The latter are of particular relevance in the case of non-preemption..

We first embark on analyzing non-idling scheduling. The optimality of the non-idling non-preemptive Earliest Deadline First scheduling policy is revisited. Then, we provide feasibility conditions in the presence of aperiodic or periodic traffic.

Second, we examine the concept of idling scheduling, whereby a processor can remain idle in the presence of pending tasks. The non-idling non-preemptive Earliest Deadline First scheduling policy is not optimal since it is possible to find feasible task sets for which this policy fails to produce a valid schedule. An optimal algorithm to find a valid schedule (if any) is presented and its complexity analyzed.

This paper shows that preemptive and non-preemptive scheduling are closely related. However, non-preemptive scheduling leads to more complex problems when combined with idling scheduling.

**Key-words:** real-time, scheduling, non-preemptive, idling, non-idling, optimality, EDF.

# Optimalité et ordonnancement temps-réel non-préemptif réexaminés

**Résumé :** Cet article traite de l'ordonnancement non préemptif sur un monoprocesseur. Il étend des résultats antérieurs concernant l'ordonnancement préemptif et non préemptif sur un monoprocesseur. Nous examinons les problèmes d'ordonnancement dans les cas non oisifs et oisifs. L'ordonnancement oisif a un intérêt particulier dans le cas non préemptif.

Dans une première partie, l'ordonnancement non oisif est étudié. L'optimalité de la politique d'ordonnancement Echéance la plus Proche en Premier (EDF) non oisive et non préemptive est examinée. Nous dérivons ensuite des conditions de faisabilité en présence de trafic aperiodique ou periodique.

La seconde partie est dédiée à l'ordonnancement oisif, pour lequel le processeur peut rester inactif en présence de tâches en attente. La politique d'ordonnancement EDF non oisive et non préemptive n'est pas optimale car il existe des jeux de tâches faisables pour lesquels cette politique ne fournit pas un ordonnancement valide. Un algorithme optimal pour trouver une séquence valide (si elle existe) est présenté et sa complexité est étudiée.

Cet article montre que les problèmes posés par l'ordonnancement préemptif et l'ordonnancement non préemptif sont similaires. Cependant, l'ordonnancement non préemptif conduit à des problèmes plus complexes quand l'ordonnancement oisif est également considéré.

**Mots-clé :** temps-réel, ordonnancement, non-préemptif, oisif, non-oisif, optimalité, EDF.

## **1. Introduction**

This paper addresses the problem of non-preemptive scheduling over a single processor. This problem has received less attention than preemptive scheduling, which has been extensively studied for the past twenty years. In the case of non-preemptive scheduling, there is a special interest in studying both idling and non-idling scheduling policies. It is recalled that with a non-idling scheduling policy, the processor cannot be idle if there are released tasks pending. We will see that there are cases where a valid schedule can be found by an idling scheduling policy whereas no non-idling scheduling policy can find such a valid schedule.

In this paper, most of the results are related to the EDF (Earliest Deadline First) scheduling policy which has been shown to be optimal in many contexts in the case of preemptive scheduling. In fact, we will show that in most cases, results established for preemptive scheduling have a counterpart when considering non-preemptive scheduling.

Section 2 is devoted to introducing the models and the notations used throughout this paper.

Non-idling scheduling issues are addressed in section 3. The first subsection establishes the optimality of the non-idling, non preemptive Earliest Deadline First scheduling policy (NINP\_EDF) for any sequence of concrete tasks. The second subsection is concerned with feasibility conditions for the aperiodic/periodic/sporadic, concrete/non-concrete contexts.

Section 4 is devoted to the analysis of idling scheduling policies. NINP-EDF is shown to be sub-optimal. The problem of finding an optimal scheduling policie in such a context has been shown to be NP-Hard in the strong sense in [GA79]. We propose an exhaustive algorithm (exponential in the worst case) which takes advantage of the partial optimality of NINP-EDF and enables the search to be limited. The behavior of this algorithm is studied with various examples.

## **2. Notations and definitions**

Throughout this paper, we assume the following:

- the **EDF** scheduling policy uses any fixed tie breaking rule between tasks when they have the same absolute deadline (i.e. release time + relative deadline).
- NINP-EDF denotes Non-Idling Non-Preemptive EDF.
- time is discrete (tasks invocations occur and tasks executions begin and terminate at clock ticks; the parameters used are expressed as a multiples of clock ticks); see [BHR90] for a justification.
- for the sake of simplicity, we shall use  $a(i)$  to describe the tasks and its parameters. For example, we shall write  $a(i)=(r_i, e_i, d_i)$  for a concrete aperiodic task.

We consider the scheduling problem of a set  $a() = \{a(1), \dots, a(n)\}$  of  $n$  tasks  $a(i)$ ,  $i \in [1, n]$  over a single processor. By definition:

- A task is said **concrete** if its release time is known a priori otherwise it is **non-concrete**. Then, an infinite number of concrete task sets can be generated from a non-concrete task set.
- An **aperiodic** task is invoked once when a **periodic** (or **sporadic**) task recurs. Periodic and sporadic tasks differ only in the invocation time. The  $(k+1)^{\text{th}}$  invocation of a periodic task occurs at time  $t_{k+1} = t_k + p_i$  while it occurs at  $t_{k+1} \geq t_k + p_i$  if the task is sporadic.

Notations:

- a concrete aperiodic task  $a(i)$ , consists of a triple  $(r_i, e_i, d_i)$  where  $r_i$  is the absolute time the task is released,  $e_i$  the execution time and  $d_i$  the relative deadline. A concrete periodic (or sporadic) task  $a(i)$ , is defined by  $(r_i, e_i, d_i, p_i)$  where  $p_i$  is the period of the task.
- a non-concrete aperiodic task  $a(i)$  consists of  $(e_i, d_i)$ . A non-concrete periodic (or sporadic) task  $a(i)$  is defined by  $(e_i, d_i, p_i)$ .

Furthermore, by definition:

- A **non-preemptive** scheduling policy does not interrupt the execution of any task.
- With **idling** scheduling policies, when a task has been released, it can either be scheduled or wait a certain time before being scheduled even if the processor is not busy.
- With **non-idling** scheduling policies, when a task has been released, it cannot wait before being scheduled if the processor is not busy. Notice that an idle period, i.e. no pending tasks in this case, can have a zero duration.
- A concrete task set  $a()$  is said to be **synchronous** if there is a time when  $r_i = r_j$  for all tasks  $i, j \in [1, n]$ ; otherwise, it is said to be **asynchronous** (the problem of deciding whether an asynchronous task set can be reduced to a synchronous one has been shown to be NP-complete in [LM80]).
- A concrete task set  $a()$  is said to be **valid (schedulable)** if it is possible to schedule the tasks of  $a()$  (including periodic recurrences in the case of periodic or sporadic task sets) so that no task ever misses a deadline when tasks are released at their specified released times.
- A non-concrete task set  $a()$  is said to be **valid (schedulable)** if every concrete task set that can be generated from  $a()$  is schedulable.
- A scheduling policy is said to be **optimal** if this policy finds a valid schedule when any exists.

### **3. Non-idling and non-preemptive scheduling**

Section 3.1 establishes the optimality of NINP-EDF in the presence of any sequence of concrete tasks. Section 3.2 is concerned with feasibility conditions.

### 3.1 Optimality of NINP-EDF

**Theorem 3a:** NINP-EDF is optimal in the presence of any sequence of  $n$  concrete tasks.

**Proof:** Let  $s()$  be a valid schedule;  $s(1)$  is the first task scheduled and  $s(n)$  is the last one. Let us introduce a particular reranking of any two successive tasks  $s(i)$  and  $s(i+1)$  in the schedule. Let  $t_i$  (respectively  $t_{i+1}$ ) be the beginning of the execution of  $s(i)$  (respectively  $s(i+1)$ ).

$s(i)$  and  $s(i+1)$  are left unchanged if  $r_i+d_i < r_{i+1}+d_{i+1}$  or  $r_{i+1} > t_i$ . Otherwise,  $s(i)$  and  $s(i+1)$  are exchanged. The resulting schedule is still valid because  $r_i+d_i > r_{i+1}+d_{i+1}$  (see figure 1).

A full reranking procedure is obtained if we browse through the permutation starting with  $i=1$  to  $i=n-1$ . After a finite number of full rerankings, we obtain a stable (i.e. unchanged by a full reranking procedure) valid non-idling, non-preemptive schedule. The finite number comes from the maximum number of times ( $j$  for  $s(j)$ ) that a scheduled task can be reranked which leads to a complexity in  $O(n^2)$  for the complete procedure.

Let us now show that the obtained schedule is NINP-EDF. Suppose the contrary, then we have a task  $j$  such that  $r_j+d_j < r_{j+1}+d_{j+1}$  and  $s(j+1)$  is released before or at  $t_j$ . In such a case the full reranking will not leave the sequence unchanged, which is a contradiction and thus  $s()$  is exactly the sequence obtained by NINP-EDF.

**EndProof**

Valid schedule

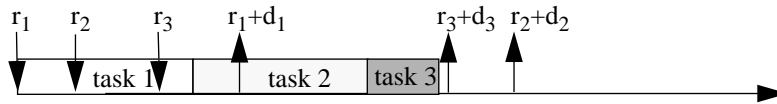
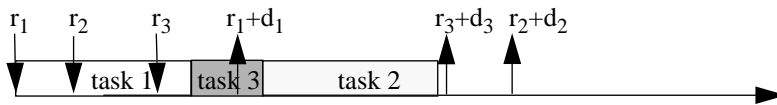


Figure 1: Reranking of  $s(2)$ ,  $s(3)$

Valid NINP EDF schedule



This theorem means exactly that, in the presence of any sequence of a concrete task set (concrete aperiodic tasks or concrete periodic/sporadic tasks during an interval), NINP-EDF is optimal among non-idling, non preemptive scheduling policies. Such a theorem has already been proved in the following cases:

- preemptive EDF for various traffics [LILA73], [LM80], [MOK83], [CHE87], [BHR90]...
- NINP-EDF with concrete/non-concrete periodic and sporadic task sets [KIM80], [JE91]. Notice that in [JE91], it is shown that the non-preemptive scheduling of concrete periodic tasks is NP-hard in the strong sense. We be-



lieve that this problem deals with the establishment of a necessary and sufficient condition but not with the optimality of NINP-EDF (in the sense that any valid sequence of schedulable concrete periodic tasks will be scheduled by NINP-EDF).

The optimality of NINP-EDF in the presence of non-concrete aperiodic task set will be addressed in 3.2.1.2.

### **3.2 Feasibility conditions**

#### **3.2.1 Aperiodic tasks**

##### **3.2.1.1 Concrete aperiodic tasks**

Let  $a()$  be a task set of  $n$  concrete aperiodic tasks  $a(i) = (r_i, e_i, d_i)$ . An obvious feasibility condition is to use directly the optimality of NINP-EDF (see section 3.1). This can be done by the following recursive algorithm  $g(t)$ .

-----  
 Initialisation:  $g(\text{first release})$

```

g(t)
put the new released tasks into the ordered (according to EDF) pending queue
IF the first task of the ordered pending queue is a valid (according to EDF)
  t <- t + duration of this task
  IF there are no more task
    success /* valid schedule */
  ELSE
    IF the ordered pending queue is not empty
      g(t)
    ELSE
      g(next release)
    ENDIF
  ENDIF
ELSE
  check /* not valid schedule */
ENDIF
  
```

-----

The complexity of  $g(t)$  is  $O(n)$  if the tasks are already sorted in increasing order of released time. For a set of randomly ordered tasks, an initial sorting would be required increasing the complexity to  $O(n \log(n))$ .

##### **3.2.1.2 Non-concrete aperiodic tasks**

Let  $a()$  be a task set of  $n$  non-concrete aperiodic tasks  $a(i) = (e_i, d_i)$ . The theorem 3.b is inspired by [JE91] (which establishes the optimality of NINP-EDF and a pseudo-polynomial necessary and sufficient feasibility condition for any non-concrete periodic/sporadic task sets) but adapted to non-concrete aperiodic task set.

**Theorem 3.b:** Let  $a() = \{a(1), a(2), \dots, a(n)\}$ , where  $a(i) = (e_i, d_i)$ , be a set of  $n$  non-concrete aperiodic tasks sorted in increasing order by relative deadline (i.e.,

for any pair of tasks  $a(i)$  and  $a(j)$ , if  $i > j$ , then  $d_i \geq d_j$ ). A necessary and sufficient condition for  $a()$  to be schedulable, using NINP-EDF is:

$$(C1) \quad \forall i, 1 < i \leq n \quad ; \quad \forall j, 1 \leq j < i: \quad d_j \geq e_i - 1 + \sum_{k=1}^j e_k$$

**Proof:** We will demonstrate first that this condition is necessary (part 1) and then sufficient (part 2). For that purpose, let us define the processor demand in the time interval  $[T_1, T_2]$ , written  $D_{T_1, T_2}$ , as the maximum amount of processing time required by a concrete task set  $b()$  (generated from  $a()$ ) in the interval  $[T_1, T_2]$ .  $D_{T_1, T_2}$  will be a function of release, execution time and deadline of the tasks. More precisely  $D_{T_1, T_2}$  will include:

- all tasks with deadlines in the interval  $[T_1, T_2]$  (complete or remaining execution time).
- some tasks with deadlines greater than  $T_2$  (if there are times when in the interval  $[T_1, T_2]$ , where only tasks with deadlines greater than  $T_2$  are pending).

$b()$  is schedulable if and only if for all intervals  $[T_1, T_2]$ ,  $D_{T_1, T_2} \leq T_2 - T_1$ .

Part 1: Condition (C1) is necessary. We will prove the contrapositive, if  $a()$  does not satisfy (C1) then there exists a concrete task set  $b()$  (generated from  $a()$ ), that is not schedulable i.e.

$$\exists i, 1 < i \leq n \quad ; \quad \exists j, 1 \leq j < i \quad \text{such that} \quad d_j < e_i - 1 + \sum_{k=1}^j e_k.$$

This leads to the concrete task set  $b()$  shown in figure 2, generated from  $a()$ , where for some value of  $i$ ,  $1 < i \leq n$ ,  $r_i = 0$  and where the other tasks are released at 1.

We then have:

$$d_j + 1 < e_i + \sum_{k=1}^j e_k = D_{0, (d_j + 1)}.$$

Indeed  $D_{0, (d_j + 1)}$  consists of the cost of:

- the execution of task  $b(i)$  (since neither preemption nor inserted idle time are allowed, task  $b(i)$  must be executed in the interval  $[0, e_i]$ ).
- plus the processor demand due to the tasks 1 through  $j$  in the interval  $[1, d_j + 1]$  (since (C1) does not hold and since the tasks are sorted in increasing order by deadline, tasks with relative deadlines greater than or equal to  $d_j$  do not contribute to this processor demand).

As (C1) does not hold then  $d_j + 1 < D_{0, (d_j + 1)}$  and hence  $b()$  is not schedulable.

The condition is necessary.

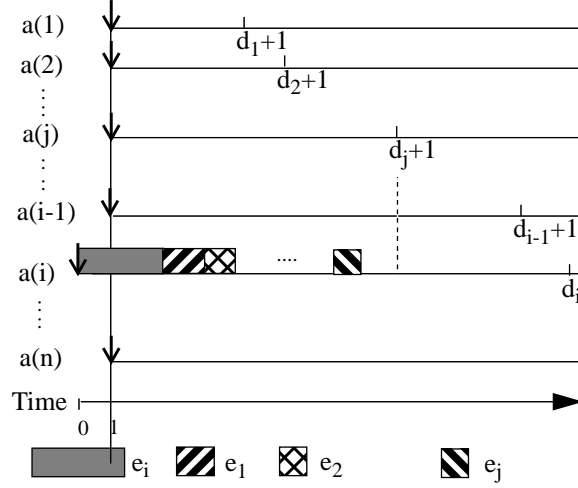


Figure 2

**Part2:** One will show now by contradiction that the condition is also sufficient. Thus, let us suppose that  $a()$  satisfies (C1) but is not schedulable. In other words, there exists at least one concrete task set  $b()$  (generated from  $a()$ ) such that  $b()$  is not schedulable,  $\forall i, 1 \leq i \leq n$   $b(i) = (r_i, e_i, d_i)$ . Let  $s()$  be a non valid schedule of  $b()$  such that the deadline of  $b(j)$  is not met at time  $T$  ( $T=r_j+d_j$ ). Consider now  $T_0$  the end of the last idle period before  $T$ . There are two cases during the busy period  $[T_0, T]$ :

- all the scheduled tasks have their absolute deadline less than or equal to  $T$ .
- the opposite; at least one of the scheduled tasks has its deadline after  $T$ .

In the first case and since  $T_0$  is the beginning of a busy period,  $D_{T_0, T}$  the processor demand during the busy period  $[T_0, T]$  is such that:

$$D_{T_0, T} \leq \sum_{k=1}^n \delta_{(T-T_0 \geq d_k)} e_k \text{ where } \delta_{(T-T_0 \geq d_k)} = 1 \text{ if } T - T_0 \geq d_k \text{ and } 0 \text{ else.}$$

Indeed,  $D_{T_0, T}$  cannot include:

- pending tasks released before  $T_0$  (by definition of  $T_0$  in our non-idling context).
- tasks released after (or at)  $T_0$  with relative deadline greater than  $T-T_0$  since we are in the first case (all the scheduled tasks during the busy period  $[T_0, T]$  have their absolute deadline less than or equal to  $T$ ).

At the same time, due to the missed deadline,  $T - T_0 < D_{T_0, T}$  and therefore one obtains:

$$(a) \quad T - T_0 < \sum_{k=1}^n \delta_{(T-T_0 \geq d_k)} e_k.$$

Moreover as  $T=r_j+d_j$  and as  $r_j < T_0$  is impossible (by definition of  $T_0$  in our non-idling

context), we just consider  $r_j \geq T_0$ . Since we are in the first case (all the scheduled tasks during the busy period  $[T_0, T]$  have their absolute deadline before  $T$ ). If  $r_j > T_0$  leads to missing the deadline of  $b(j)$  then it follows that  $r_j = T_0$  leads to missing the deadline of a task  $b(f)$  with  $f \geq j$  (by iteration on  $j$ , this problem will be detected later). Then, we examine only  $r_j = T_0$  (i.e.  $T - T_0 = d_j$ ) and one obtains

$$\sum_{k=1}^n \delta_{(d_j = T - T_0 \geq d_k)} e_k = \sum_{k=1}^j e_k.$$

As a consequence (a) is equivalent to (a')  $d_j < \sum_{k=1}^j e_k$ .

Since (C1) implies that  $\forall j, 1 \leq j \leq n: d_j \geq \sum_{k=1}^j e_k$ : (a') contradicts (C1).

In the second case (see figure 2), let us consider the scheduled task  $s(i')$  as being the last task which is scheduled during the busy period  $[T_0, T]$  having a deadline after  $T$  (by bijection there exists  $a(i)$  such as  $s(i')$  is the execution of  $a(i)$ ). Notice  $T_i$  the start time of the execution of  $a(i)$ . Similarly to the first case, the processor demand  $D_{T_i, T}$  during the busy period  $[T_i, T]$  is such that:

$$D_{T_i, T} \leq e_i + \sum_{k=1}^n \delta_{(T - T_i - 1 \geq d_k)} e_k.$$

Indeed, if we use NINP-EDF,  $D_{T_i, T}$  cannot include:

- pending tasks at  $T_i$  (except  $a(i)$ ) such as their relative deadline are:
  - . less than  $d_i$  (otherwise, due to NINP-EDF, they should have been executed instead of  $a(i)$ ).
  - . greater than or equal to  $d_i$ . Since  $a(i)$  is the last scheduled task during  $[T_0, T]$  with a deadline greater than  $T$ .
- tasks released after  $T_i$  with relative deadlines greater than  $T - T_i - 1$  since we are in the second case and that any scheduled task after  $T_i$  has its absolute deadline less than or equal to  $T$ .

At the same time, due to the missed deadline,  $T - T_i < D_{T_i, T}$  and therefore one obtains:

$$(b) \quad T - T_i < e_i + \sum_{k=1}^n \delta_{(T - T_i - 1 \geq d_k)} e_k.$$

Moreover as  $T = r_j + d_j$  and as  $r_j < T_i + 1$  is impossible (otherwise, due to NINP-EDF,  $a(j)$  would have been executed instead of  $a(i)$ ), we just consider  $r_j \geq T_i + 1$ . Since we are in the second case and that any scheduled task after  $T_i$  has its absolute deadline less

than or equal to  $T$ . If  $r_j > T_i + 1$  leads to missing the deadline of  $b(j)$  then it follows that  $r_j = T_i + 1$  leads to missing the deadline of a task  $b(f)$  with  $f \geq j$  (by iteration on  $j$ , this problem will be detected later). Then, we examine only  $r_j = T_i + 1$  (i.e.  $T - T_i - 1 = d_j$ ) and one obtains

$$e_i + \sum_{k=1}^n \delta_{((d_j = T - T_i - 1) \geq d_k)} e_k = e_i + \sum_{k=1}^j e_k.$$

As a consequence (b) is equivalent to  $d_j < e_i - 1 + \sum_{k=1}^j e_k$  which contravenes to our initial conditions (C1).

As any non-concrete aperiodic task set  $a()$  verifying the necessary conditions is scheduled by NINP-EDF, it follows that:

- the condition is also sufficient.
- NINP-EDF, is optimal in presence of any non-concrete aperiodic task set.

**EndProof**

Notice that the condition (C1) is in  $O(n^2)$  and that in [CHE87], a feasibility condition is given for aperiodic preemptive task set which could have been used to establish the above theorem.

### 3.2.2 Concrete periodic Tasks

The problem of knowing whether in non-idling context, a non-preemptive set of concrete periodic tasks (defined as  $a(i) = (r_i, e_i, d_i, p_i)$  for  $i=1..n$ ) is schedulable has been shown NP-Complete in the strong sense by [JE91]. More precisely, with  $0 < e_i \leq d_i = p_i$ , a given pseudo-polynomial feasibility condition is shown to be:

- necessary and sufficient for any non-concrete periodic/sporadic task set and for any concrete sporadic task set.
- sufficient but not necessary for any concrete periodic task set since by construction the proof characterizes the worst case which is not necessarily the case for concrete periodic tasks.

In this subsection we show that the feasibility conditions established with preemptive EDF for concrete periodic task sets can be adapted with any non-preemptive, non-idling optimal scheduling policy (e.g. NINP-EDF as shown in theorem 3.a). We illustrate this idea on two well known results:

- First, we extend (section 3.2.2.1) the result of [LM80] to non-preemptive scheduling in order to provide a necessary and sufficient, but exponential, feasibility condition. This condition makes it possible to determine if a non-preemptive concrete periodic task set is schedulable.
- Then, we study (section 3.2.2.2) the special case of synchronous tasks. In this particular context we adapt the result of [BHR90] in order to obtain a pseudo polynomial necessary and sufficient feasibility condition.

Let us introduce:

- $P$  = least common multiple of  $\{p_1, \dots, p_n\}$  the periods of a task set  $a()$ .
- $r = \max\{r_1, \dots, r_n\}$  (without loss of generality, we assume that  $\min\{r_1, \dots, r_n\} = 0$ ).

### 3.2.2.1 Asynchronous case

This part is an adaptation of [LM80] to any non-idling, non-preemptive optimal scheduling policy.

Let  $C_{s()}(a(), t) = \sum_{i=1}^n e_{i,t}$  denote the configuration, at time  $t$ , of the schedule  $s()$  for

the task set  $a()$ .  $e_{i,t}$  is the amount of time for which task  $T_i$  has executed from its last release time up until time  $t$  ( $e_{i,t} = 0$  if  $t < r_i$ ).

Notice that if  $C_{s()}(a(), t)^-$  and  $C_{s()}(a(), t)^+$  denote the amount of work done respectively just before and just after time  $t$ , generally we have  $C_{s()}(a(), t)^- = C_{s()}(a(), t)^+$  except if tasks arrive at time  $t$ . More precisely, as  $d_i \leq p_i$  (only the last release of any task  $a(i)$  matters) and if  $A_t$  is the set of tasks arriving at time  $t$ , we have:  $C_{s()}(a(), t)^+ = C_{s()}(a(), t)^- - \sum_{i \in A_t} e_i$ .

**Lemma 3.a:** Let  $s()$  be the schedule of an asynchronous concrete periodic task set  $a()$  (defined as  $a(i) = (r_i, e_i, d_i, p_i)$  for  $i=1 \dots n$  with  $0 < e_i \leq d_i \leq p_i$ ) constructed by any given non-preemptive, non-idling optimal scheduling policy. Then for each instant  $t \geq r$ , we have  $C_{s()}(a(), t) \geq C_{s()}(a(), t + P)$ .

**Proof:** In a non-preemptive context, it is possible to execute a task  $a(i)$  even if a pending task  $a(j)$  has a deadline less than  $a(i)$ . This occurs if  $a(j)$  is released during the execution of  $a(i)$ . Therefore for this Lemma, unlike [LM80], we are obliged to consider, at any time  $t$ , the complete configuration of the released task ( $C_{s()}(a(), t)$ ) and not tasks one by one.

Due to the first releases times, the pattern of the arriving tasks during  $[0, r]$  is not similar to the pattern of the arriving tasks during  $[P, P + r]$ . More precisely, the scheduled tasks during  $[0, r]$  can be deduced from the scheduled tasks during  $[P, P + r]$  by removing for every task  $a(i)$  each execution during  $[P, P + r_i]$  (not arrived in  $[0, r_i]$  due to the first release time of  $a(i)$ ).

As the given scheduling policy is non-idling and as  $d_i \leq p_i$  (only the last release of any task  $a(i)$  matters), this removing operation can only increase, at time  $t=r$ , the sum of the execution time devoted to the execution of the configuration. Then:

$$C_{s()}(a(), r) \geq C_{s()}(a(), r + P).$$

As the pattern of the releases after  $t \geq r$  is the same as after  $t + P$ , it follows that:

$$C_{s()}(a(), t) \geq C_{s()}(a(), t + P).$$

**EndProof**

Lemma 3.a means that as the time elapses there is a tendency to be late. We are now ready to obtain the result which is again a fairly intuitive result: if an asynchronous concrete periodic task set is feasible then the amount of work executed in similar conditions does not strictly decrease.

**Lemma 3.b** Let  $s()$  be the schedule of an asynchronous concrete periodic task set  $a()$  (defined as  $a(i)=(r_i, e_i, d_i, p_i)$  for  $i=1\dots n$  with  $0 < e_i \leq d_i \leq p_i$ ) constructed by any given non-preemptive, non-idling optimal scheduling policy. If  $a()$  is feasible on one processor, then there is some instant  $t$  in the interval  $[r+P, r+2P]$  when

$$C_{s()}(a(), t_1) = \sum_{i=1}^n e_i.$$

In other words, at  $t$  all the released tasks are executed i.e.  $C_{s()}(a(), t)$  reaches its maximum.

**Proof:** First, we show that a processor idle at  $t$  cannot be busy during  $]t, t+P]$  and still has work to do at  $t+P$ . For that purpose, let  $M(T)$  be the maximum processing time requested by the tasks during  $[0, T]$ . By simple algebra,  $M(T)$  is given by:

$$M(T) = \sum_{i=1}^n \max\left(0, \left\lceil \frac{T - r_i}{p_i} \right\rceil\right) e_i.$$

The processor is idle at  $t$  then the processing time used to execute all the tasks is equal to  $M(t)$ . Suppose now that the processor is continuously busy (no idle period) in the interval  $]t, t+P]$ , then the processing time used to execute all the tasks at  $t+P$  is exactly  $M(t+P)$ . If at  $t+P$  the processor is not idle then we have

$$M(t+P) > M(t) + P \quad (1)$$

$$\text{With } M(t+P) = \sum_{i=1}^n \max\left(0, \left\lceil \frac{t+P - r_i}{p_i} \right\rceil\right) e_i.$$

As  $P$  is the least common multiple of the periods,  $M(t+P)$  can be reformulated as:

$$M(t+P) = P \sum_{i=1}^n \frac{e_i}{p_i} + M(t)$$

$$\text{then (1) becomes: } P \sum_{i=1}^n \frac{e_i}{p_i} + M(t) > P + M(t) \quad \text{leading to } \sum_{i=1}^n \frac{e_i}{p_i} > 1$$

which is not possible on one processor.

We are now ready to complete the proof. Consider  $t$  the end of the last idle period before  $r+2P$  (i.e.  $]t, r+2P]$  is a busy period). If  $t < r+P$ , it comes that

$$\sum_{i=1}^n \frac{e_i}{p_i} > 1$$

which is impossible on one processor. Thus, if  $a()$  is feasible,  $C_{s()}(a(), t)$  reaches its maximum during  $[r+P, r+2P]$ .

**EndProof**

**Theorem 3.c:** Let  $s()$  be the schedule of an asynchronous concrete periodic task set  $a()$  (defined as  $a(i)=(r_i, e_i, d_i, p_i)$  for  $i=1\dots n$  with  $0 < e_i \leq d_i \leq p_i$ ) constructed by any given non-preemptive, non-idling optimal scheduling policy.  $a()$  is feasible on one processor if and only if (1) all deadlines in the interval  $[0, r + 2P]$  are met in the schedule  $s()$  and (2) there is some instant  $t$  in the interval  $[r+P, r+2P]$  when all the pending tasks are executed.

**Proof:** (the only if part) If  $a()$  is feasible on one processor and since we use a non-idling optimal scheduling policy then  $s()$  must be a valid schedule. Thus all deadlines in the interval  $[0, r+2P]$  are met in the schedule  $s()$  (condition (1)), see [LM80] for a justification of the interval. Furthermore, if  $a()$  is feasible, by Lemma 3.b, we have some instant  $t$  in the interval  $[r+P, r+2P]$  when all released tasks are executed (condition(2)).

(the if part) If conditions (1) and (2) hold it follows that, as the schedule  $s()$  is valid in the interval  $[0, t]$  and as the pattern of arrival is the same every  $P$  after  $t$  (i.e. the configuration reaches its maximum every  $C_{s()}(a(), t + k \cdot P)$  for each non-negative integer  $k$ ),  $a()$  is feasible on one processor.

**EndProof**

Theorem 3.c makes it possible to determine whether a non-preemptive set  $a()$  of concrete periodic tasks is schedulable. The complexity of this test is exponential, indeed the problem is known to be NP-Complete in the strong sense (see [BHR90]), but our aim was to show that the feasibility conditions established with preemptive EDF for concrete periodic task sets could be reformulated with any non-preemptive, non-idling optimal scheduling policy.

Let us now show that, as in the case of preemptive scheduling, more refined results can be obtained in the case of synchronous tasks where the total load is known to be under 100%.

### 3.2.2.2 Synchronous case

The following results are close to [BHR90] since the non-preemptive behavior does not modify the strategy used. More precisely, whatever the context (preemptive or non-preemptive), if the density is less than 1 and if the system is not feasible then there must be a time interval in which too much execution time is required. This



interval can be combined, whatever the context, with any optimal scheduling policy to establish the proofs.

Let us first show (theorem 3.d) a simple but not optimized w.r.t complexity result in a synchronous non-preemptive context. We will show subsequently (theorem 3.e) that this result, as in [BHR90], can be improved w.r.t complexity.

**Theorem 3.d:** Let  $s()$  be the schedule of a synchronous concrete periodic task set  $a()$  (defined as  $a(i)=(r_i, e_i, d_i, p_i)$  for  $i=1\dots n$  with  $0 < e_i \leq d_i \leq p_i$ ) and all the release times synchronized at 0) constructed by any given non-preemptive, non-idling optimal scheduling policy.  $a()$  is feasible on one processor if and only if all deadlines in the interval  $[0, P]$  are met in the schedule  $s()$  (condition 1) and at time  $P$ , all the pending tasks are executed (condition 2).

**Proof:** This context is a particular case of theorem 3.c in a synchronous context where:

- the interval is limited to  $[0, P]$  instead of  $[0, r+2P]$  (see [LM80]).
- all the release times are synchronous every  $k \cdot P$  for each non-negative integer  $k$ .

(only if part) If  $a()$  is feasible on one processor and since we use a non-idling optimal scheduling policy then  $s()$  must be a valid schedule. Thus all deadlines in the interval  $[0, P]$  are met in the interval  $s()$  (condition (1)). Furthermore, if  $a()$  is feasible, by Lemma 3.b, all the released tasks are executed every  $k \cdot P$  for each non-negative integer  $k$  otherwise at least one deadline is missed (condition (2)).

(if part) If condition (1) and (2) hold it follows that, as schedule  $s()$  is valid in the interval  $[0, P]$ , as the pattern of arrival is the same every  $P$  and then as the configuration reaches its maximum every  $C_{s()}(a(), k \cdot P)$  (for each non-negative integer  $k$ ),  $a()$  is feasible on one processor.

**Endproof**

Let denote  $\eta_i(t_1, t_2)$  ( $0 \leq t_1 < t_2$ ) the total number of times such that a task  $a(i)$  must be completely scheduled during the interval  $[t_1, t_2)$ . In [BHR 90] it is shown in the synchronous case with simple algebra that:

$$\eta_i(t_1, t_2) = \max \left\{ 0, \left\lfloor \frac{t_2 - d_i}{p_i} \right\rfloor - \left\lfloor \frac{t_1}{p_i} \right\rfloor + 1 \right\}.$$

This result will now allow us to produce a better feasibility condition than Theorem 3.d.

**Theorem 3.e:** A task system  $a()$ , satisfying the context of theorem 3.d and

$$\sum_{i=1}^n \frac{e_i}{p_i} = c, \text{ with } c < 1 \text{ is feasible iff:}$$

$$(1) \quad \sum_{i=1}^n \eta_i(t_1, t_2) e_i \leq t_2 - t_1,$$

with

$$0 \leq t_1 < t_2 < \frac{c}{1-c} \max(p_i - d_i).$$

**Proof:** The proof is based on the fact that  $t_1$  may be chosen to be 0. From the previous algebra, we have:

$$\eta_i(0, t_2 - t_1) = \max \left\{ 0, \left\lfloor \frac{t_2 - t_1 - d_i}{p_i} \right\rfloor + 1 \right\}$$

As  $\lfloor a - b \rfloor \geq \lfloor a \rfloor - \lceil b \rceil$ , it follows:

$$\eta_i(0, t_2 - t_1) \geq \max \left\{ 0, \left\lfloor \frac{t_2 - d_i}{p_i} \right\rfloor - \left\lceil \frac{t_1}{p_i} \right\rceil + 1 \right\} = \eta_i(t_1, t_2)$$

If condition (1) does not hold,  $\sum_{i=1}^n \eta_i(t_1, t_2) e_i > t_2 - t_1$  and then

$$\sum_{i=1}^n \eta_i(0, t_2 - t_1) e_i > t_2 - t_1.$$

Hence  $t_1$  may be chosen to be 0 and as  $t_2 < \sum_{i=1}^n \eta_i(0, t_2) e_i$ , it follows that:

$$t_2 < \sum_{i=1}^n \left( \left\lfloor \frac{t_2 - d_i}{p_i} \right\rfloor + 1 \right) e_i.$$

Removing the floor function, we have:

$$t_2 < \sum_{i=1}^n \frac{t_2 - d_i + p_i}{p_i} e_i = \sum_{i=1}^n \left( \frac{t_2 e_i}{p_i} + \frac{(p_i - d_i) e_i}{p_i} \right) \text{ which leads to:}$$

$$t_2 < ct_2 + c \max \{ p_i - d_i \}.$$

Finally:

$$t_2 < \frac{c}{1-c} \max \{ p_i - d_i \},$$

i.e., if condition (1) does not hold then there exists  $t_2 < \frac{c}{1-c} \max \{ p_i - d_i \}$  such that a deadline is missed.

**Endproof.**

Notice that:

- A consequence, theorem 3.e can be checked in  $O(n \cdot \max\{p_i - d_i\})$  if

$$\sum_{i=1}^n \frac{e_i}{p_i} \leq c, \text{ with } c < 1.$$

- Unlike the preemptive case, the worst pattern of arrival in the non-preemptive case is not the synchronous one. Indeed, due to the non-preemptive behavior, a high priority task can be delayed by any task starting its execution just before it (see the pattern examined in theorem 3.b for aperiodic feasibility condition of non-concrete task set).
- In the periodic section, we have shown on two examples that it was easy to derive existing feasibility conditions established for preemptive traffics to non-preemptive ones (our aim was not to address every existing improvement [Mok94]).

## 4. Idling and non-preemptive scheduling

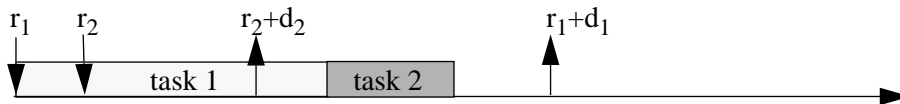
In this section we schedule a set of  $n$  concrete aperiodic tasks  $a()$  ( $a(i) = (r_i, e_i, d_i)$ ), (see section 2) sorted in non-decreasing order of released time ( $r_1=0$  by convention).

### 4.1 Overview

The general problem of finding a feasible schedule in an idling and non-preemptive context is known to be NP-complete [GA79, annex 5]. An exhaustive search leads to examining  $n!$  different schedules in the worst case.

Heuristic techniques can be used [MA84], [MOK83], [ZHAO87] to reduce the complexity. However, this reduction is achieved at the cost of obtaining a potentially sub-optimal solution. For example, NINP-EDF is not optimal for idling scheduling otherwise this would have contradicted the NP-completeness, e.g. the following task set is feasible but NINP-EDF is unable to find a valid schedule (see figure 3).

non-valid schedule obtained by NINP-EDF



Valid schedule

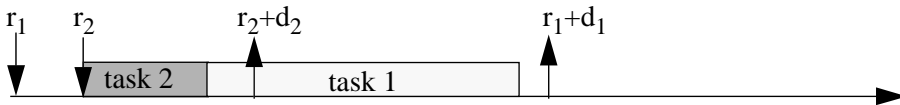


Figure 3: NINP-EDF is not optimal in idling context

Optimal decomposition approaches can be used [YUA91], [YUA94], [PC92] to reduce the complexity by dividing the  $n$  tasks into  $m$  subset. Decomposition, however, is not possible for any task sets.

We now propose a branch and bound scheduling algorithm (see section 4.2) which efficiently limits an exhaustive search. This algorithm always finds a solution when any exists. Although the theoretical complexity is still in  $n!$  in the worst case, this algorithm shows good performances at run time (see section 4.3).

## 4.2 A scheduling algorithm

### 4.2.1 Description of the algorithm

The basic idea is that NINP-EDF is still optimal if it is applied during specific intervals. For this purpose, we will show that any valid schedule can lead to at least one valid prompt schedule (see lemma 4.a) which can itself lead to one valid prompt EDF schedule (see lemma 4.b). Then a branch and bound scheduling algorithm will be presented (see lemma 4.c and theorem 4.a).

**Lemma 4.a:** If any valid schedule  $s()$  of a task set  $a()$  exists then it is possible to find at least one valid schedule  $s'()$  for which every task starts either at a released time or at the end of the execution of the last scheduled task. We call  $s'()$  a valid prompt

schedule of  $a()$  (see figure 4).

**Proof:** The proof comes from the simple fact that if one can advance the execution time of the tasks then the schedule is still valid.

To do so consider a task  $s(i)$  of  $s()$  starting its execution at  $t1$ . If  $s(i)$  does not start either:

- at  $t2$ , the end of the execution of  $s(i-1)$ , the previous executed task,
- or at  $r_j$ , one of the possible release times such that  $t2 < r_j$  and  $r_j > r_i$ ,

then it is possible to advance the execution of  $s(i)$  at  $t3$  such that:

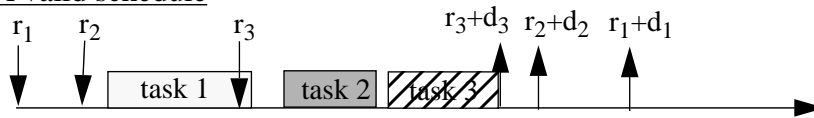
$$(t3 = t2 \text{ or } t3 = r_j) \text{ and } t3 > r_i.$$

We now call this task  $s'(i)$ . If we apply this consideration from the first to the last executed task of  $s()$ , we are sure that all these tasks will satisfy this lemma. Then we obtain  $s'()$  a valid prompt schedule of  $a()$ .

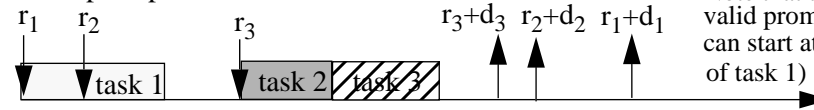
Note that a valid schedule can lead to several valid prompt schedules, as each task can lead to several possible choices for  $t3$ .

**EndProof**

A valid schedule

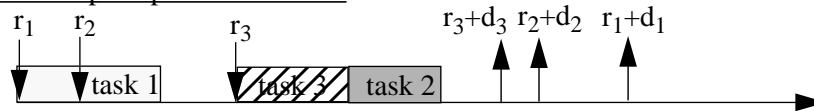


A valid prompt schedule



Note that there are several possible valid prompt schedules (e.g. task 2 can start at the end of the execution of task 1)

A valid prompt EDF schedule



**Figure 4:** obtaining a valid prompt EDF schedule using lemma 4.a and lemma 4.b

**Lemma 4.b:** Let us now suppose that we know a valid prompt schedule (see lemma 4.a), then it is possible to derive another valid prompt schedule satisfying the following property: during any EDF-period, the executed tasks are scheduled according to NINP-EDF. An EDF-period is delimited by two successive release times (notice that, due to the non-preemptive behaviour, if a release time occurs during an execution, the EDF-period is postponed until the end of this execution).

We call such a valid prompt schedule a valid prompt EDF schedule (see figure 4).

**Proof:** Following lemma 4.a, the valid prompt schedule is composed of several EDF-periods. As there are no idling possibilities during an EDF-period then each EDF-period can be individually reranked according to the non-preemptive EDF scheduling policy (see theorem 3.a).

**EndProof**

**Lemma 4.c:** If a valid schedule exists then we can derive at least one valid prompt EDF schedule.

**Proof:** It is straightforward from lemma 4.a and lemma 4.b (see figure 4).

**EndProof**

We are now ready to give a necessary and sufficient condition concerning the feasibility of a concrete task set in idling non-preemptive context.

**Theorem 4.a:** The following recursive algorithm  $f(t)$  finds out whether a valid schedule exists.

-----  
 Initialization :  $f(\text{first release})$

$f(t)$

**A:** put the new released tasks into the ordered (according to EDF) pending queue

**IF** there is at least one new pending task

set the current pending task as the first of the ordered (according to EDF) pending queue

**ENDIF**

**B:** IF the **current pending task** is valid (according to EDF)

$t < t + \text{duration of this task}$  /\* case a \*/

IF there are no more task

success /\* valid schedule \*/

ELSE

IF the ordered pending queue is not empty

$f(t)$

ELSE

$f(\text{next release})$

ENDIF

ENDIF

ELSE

check /\* not valid schedule \*/

ENDIF

/\* this part is implied by the idling mode \*/

reset the ordered pending queue and the current pending task as it was before B:

**IF** the current pending task is valid (according to EDF) /\* otherwise no idling can be successful \*/

IF the ordered pending queue is not empty /\* otherwise go to the next release \*/

set the current pending task as the next task of the ordered pending queue /\* case b)

$f(t)$

ELSE

$f(\text{next release})$  /\* case c \*/

ENDIF

ENDIF

reset the ordered pending queue and the current pending task as it was before A:

-----

**Proof:** By its very nature, the previous algorithm searches all the valid prompt EDF schedules (i.e. all the valid schedules satisfying lemma 4.c). Indeed if a valid schedule exists then at least one valid prompt EDF schedule satisfying lemma 4.c also exists and therefore the previous algorithm finds it.

**End Proof**

### 4.2.2 Comments and example

This recursive algorithm  $f(t)$  is an extension, in idling context, of the algorithm  $g(t)$  presented in section 3.2.1.1 (the updated parts are in bold). In order to search all the idling possibilities satisfying lemma 4.c the algorithm maintains (see figure 5),

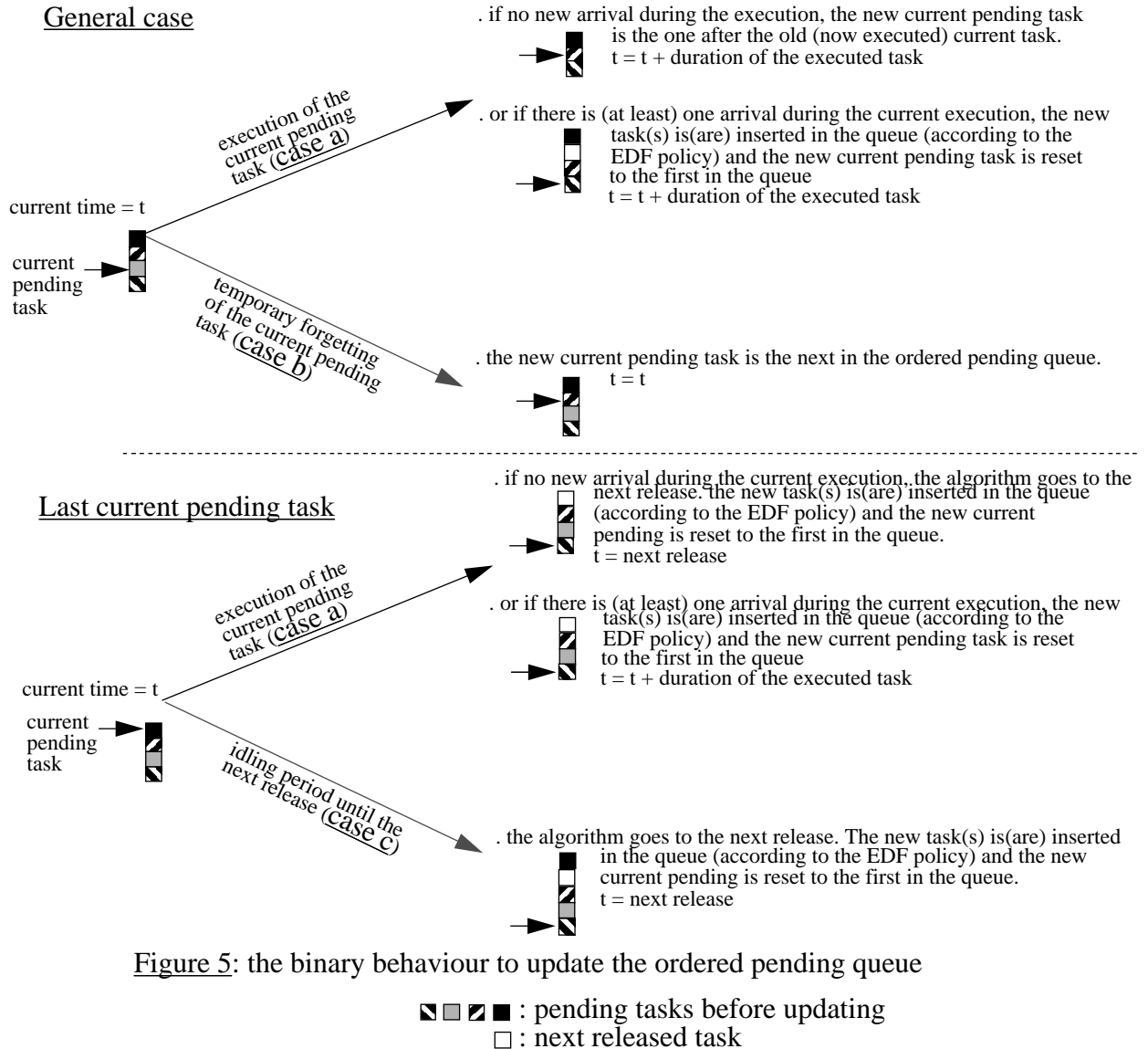


Figure 5: the binary behaviour to update the ordered pending queue

at each call, an ordered queue (according to EDF) of the pending tasks and a pointer of the current pending task (which will be the next possible task executed in the current EDF-period). Then the ordered pending queue is updated by one of the three following cases:

- the current pending task is executed (case a). The ordered pending queue is then updated by removing the executed task and by taking account of possible new release tasks during the execution
- the current pending task is temporarily forgotten (until the next EDF-period). There are two possibilities:
  - . the new current pending task is the next one in the ordered pending queue (case b).
  - . there are no more tasks in the ordered pending queue. This leads to an idling period until the next EDF-period (case c).

Of course, in order to allow the search of all the possible valid prompt EDF, at each new release time (which leads to a new EDF-period) the current pending task is reset to the first task of the ordered pending queue.

Therefore the whole algorithm produces a binary tree where every leaf may provide (if there is no deadline failure) a valid prompt EDF schedule:

Let us give now a short example (see figure 6) for three tasks. At run time, we find:

- four valid prompt EDF schedules
- and one non-valid schedule

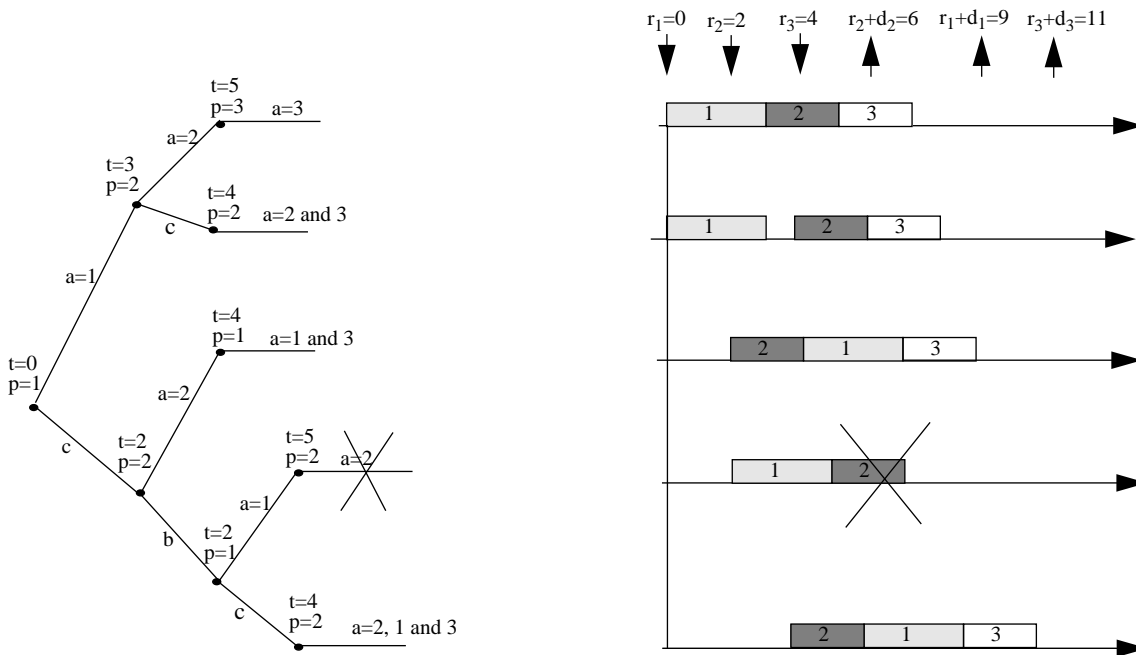


Figure 6: execution of 3 tasks ( $a() = \{a(1)=(0,3,9), a(2)=(2,2,4), a(3)=(4,2,7)\}$ ).

- above each node,  $t$  is the current time and  $p$  is the current pending task.
- between two nodes, the case considered (a,b,c see figure 5). The case a is followed by the number of executed tasks.



### 4.3 Performances

Although the potential number of solutions for the previous algorithm  $f(t)$  (see section 4.3.1) is  $n!$ , we will see in 4.3.2 that this number is generally much less.

#### 4.3.1 Complexity

In order to compute the complexity of this tree algorithm, we will establish the theoretical number of solutions generated by  $n$  tasks. Intuitively this number is maximized when the idling behavior is not limited (fully developed tree) i.e. when it is always possible to compute the pending tasks between two release-times.

More formally let  $\overset{f-s+1}{C_f} \underset{(s,d+1)}{\circ}$  be a node  $(s,d+1)$  generated from a node  $(f,d)$  where:

- $d+1$  is the current depth (i.e. the release time of the node  $(s,d+1)$ ).
- $f$  is the number of pending tasks at depth  $d$  (the previous release time).
- $s$  is the number of pending tasks ( $1 \leq s \leq f + 1$ ) at depth  $d+1$  (the current release time).

It follows that:

- a node  $(f,d)$  can generate  $f+1$  nodes  $(s,d+1)$  where  $f-s+1$  is the number of executed tasks from depth  $d$  to depth  $d+1$  (i.e. from the previous release time to the current release time).
- a node  $(s,d+1)$  is weighted by  $C_f^{f-s+1}$  the number of non-ordered sequences of  $f-s+1$  executed tasks among the  $f$  pending task. Indeed, due to the EDF policy during an EDF-period (see lemma 4.b, section 4.2), only one order is available for each sequence of executed tasks.

Let us see an example of the exhaustive graph (see figure 7) built with 3 tasks. Note that, as we search the number of possible leaves and not the way to obtain them (case a, b and c of the algorithm), we directly use the node  $(f,d)$  formalism which is more concise (then every node is weighted by the number of possible ordered

sequences i.e. by  $C_f^{f-s+1}$ ).

For 3 tasks, there are  $3! = 6$  potential valid prompt EDF schedules that may be compared with the real example (see figure 6) which leads only to four valid prompt

schedules and one non-valid schedule (see the discussion in section 4.3.2).

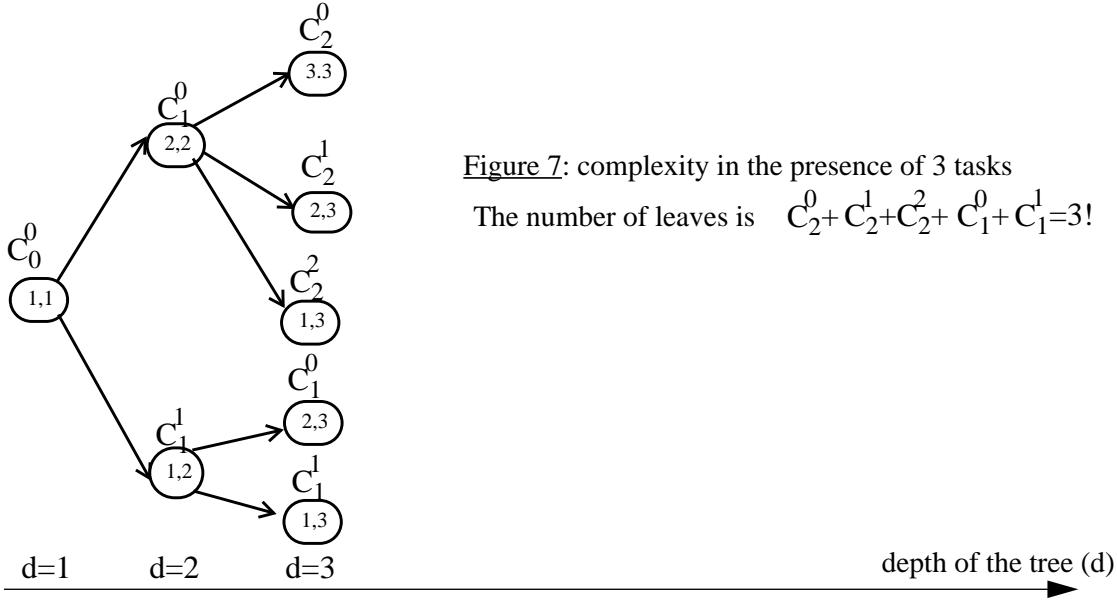


Figure 7: complexity in the presence of 3 tasks  
The number of leaves is  $C_2^0 + C_2^1 + C_2^2 + C_1^0 + C_1^1 = 3!$

More generally, we shall now prove that the number of leaves generated by an original node(1,1) on depth n is n!. For that let us first consider the following theorem proved by induction

**theorem 4.b:** The number of leaves generated by a node(f,d) at depth  $d' \geq d$  is:

$$P(f,r) = r!(r)^{f-1} \quad (\text{with } r=d'-d+1).$$

**proof:** Clearly, the original node(1,1) generates at depth 1:  $P(1,1) = 1$  (i.e.  $1!(1)^{1-1}$ ) leaf.

Assume now that the theorem is valid  $\forall f \in \mathbb{N}$  at depth  $d'$ , then  $P(f,r) = r!(r)^{f-1}$ .

As a node(f,d) generates at depth  $d+1$ ,  $f+1$  nodes(s,d+1) weighted by:  $C_f^{f-s+1}$

with  $1 \leq s \leq f+1$ . As each node(s,d+1) can generate,  $P(s,r) = r!(r)^{s-1}$  leaves.

Then a node(f,d) can generate at depth  $d'+1$  (relative depth  $r+1$ ):

$$P(f,r+1) = \sum_{s=1}^{f+1} C_f^{f-s+1} r!(r)^{s-1} = r! \sum_{s=0}^f C_f^{f-s} (r)^s = r! \sum_{s=0}^f C_f^s (r)^{f-s}$$

And then from binomial theorem,

$$P(f,r+1) = r!(r+1)^f = (r+1)!(r+1)^{f-1}.$$

It follows that the assumption is still valid at the relative depth  $r+1$ .

**End Proof**

As a result the number of leaves generated by a node(1,1) at depth n is  
 $P(1,n) = n!(n)^0 = n!$ .

### 4.3.2 Discussion

Of course a theoretical number of solutions in  $n!$  is not satisfactory yet the algorithm  $f(t)$  can be considered for the following reasons:

- due to the lemmas 4.a and 4.b (see section 4.2) the search is not exhaustive. More precisely the idling behavior is strongly limited (only the potential valid prompt EDF schedules are considered) without the risk of missing a solution.
- at run-time a tree search is limited by the tasks parameters leading to:
  - . deadline failures which stop the current tree search path. It follows (see 4.3.1) that if the search stops on a node(f,d),  $P(f,r)=(r)!(r)^{f-1}$  potential schedules are not explored (with  $r=n-d+1$ ).
  - . gathering release time disables idling development. Indeed, suppose a node(f,d). This node will generate from theorem 4.b  $P(f,r)=(r)!(r)^{f-1}$  leaves (with  $r=n-d+1$ ). Suppose now that the node(f,d) at the next release time sees not only one task but k tasks. Then it comes that the number of leaves generated by the node(f,d) is:

$$\sum_{s=0}^f C_f^s P(f+k-1-s, r-k) = (r-k)!(r-k)^{k-1} \sum_{s=0}^f C_f^s (r-k)^{f-s} = (r-k)!(r-k)^{k-1} (r-k+1)^f$$

Which is inferior to  $(r)!(r)^{f-1}$  for  $k>1$  (and equal for  $k=1$ ). Indeed, for  $k>1$

$$\frac{(r-k)!(r-k)^{k-1}(r-k+1)^f}{r!(r)^{f-1}} < \frac{(r-k)^{k-1}(r-k+1)^f}{(r-k+1)(r-k+2)\dots(r)(r)^{f-1}} < \frac{(r-k)^{k-1}(r-k+1)^f}{(r-k-1)^k(r)^{f-1}} < \frac{(r-k+1)^{f-1}}{(r)^{f-1}} < 1$$

Then the gathering of release time limits the tree search path.

In order to evaluate the performances of the algorithm  $f(t)$  at run time (for any concrete aperiodic task set  $a()=\{a(1),\dots,a(n)\}$  where each task  $a(i)= (r_i, e_i, d_i)$  is as defined in section 2 and where the tasks are sorted in non-decreasing order by release time) let us define the following parameters:

- $s_{a()}$  which is the number of valid prompt schedules of  $a()$  found at run time.
- $c_{a()}$  which is the cost i.e. the number of valid and non-valid EDF prompt schedules of  $a()$  found at run time (remember that a non-valid EDF prompt schedule is necessarily incomplete due to the deadline failure).
- the cost ratio  $cr_{a()} = c_{a()}/n!$ .
- the efficiency ratio  $er_{a()} = s_{a()}/c_{a()}$ .

Let's start the performance analysis by describing two sufficient conditions leading to good performances:

- If  $\forall i, 1 \leq i \leq n-1, r_{i+1} + e_i \geq r_i + d_i$  and  $r_{i+1} \geq r_i + e_i$  then  $a()$  leads to

only one valid schedule.

Indeed this situation means that any idling search lead to invalid schedules, only the pure prompt and non-idling EDF policy leads to a valid schedule (see figure 8). Therefore the tree is reduced to a unique path from the root to an unique leaf (we call this situation an efficient limited tree). This unique path is exactly the one studied in section 3.2.1.1.

- If  $\forall i, 1 \leq i \leq n$  ,  $d_i = r_n + \sum_{k=1}^i e_k$  then every potential prompt EDF schedule is valid.

Indeed this situation means that, due to the late deadline, all the tree search paths lead to valid schedules. Therefore the tree is fully developed (we call this situation an efficient large tree)..

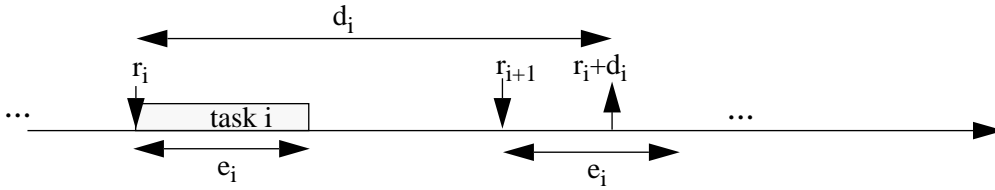


Figure 8: efficient limited tree

The efficient limited tree situation leads to one solution with a very limited search ( $s_{a()}=1$ ,  $c_{a()}=n$  and then  $cr_{a()}=n/n!$ ,  $er_{a()}=1/n$ ) when the efficient large tree leads to a search of exponential complexity but also to many valid schedules ( $s_{a()}=n!$ ,  $c_{a()}=n!$  and then  $cr_{a()}=1$ ,  $er_{a()}=1$ ).

Of course these two sufficient conditions seem strongly restrictive however it is possible to describe a lot of efficient scenarios like those.

The real conjecture is to know if there exist situations leading to an inefficient large tree i.e. search of exponential complexity leading to few valid schedules (in worst case  $s_{a()} = 0$ ,  $c_{a()} = n!$  and then  $cr_{a()} = 1$ ,  $er_{cost} = 0$ ). In order to examine this possibility, let us now give some simulations (see table 1) to estimate this efficiency in a real context. We make use of random task sets and give, for each task number, an average on ten runs. These results show that:

- the limitation of the tree search at run time (due to deadline failures, regrouping of release time...) is strongly marked when the number of task increases (the cost ratio  $cr_{a()}$  decreases). This limits the computation time.
- the efficiency ratio  $er_{a()}$  is non-negligible (around 0.5) and is not impacted by the task number.

This lead (when if it is sufficient to obtain only one valid schedule for a given task set) to propose the following heuristic: search at random some path in the tree until finding one valid schedule. Technical results concerning the efficiency of the

algorithm  $f(t)$  will be developed in further papers.

**Table 1: Simulation results**

Task number	$n!$	success $s_a()$	cost $c_a()$	cost ratio $cr_a()$	efficiency ratio $er_a()$
3	6	2	4	0.67..	0.5
6	720	88	174	0.24..	0.51..
8	40320	1892	3668	0.09..	0.52..
10	$3.63.. 10^6$	10658	30046	0.008..	0.35..
12	$479.. 10^6$	670240	2421600	0.005..	0.28..

## **5. Conclusion**

EDF is extensively studied in this paper for non-preemptive scheduling.

In the case of non-idling scheduling, EDF is shown to be optimal. Feasibility conditions are established for many patterns of arrival laws (aperiodic/periodic, concrete/non-concrete, synchronous/asynchronous...). This paper shows that preemptive and non-preemptive scheduling are closely related.

The case of idling scheduling opens the door to more complex problems. Indeed, any valid non-idling schedule is also valid in idling scheduling but the reverse is not true. Although the theoretical number of valid schedule is  $n!$ , we have shown that non-idling EDF could be applied during specific intervals, called EDF-periods. This property allows to propose an algorithm which efficiently finds valid schedules.

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Éditeur

Inria, Domaine de Voluceau, Rocquencourt, BP 105 LE CHESNAY Cedex (France)

ISSN 0249-6399