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Towards Realistic Soft Tissue Modeling in Medical Simulation

Hervé Delingette

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————— THÈME 3 —————



*Rapport
de recherche*

Towards Realistic Soft Tissue Modeling in Medical Simulation

Hervé Delingette^{*}

Thème 3 — Interaction homme-machine,
images, données, connaissances
Projet Epidaure^{**}

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Abstract: Most of today's medical simulation systems are based on geometric representations of anatomical structures that take no account of their physical nature. Representing physical phenomena and, more specifically the realistic modeling of soft tissue will not only improve current medical simulation systems but will considerably enlarge the set of applications and the credibility of medical simulation, from neurosurgery planning to laparoscopic surgery simulation.

In order to achieve realistic tissue deformation, it is necessary to combine deformation accuracy with computer efficiency. On the one hand, biomechanics has studied complex mathematical models and produced a large amount of experimental data for representing the deformation of soft tissue. On the other hand, computer graphics has proposed many algorithms for the real-time computation of deformable bodies, often at the cost of ignoring the physics principles.

In this paper, we survey existing models of deformation in medical simulation and we analyze the impediments to combining computer-graphics representations with biomechanical models. In particular, the different geometric representations of deformable tissue are compared in relation to the tasks of real-time deformation, tissue cutting and force-feedback interaction. Finally, we inspect the potential of medical simulation under the development of this key technology.

Key-words: surgery simulation, deformable models, finite element method, medical imaging

(Résumé : tsvp)

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Modélisation de tissus mous pour la simulation de chirurgie

Résumé : La plupart des simulateurs médicaux actuels sont fondés sur des modèles géométriques de structures anatomiques, sans tenir compte de leur réalité physique. La représentation des phénomènes physiques et, plus spécifiquement, la modélisation réaliste des tissus mous, n'améliorera pas uniquement la qualité des simulateurs médicaux, mais permettra d'élargir considérablement le champ d'application de ces simulateurs, depuis la planification de la neurochirurgie vers la simulation de la chirurgie minimalement invasive.

Afin de simuler de manière réaliste la déformation des tissus mous, il est nécessaire de modéliser avec précision ces déformations mais aussi de les calculer efficacement. Pour le premier aspect, la biomécanique est le domaine scientifique qui étudie des modèles mathématiques complexes de comportement des tissus vivants. Il existe une quantité importante de données expérimentales concernant le comportement de quelques structures anatomiques. Pour le second aspect, la synthèse d'images a proposé plusieurs algorithmes pour le calcul temps-réel de corps déformables, souvent en respectant de manière approchée les lois de la physique. Une confrontation entre ces deux domaines est donc nécessaire.

Dans cet article, nous passons en revue les différentes modélisations de tissus mous utilisées en simulation médicale et nous analysons les difficultés pour combiner les modèles issus de la biomécanique avec les algorithmes issus de la synthèse d'images. Plus précisément, différentes représentations de tissus mous sont comparées en fonction de leur performance pour le calcul temps-réel de leur déformation, la simulation de découpes et la simulation du retour d'effort.

Mots-clé : simulation de chirurgie, modèles déformables, éléments finis, imagerie médicale

1 Introduction

During the past five years, there has been growing interest in the medical and computer science field, around the simulation of medical procedures. Under the terminology proposed by Satava [Sat96], the first generation of medical simulators applied the concept of navigation and immersion to tridimensional anatomical datasets. Those techniques, borrowed from *Virtual Reality*, only considered the geometrical nature of the human body. Despite their limited user interaction, those simulators found many interesting applications in the field of education or training.

The second generation of simulators aims at modeling the physical interaction of each anatomical structure. For osseous structures, those simulators model the coupling between kinematic constraints and muscle deformation [DL95]. For soft tissue, it is necessary to model their deformability under the influence of neighboring structures or surgical instruments.

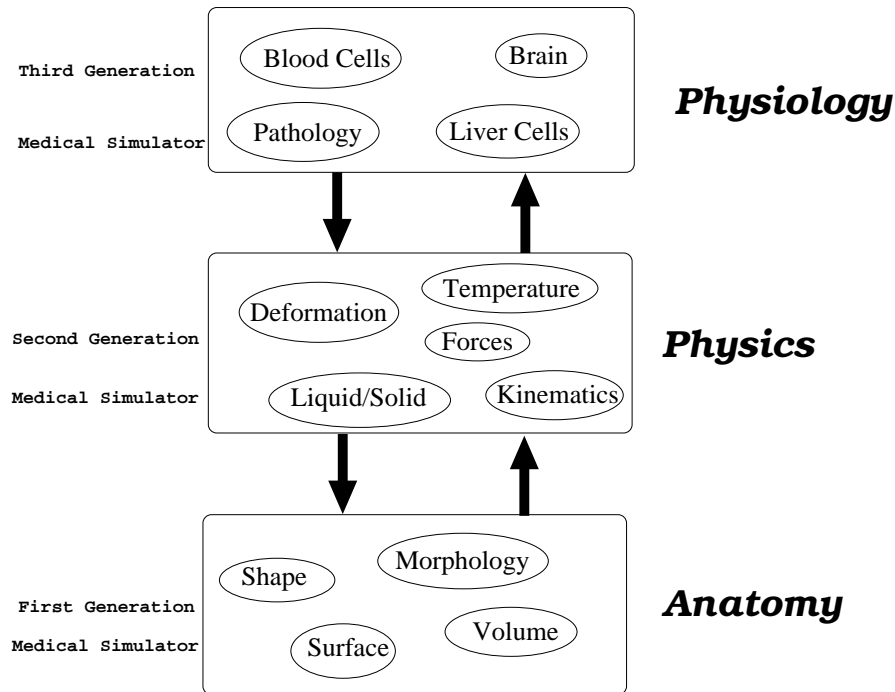


Figure 1: The different generations of medical simulators.

The third generation of simulators takes into account the functional nature of human organs. Figure 1 shows how the different levels of simulation (anatomy, physics or physiology) interact with each other [DKMC95]. For instance, cutting a vessel (physical phenomenon) has an influence on the blood pressure and therefore the function of other organs. On the contrary, the development of tumorous lesions (physiological phenomenon) modifies locally the tissue mechanical properties.

To achieve such advanced simulations, it is essential to model the phenomena occurring at the geometrical, physical and physiological levels. If the 3D recovery of anatomical structures from medical images is a relatively mature technology, a lot of research efforts is still need for the physical modeling of human tissue. In particular, *soft tissue modeling*, *i.e.* the modeling of soft tissue mechanics and deformation, has been identified as a key technology for the development of the second and third generations of medical simulators [Sec96]. Since the human body is

mainly made up of soft tissue, the medical consequence of soft tissue modeling is very important, ranging from neurosurgery, plastic surgery, musculoskeletal surgery, heart surgery, abdominal surgery, minimally-invasive surgery . . .

For a given surgical simulation, soft tissue *deformation accuracy* and *computation time* are the two main constraints for the modeling of soft tissue. We have summarized the different types of applications according to those two criteria [Sec96] in figure 2. *Scientific analysis* aims at validating physical hypothesis of soft tissue for the design of new procedures or implants. In such cases, the accuracy of deformation is far more important than the computation time. On the other hand, *surgery planning* for predicting the outcome of surgery or rehearsing complex operations, requires less computation time (from 30s to one hour) since several trials may be necessary. Furthermore, it is essential to evaluate of the accuracy of the method. Finally, for *surgery procedure training* computation time of the order of 0.1s are required to achieve smooth user interaction whereas accuracy of deformation is not of primary importance.

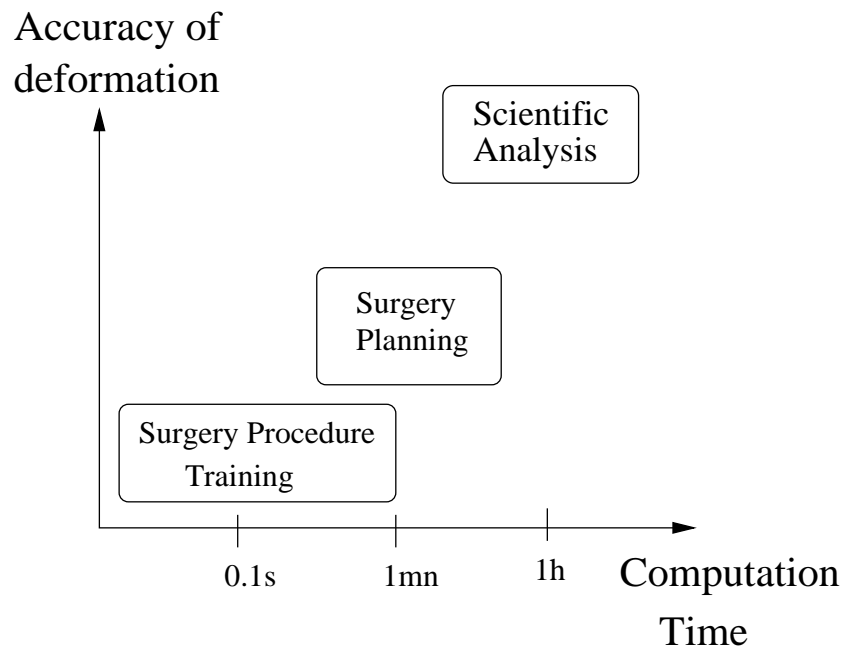


Figure 2: The time/accuracy requirements of soft tissue modeling.

In this paper, we first study the different criteria for implementing a surgical simulator involving soft tissue deformations. We then evaluate the existing models of soft tissue and analyze the obstacles to achieving realistic soft tissue models.

2 Constraints for Soft Tissue Modeling

We present the main technical difficulties for designing a realistic medical simulator. The technologies enabling an operational surgical simulator need to solve to some extent all of those problems. The difficulties associated with each constraint are not necessarily of the same order of magnitude, and we discuss in section 4 the possible compromises.

2.1 Biomechanical Model

The obvious constraint for soft tissue modeling is that it correctly represents the deformability of real tissue. More precisely, we need to control the accuracy of deformation and therefore compare the computed soft tissue model with the actually deformed tissue. The level of accuracy typically depends on the application. For surgery training for instance, realistic visual and haptic display is more important than the accuracy of deformation. However, if the difference of behavior is too great under large deformations, it could result in learning inappropriate procedures.

In all cases, it is vital to have quantitative knowledge of the biomechanical behavior of soft tissue. We distinguish between knowledge of soft tissue deformable properties and knowledge of interaction with surrounding tissues.

The study of soft tissue deformability belongs to the field of biomechanics. There exists a large bibliography on the study of soft tissue deformation [Fun93]. Those studies range from the determination of qualitative behavior of tissues to the recovery of quantitative parameters governing their deformation. The tissues that have been mainly studied include skin [Lar86], vessels [Fun93], muscles, brain [CM96] and heart [HS88].

Many mathematical models of soft tissue deformations have been proposed [Fun93], and an extensive description would fall outside the scope of this paper. However, we briefly present the simplified models that are commonly encountered for surgery simulation purposes.

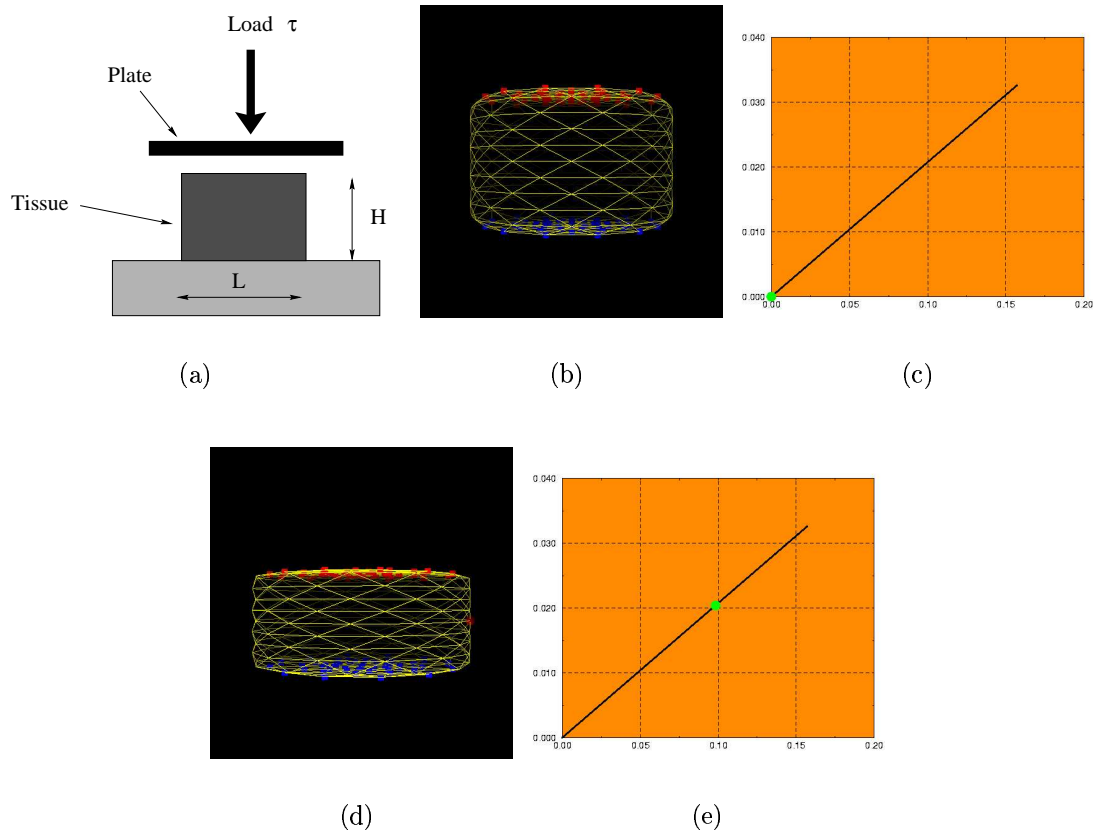


Figure 3: Behavior of linear elastic materials.

The simplest model of static reversible elastic deformation corresponds to the *linear elastic* model. According to this model, a cylindrical tissue sample of height H and diameter L under a load τ (see figure 3) follows Hooke's law :

$$\epsilon_1 = \frac{\Delta H}{H} = \frac{1}{E}\tau \quad \epsilon_2 = \frac{\Delta L}{L} = \frac{\sigma}{E}\tau \quad (1)$$

where :

- ϵ_1 and ϵ_2 represent the longitudinal and transverse elongation of the tissue under the load τ .
- E and σ are the Young modulus and the Poisson coefficient that characterize the nature of the material.

Therefore a linear elastic material has a linear *stress/strain* relationship and the trajectories of physical points under a load of varying intensity are straight lines. Because of this linear relationship, linear elastic materials have been used for fast surgery simulation [BN95, CDA96, PRZ92, KE96].

For most materials, the linear elastic model is only valid for small displacements. For large displacements, more complex non-linear models have been introduced such as the Mooney-Rivlin model [BN96, SBMH94] or the St. Venant Kirchoff [TF88b, BN95, Kai96] where the stress/strain or stress/displacement relationships are no longer linear. Additional physical constraints may be considered, such as incompressibility.

Furthermore, for many tissues, plastic deformations, where the strain does not reverse to zero after unloading (see figure 4 (a)), occur when the material reaches its elastic limit. Similarly, the material may have a non-reversible elastic behavior (see figure 4 (b)).

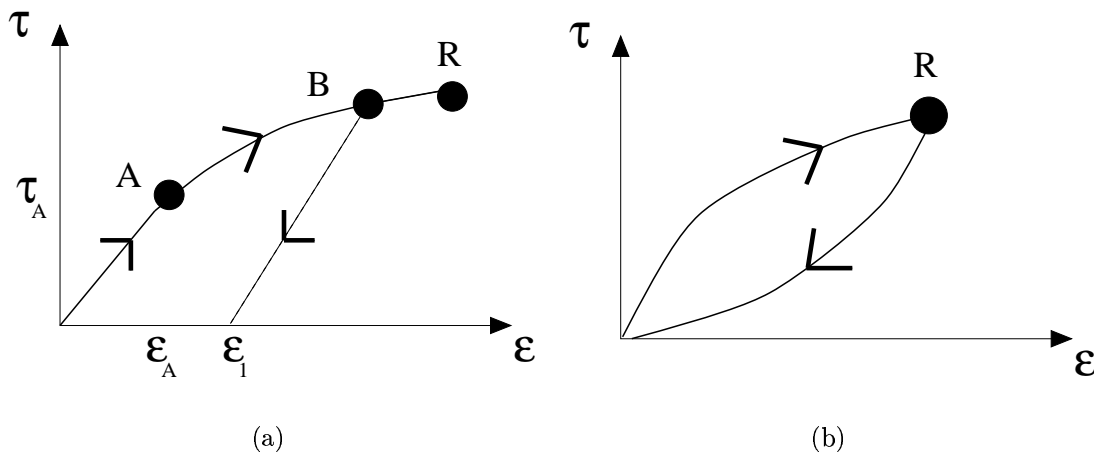


Figure 4: Stress/strain relationships for plastic material (a) and hysteresis elastic material (b).

The stress/strain relationship characterizes the static behaviors of a material. However, in general, the stress τ is also related to the speed of deformation $\dot{\epsilon}$ (the *strain rate*) and therefore tissue material can be considered *viscous*. This implies that the deformation of a viscous material depends on the history of the applied forces and not only on their instantaneous values.

The simplest viscous material is the *Newtonian* viscous fluid, where the stress is proportional to the strain rate (Equation of Newton) :

$$\tau = \eta \dot{\epsilon} \quad (2)$$

Most soft tissues are *viscoelastic*, combining elastic and viscous behaviors. For instance, the Maxwell-Voigt viscoelastic model associates linear elasticity with constant viscosity. Those materials are characterized by their *creep* and *relaxation* functions occurring when the material is subjected to a constant load or deformation (see figure 5). Finally, many anatomical tissues can be considered as biphasic, where the tissue is a mixture of a solid porous matrix and an incompressible fluid.

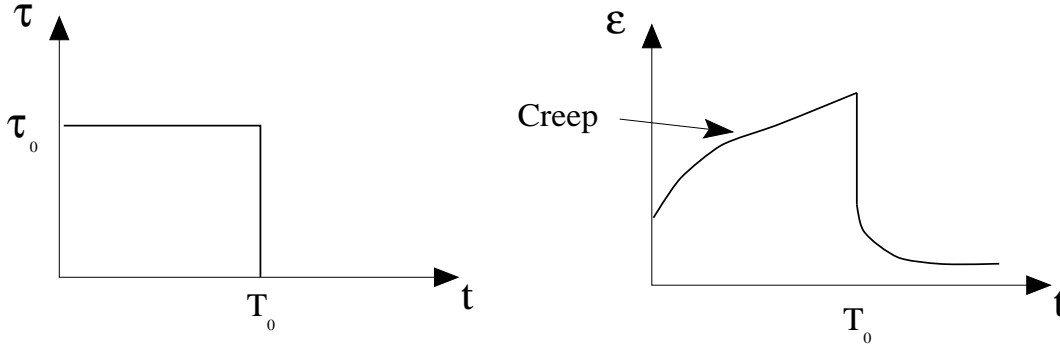


Figure 5: Deformation response of a linear viscoelastic material under loading and unloading

2.2 Interaction with rigid or soft bodies

The interaction between a soft tissue and surrounding bodies (surgical instruments, bones, soft tissue,..) can be decomposed into two different tasks : collision detection and computation of interaction forces (or displacements). Many algorithms have been proposed for detecting the collision between a moving point object and a static mesh. They are sufficiently efficient to sustain a 30Hz refresh rate with meshes of reasonable size (between 10 000 to 40 000 polygons).

However, in surgical simulation, those algorithms are not applicable because soft tissue models cannot be considered as static since several surgical instrument interact at the same time. Furthermore, for many surgical instruments, the interaction with tissue not only occurs at their extremity, but also along their length. Therefore, it is necessary to consider the surgical instruments as rigid moving meshes and soft tissue as deformable moving meshes. When detecting the collision between two moving objects, pre-computation is not possible and the computational time rises sharply.

When considering the collisions between two deformable meshes or the self-collisions of a deformable mesh, the computational complexity becomes prohibitive with current algorithms for real-time processing. However, alternative soft tissue representations such as implicit surface modeling [CGD97], offers possible research directions to overcome those major limitations.

Once a collision has been detected, the second task consists of computing the resulting interaction forces. Very little is known about the real interaction between tissues. The main difficulty is that, unlike material characterization, the experiments on tissue interaction must be performed *in vivo*.

2.3 Real Time Deformation

The real-time deformation of soft tissue is an important constraint for medical virtual reality systems. It has been clearly established that the immersion of the operator, therefore its ability to learn from a computer simulated system, is directly linked to the bandwidth of the simulator. An acceptable bandwidth for visual display is in the range of 20-60Hz while the acceptable bandwidth for haptic display is on the range if 300-1000Hz (300Hz is the free hand gesture frequency). Two numbers are particularly important for accurate perception by the user : *latency* and *computation time* . Latency measures the time between sensor acquisition (for instance the position of the surgical instrument) and action (visual or haptic display). The computation time is that needed to update the geometric model. On multi-processor computers, those two numbers are not necessarily correlated.

Latency is critical for the user immersion. The hardware configuration of the system can greatly influence latency since communication between elements may be responsible for additional delays. In figure 6, we show the architecture of the simulation system used at INRIA [CDA96]. It is composed of one haptic display, a PC and a graphics workstation. There are several contributing causes to latency : communication between the haptic display and the PC, communication between the PC and the graphics workstation, the time taken by the graphics display, the computation time for collision detection, force feedback and deformation. Since much of the communication between elements are asynchronous, the total latency is not the sum of those delays but it is important to reduce them to their minimum values. The latency depends greatly on hardware, specifically on computation and graphics performance.

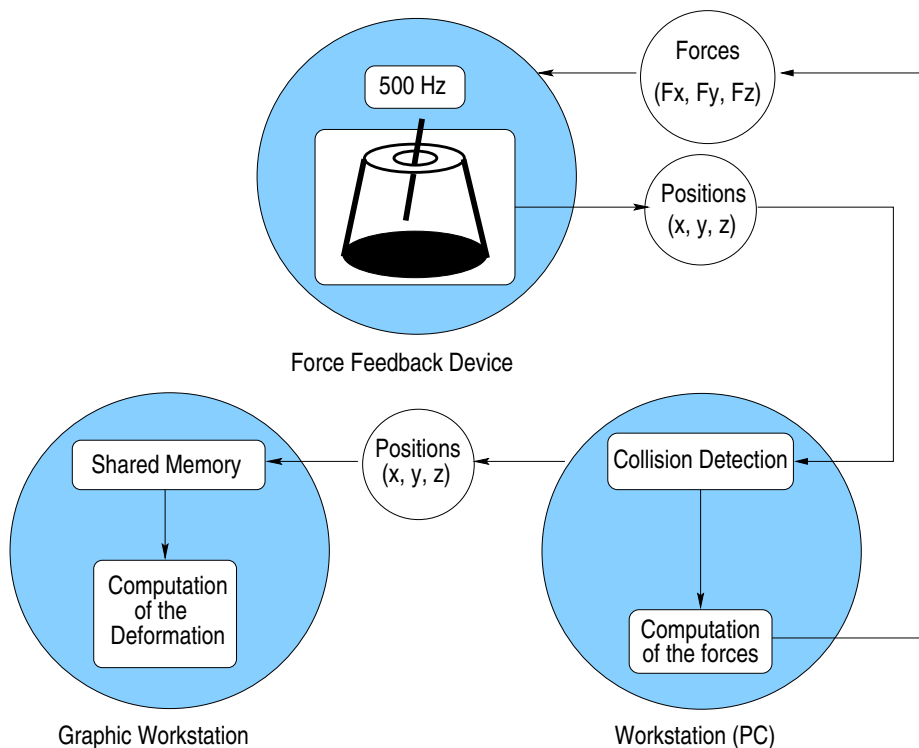


Figure 6: Example of simulation systems.

The computation time depends on the choice of the geometric and physical model of soft tissue (see section 3). If we write X_t as the position of the tissue model at iteration t , we will

write C_t the computation time needed to compute the new position X_{t+1} . The computation time must be bounded ($C_t < C^*$) in order to guarantee minimal bandwidth.

We distinguish between a *static* equilibrium equation and a *dynamic* law of motion depending whether X_{t+1} depends on the previous position X_t . When using static equations of the form $F(X) = 0$, the computed shape corresponds to the state of equilibrium, so takes no account of inertia or viscoelasticity.

With dynamic laws of motion, the current position of the tissue has an influence on its future position. Mostly, this can be modeled using a Newtonian law of motion having the corresponding differential equation :

$$m \frac{\partial^2 X}{\partial t^2} = -\gamma \frac{\partial X}{\partial t} + F(X) \quad (3)$$

Depending on the complexity of the deformation model, the computation of X_{t+1} may be performed interactively. The number of iterations must be bounded since the computation time C_t must be less than C^* . A major difficulty consists of ensuring *synchronicity*, *i.e.* that the *numerical time* used for computation matches the *user time* used for interaction. For instance to correctly estimate the speed of a vertex x_t , one should divide the position difference $x_t - x_{t-1}$ by the real time spent between two iterations.

Static equations have two advantages. First, they are faster to compute because no time integration is needed. Second, they are well suited for parallel algorithms or for asynchronous computation. For instance, in Cotin *et al.* [CDA96], we use a static formulation motion that enable us to decouple the force c with the deformation computation. This is highly advantageous because, as noted before, the force computation must run at least at 300Hz but requires few operations (product of a matrix with a vector) whereas the deformation computation must run at 30Hz with a large amount of computing.

However, static laws are not able to model realistic deformations such as inertia or viscoelasticity. Dynamic laws of motion on the other hand, can model more accurate deformations but are more difficult to handle and computationally more expensive.

2.4 Tissue Cutting and Suturing

The ability to cut and suture tissue is of primary importance for designing a surgery simulation system. The impact of those operations in terms of tissue modeling is considerable. In fact, they imply that the geometric representation of tissue must change its topology over time. The cost of such a topological change depends largely on the chosen representation (see section 3).

In addition, the behavioral model of the tissue must be adapted at parts where cutting or suturing occurs. Little is known about the stress/strain relationship occurring during and after cutting. The basic assumption is that the physical properties of tissue are only modified locally. However, in practise, cutting can greatly modify the boundary conditions between tissue and the surrounding organs which entails considerable change in terms of deformability.

Finally, when cutting volumetric or surface models, it is very likely that the new geometric and physical representation of tissue leads to self-intersections. The detection of self-intersections is computationally extremely expensive, therefore repulsive force between neighboring vertices are sometimes added to prevent self-intersections.

2.5 Force-Feedback Computation

Haptic display serves at least two purposes in a surgical simulator : kinesthetic and cognitive. First, it provides the sensation of movement to the user and therefore it greatly enhances its surgical performance. Second, it is used to distinguish between tissues by testing their mechanical properties.

However, the addition of a haptic display in a simulation system increases by a large factor its complexity and the required computational power [MRFV96] : an increase by a factor 10 of the required bandwidth, synchronisation between visual and haptic displays, force computation, . . . Few papers have assessed the importance of haptic feedback in surgery [Mar96]. In general, it is accepted that the combination of visual and haptic displays are optimal for surgery training or pre-planning.

In video-surgery, the surgical instruments slide inside a trocar and are constrained to go through a fixed point. This entails substantial friction, specifically in laparoscopy where airtightness must be enforced. The friction of the instruments inside trocars perturbs the sensing of forces by the end-user. Despite those perturbations, it appears that it is still necessary to provide force-feedback for realistic user immersion.

The force computation depends on the chosen deformation model. Deformation models based on biomechanics naturally lead to physically meaningful forces and are very likely to provide an intuitive feeling to the end-user. On the other hand, with more *ad hoc* deformation models, for instance spring models, the force computed at each iteration may not correspond to intuitive sensation, especially if there is no continuous representation of the tissue.

Another problem with the use of haptic displays is the small number of commercially available systems. They often correspond to modified versions of joysticks dedicated to different applications (teleoperation, video games, general user interface, . . .). Therefore, they rarely meet the specific constraints of open and video surgery, in terms of workspace or encoder resolution. Current commercial equipment read the position of the end-effector and are force-controlled.

2.6 Visualization

Visual feedback is the most powerful perception channel. The quality of the visual rendering greatly influences user immersion and therefore the effectiveness of the simulator. In the past few years, substantial technological advances have enabled sharp decreases in the price of efficient graphics boards. With the existence of standard programming environment (such as OpenGL, Direct3D, . . .) and standard platform configurations, this trend should grow over the next five years to deliver high performance at reasonable price. It will have the effect of increasing the number of polygons that can be drawn at each frame. By combining improved graphics performance with anti-aliased texture mapping with large texture memory, a major improvement of the realism can be foreseen.

However, there are many problems in visualization for surgery simulation that cannot be solved simply by improving the raw graphics processing. For instance, after cutting or suturing the tissue model, modification of the texture coordinates must occur to reflect the change of topology. The development of volumetric texture images could be used instead of two-dimensional texture images. When simulating video-surgery, the optical aberration of large angled endoscopes, as well as the intense light of the optical fiber, must be modeled for more realistic display. Finally, techniques for depicting bleeding, and the rendering of semi-translucent and filandrous structures, must be improved.

3 Deformable Tissue Models

In this section, we review the main existing models of soft tissue.

3.1 Surface or Volumetric Tissue Models

The geometric representation of deformable tissue may consist of surfaces or volumes. The choice between surface and volume based models is governed by two factors : computer efficiency and physical accuracy. In terms of computation, surface models are advantageous because they have less vertices than volumetric models for representing the same shapes.

However, most biomechanical models such as those presented in section 2.1, naturally call for volumetric representations rather than surface models. Surface models tend to give physically invalid deformations especially in regions that are thin. In figure 7, we show the deformation of a volumetric plate model. Similar experiments with a surface plate model would have entailed self-intersections. Furthermore, the behavior of volumetric models may take into account physical inhomogeneities, for instance due to the presence of lesions.

Finally, volumetric models are better-suited to the simulation of cutting or suturing operations. This is because those operations changes the geometrical and the physical nature of the model.

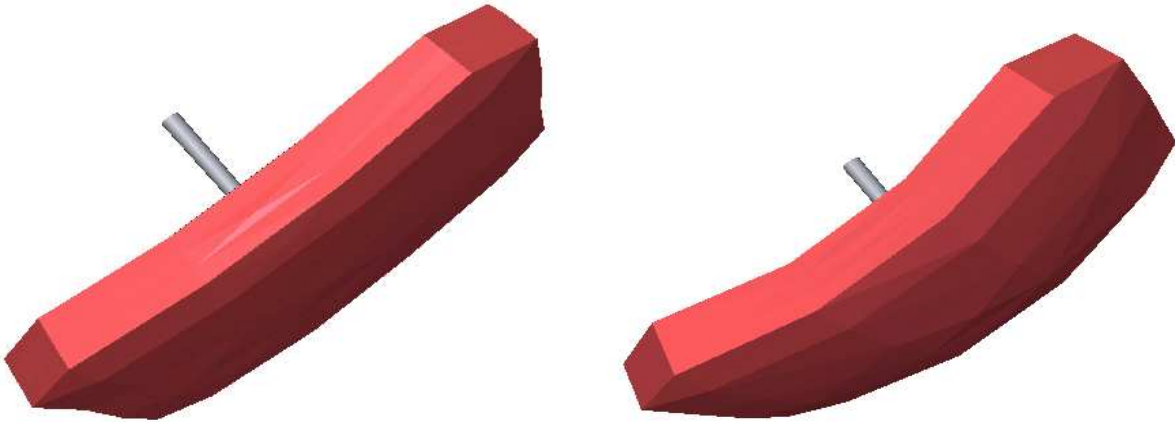


Figure 7: Volumetric deformable model behavior.

However, surface models may be relevant for modeling cavernous tissues such as vessels or the gallbladder. In this case, the physical model of deformation can incorporate a representation of a liquid or of a gaseous pressure combined with surface tensions.

3.2 Springs and Particles

Spring models consist of a set of points linked by springs and dampers. In the simplest formulation, the equation of motion of a point i is :

$$\mu \frac{d^2 \mathbf{r}_i}{dt^2} = -\gamma \frac{d\mathbf{r}_i}{dt} + \sum_{j \in N(i)} K_{i,j} \frac{(l_{i,j}^0 - \|\mathbf{r}_i - \mathbf{r}_j\|) \mathbf{r}_i \mathbf{r}_j}{\|\mathbf{r}_i - \mathbf{r}_j\|} \quad (4)$$

where μ is the mass, γ the damping factor and $K_{i,j}$ the stiffness of the spring connecting point i and points j in the neighborhood $N(i)$ of point i .

Spring models have been used extensively for simulating the elasticity of soft tissue. Waters [Wat92] has defined springs on regular lattices for modeling facial tissue. He derived the two stiffness parameters of biphasic springs from stress/strain curves described in a biomechanical study [KGEB75]. Similarly, Delingette *et al.* [DSCP94] represented fat tissue elasticity as a network of springs on a 3-simplex mesh. Keeve *et al.* [KE96] proposed a similar approach for modeling fat tissue in a craniofacial surgery simulation system but with the addition of a volume-preservation force intended to model the incompressibility of human tissue. Koch *et al.* [KGCvB96] combined a finite element model for representing the skin surface, with a spring model to represent the fat tissue. The stiffnesses of the springs are derived from the intensity of voxels in a CT-scan image. The underlying assumption is that stiffness is proportional to tissue density and therefore to the Hounsfield units.

The main advantage of spring models is their ease of implementation since they do not require continuous parameterization. They have been used for static as well as for dynamic computation. Another advantage is their ability to model cutting or suturing simply by removing or adding connections between vertices.

However, for the soft tissue simulation, they suffer from the following problems :

- **Topological design.** The topology of springs and masses is of great importance. Since a spring constrains the length between two vertices, the number of springs per vertex conditions the global behavior of the system. If the system is under-constrained, several rest positions are possible, and the system can fall into unwanted local minima. If the system is over-constrained, it tends to decrease the range of deformation. For tetrahedral meshes, the number of springs per vertex should be as close as possible to 6. Because of this difficulty in designing topologically a network of springs and masses, authors have usually organized the springs on regular lattices [Wat92] or on prisms with triangular base [KGCvB96, KE96]. Those choices however, impose the restriction of the geometric representation that it be organized in sets of parallel layers.
- **Validity of deformations.** The deformation induced by springs cannot easily be compared with those given by biomechanical studies because springs do not rely on continuum mechanics. However, for small deformations, a spring model behaves similar to a linear elastic finite element model, as verified by Keeve *et al.* [KE96]. By linearizing equation 4, the stiffness parameter can be identified with the stiffness of a linear elastic model. However, for large deformations, spring models do not behave like a linear elastic material and comparison with non-linear elastic finite element models is difficult. In order to identify spring parameters, given a behavior model, several algorithms have been proposed. In [JL95], Louchet uses a genetic algorithm for identifying the spring parameters to represent a cloth animation model. Deussen *et al.* [DKT95] bases the search for optimal parameters on simulated annealing.
- **Dynamic Behavior.** For dynamic spring models consisting of n nodes, for a given time step Δt and a given mass $\mu = m_{total}/n$, there is a critical stiffness K_c above which the numerical system is divergent. The relationship between K_c and the time step Δt is :

$$K_c \approx \frac{\mu}{\pi^2(\Delta t)^2} \approx \frac{m_{total}}{n\pi^2(\Delta t)^2} \quad (5)$$

This relation, which is also valid for explicit linear elastic models, implies that in order to increase the stiffness of the model, it is necessary to decrease the time step. Since the computation time C_t is independent of the time step Δt , to maintain the refresh rate as high as possible, it is necessary either to decrease the stiffness K or to decrease the number of nodes n . Stiffness controls the propagation of constraints along the tissue model. High stiffness models tend to exhibit global behavior which is desirable for most soft tissues. In practise, we have found that greater time steps could be used with explicit linear elastic finite element models than for spring models. This implies that the range of possible dynamic behaviors of spring models is more limited than those of finite element models.

- **Visualization** Spring models are composed of mass-points and edges represented by springs. To visualize the tissue surface, it is necessary to define polygons from the set of edges and vertices. However, when cutting or suturing occurs it is necessary to update the set of visible faces. This suggests that the network of springs and masses should be built upon a manifold (surface or volume). A complete data structure of the manifold must be used in a way similar to finite element models.

Several improvements to spring models have been proposed, specifically with regard to their dynamical behavior. Luciani [LJFC91] developed an animation system (CORDIS/ANIMA) based on particle systems for modeling complex physical phenomenon. The system is coupled with a force-feedback device for real-time interaction. Provot *et al.* [Pro95] defined super-elongated springs to increase the stiffness of its cloth model but with no guarantee of convergence. Cover *et al.* [CEO93] used springs to represent the surface of the gall bladder. They combined *home* forces with internal forces to enforce shape constraints. Similarly, Kuhn *et al.* [KKKN96], in the KISMET project, developed a surface model of the gall bladder based on springs, dampers and plastic elements where mass-points are connected to parent nodes. In that system, a NURBS representation is attached to points, giving a realistic rendering of the tissue. Stone *et al.* [RRF96] added a slip and a split threshold to a spring network, in order to model plasticity and fracture. Meseure *et al.* [MC97] proposed a surface representation of tissue similar to the hybrid model proposed by Terzopoulos [TF88a]. Springs of zero length are attached to a virtual rigid component. During deformation, both the positions of mass-points and the translation/rotation parameters of the rigid component are updated. This framework has the advantage of decoupling stiff behavior modeled by the rigid component and local deformation modeled by springs and dampers.

Gibson *et al.* [GSMF97] proposed a "ChainMail" model that does not derive from the equation of dynamics 3. Instead, the deformation model governs the displacement of all nodes, given a displacement on its boundary. Tissue is represented as a set of deformable voxels linked to their 6 nearest neighbors. When a node is pulled or pushed, neighboring links absorb the movement by moving slightly. If a link between two nodes is stretched or compressed to its limit, displacements are transferred to neighboring links. Within this framework, stiff behavior can be modeled for large displacements whereas compliant motion will be observed for small displacements. The "ChainMail" model is well-suited for real-time deformation and cutting operations, However, it is not clear how realistic the deformation is nor how feasible the real-time rendering of the tissue model is.

3.3 Finite Element Models

Finite element models are the surface and volume representation most widely used in engineering. Finite element methods describe a shape as a set of basic elements (triangles, quadrilaterals, tetrahedra,...) where shape functions with limited support are defined [Zie77]. This leads to continuous representations with varying levels of continuity. A finite element model is fully defined by the choice of its elements, its shape function and its global parameterization between parameter space Ω and \mathbb{R}^3 ($\Omega \subset \mathbb{R}^2$ for surfaces and $\Omega \subset \mathbb{R}^3$ for volumes). For surfaces that are neither topologically planar or cylindrical, the parameterization can be problematic.

Finite elements with C^0 continuity, where the shape node consists of a vertex position are similar to finite difference methods. Similarly, Bézier splines, B-splines or Hermite splines can be seen as finite elements with specific shape functions.

A finite element model is represented by the node vector \mathbf{X} . For static computation, the stress-strain relationships leads to $f(\mathbf{X}) = 0$, whereas for dynamic computation, the following Newtonian formulation is often used :

$$m\ddot{\mathbf{X}} + \gamma\dot{\mathbf{X}} + f(\mathbf{X}) = \mathbf{0} \quad (6)$$

Those equations can equivalently be derived by minimization of the bending energy through the principle of virtual work. The integration of this differential equation can be performed using an semi-implicit or explicit scheme. In general, implicit schemes are unconditionally stable whereas explicit schemes are conditionally stable. This implies that smaller time steps must be used with explicit schemes. However, explicit schemes are simpler to compute and may not require the matrix inversion.

Membrane and thin plate energies have been largely used in computer vision [TWK87], computer graphics [TPBF87] and for modeling elastic tubular surface tissue [Meg96]. These two belong to a family of regularizing energies : the controlled-continuity generalized spline kernels [Ter86]. Those quadratic energies have been used extensively because of their numerical properties (they lead to linear elastic forces). However they do not correspond to physical elastic energies. The membrane energy is a linearized version of the surface tension energy on soap films [Hil44]. The thin plate energy is a linearized version of the isotropic thin shell flexion energy [Ber96]. In particular, those energies are not invariant with respect to a change of parameterization. However, for small deformations, they can be considered as valid approximations. In [TF88a], Terzopoulos proposes a more general bending energy consisting of the sum of the square of the metric tensor and the curvature variation. He defines a hybrid formulation of deformation that includes a rigid component and a deformable component. With this new parameterization of deformation, invariant to rigid transformation, thin plate bending energy is used to model large deformations. In [CYTT92], a dynamic model similar to [TF88a] is proposed but the damping factor is replaced by the time derivative of the strain tensor.

The most advanced surface deformation model has been proposed in [TF88b]. In this paper, a Maxwell and Voigt viscoelastic model has been implemented using a semi-implicit scheme with finite differences on a regular grid. The addition of plastic units enabled non reversible behavior to be modeled. However, those results required substantial computational power for relatively small grids (around 30×30).

Linear elastic volumetric finite element models have been widely used to model the deformation of soft tissue. In such cases, the stress/strain relationship is represented by a linear equation : $\mathbf{F} = \mathbf{K}\mathbf{X}$. The rigidity matrix \mathbf{K} depends on the rest shape geometry, the Young modulus E and the Lamé parameter λ . In most cases, only C^0 elements are used, leading to simple shape functions.

Pieper [PRZ92] and Keeve [KE96] simulated fat tissue elasticity for plastic surgery with prismatic finite elements. Chen [CZ92] built a sophisticated muscle model based on biomechanical data where a linear elastic muscle is submitted to non-linear tendon forces. Despite the approximation on the muscle elastic behavior of the muscle, good correspondence with biomechanical has been observed. Gourret [GTT89] computes the skin deformation of human fingers during the grasping of a soft object. The simulation models the interaction between two deformable bodies.

More complex elastic behaviors have been proposed by Terzopoulos. In [TF88b], he defines the square norm of the metric tensor as the potential energy, which can be shown [BN96] to be equivalent to a St. Venant Kirchoff deformation model. Bro-Nielsen [BN95] used this non-linear elastic model for simulating craniofacial surgery with a finite difference scheme defined on cubic lattices. In [Kai96], Kaiss *et al.* proposes a complete model of the eye/trepan contact for predicting eye deformations under surgery. The sclera and cornea are modeled as St. Venant-Kirchoff materials and transversely isotropic materials. The penetration of the trepan requires the re-meshing of the cornea represented by hexahedral elements with 27 nodes. The computation time was of the order of an hour on a powerful workstation.

Finite element models have been used widely to compute soft tissue deformations under mechanical constraints. In the past few years, real-time finite element models have been developed as an alternative to spring models. Sagar [SBMH94] developed a virtual environment for eye surgery simulation where the cornea deformation is modeled as a non-linear elastic material (Mooney-Rivlin material). The finite element solver computed the cornea deformation every second while the graphics module was able to provide a 10Hz refresh rate. In [CDA96], Cotin *et al.* describes a hepatic surgery simulator where the liver is represented as a linear elastic volumetric model with static constraints. By pre-computing the response of surface vertices to position constraints, the liver model can be deformed in real-time. Furthermore, the force-feedback computation and the liver deformation computation can be decoupled to achieve optimal haptic display (500Hz) and visual display (30Hz). Similarly, Bro-Nielsen [BNC96, BN96] decreased the computation time of a linear elastic model with a semi-implicit scheme, by condensing and explicitly inverting the reduced stiffness matrix in a preprocessing stage. Video frame-rates of 15-20 frames/second were obtained with this method.

By taking advantage of the linear nature of the static or dynamic equation, the methods of Cotin [CDA96, Cot97] and Bro-Nielsen [BNC96] decreases the computation time of finite element models by at least factor of 100. However, such optimizations are not compatible with the topological change entailed by suturing or cutting where the stiffness matrix must be updated.

In figure 8 we show an example of real-time hepatic simulator developed at INRIA [CDA96, Cot97]. The user manipulates a force-feedback simulation platform and feels the contact between the virtual instrument and a liver model.

As a conclusion, finite element models are well suited to compute accurate and complex deformation of soft tissue. However, it is extremely difficult to get real time performance on a moderately powerful workstation using finite element models. But for linear elastic models, only valid for small displacements, it is possible to achieve real-time deformations. Unlike spring models (see section 3.2), there is no restriction on the stiffness value of the model with respect to the time step Δt when using semi-implicit or static schemes.

The cutting or suturing operation requires the finite element model to be re-meshed. When using structured elements such as rectangular, prismatic or hexaedral elements, the cutting is often constrained to occur along a given direction [Kai96, SR95]. With unstructured elements

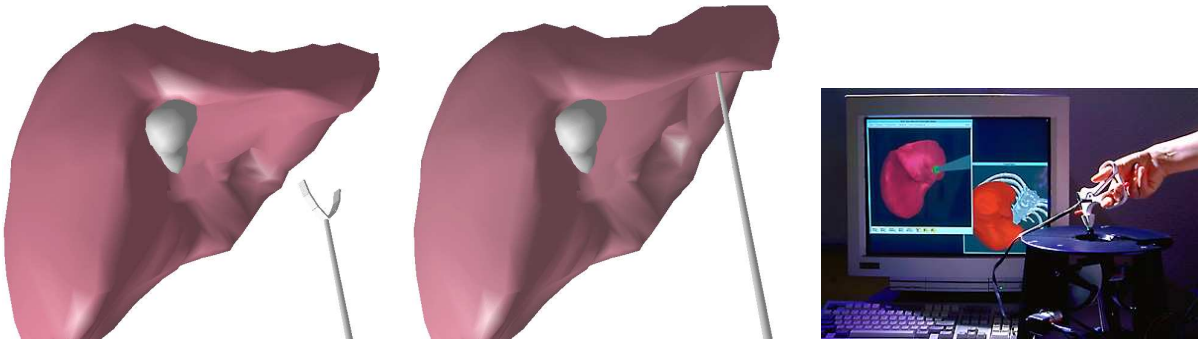


Figure 8: Hepatic surgery simulation platform at INRIA

such as triangular or tetrahedral elements, more general cut planes may be designed at the cost of greater complexity. Finally, visualization of finite elements is well suited for graphics hardware since it consists a rendering visible elements. After cutting or suturing a volumetric model, it is necessary to update the list of visible facets. Figure 9 shows the real-time cutting of a tetrahedral finite element mesh of a kidney[Cot97]. The kidney has a linear elastic behavior and an explicit computation scheme prevents the re-computation of the global stiffness matrix.

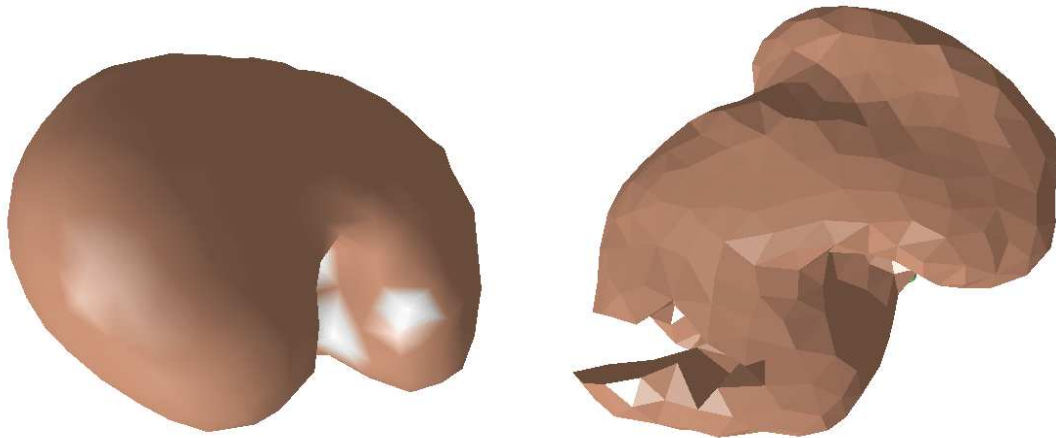


Figure 9: Cutting of a kidney finite element model.

3.4 Other Deformable Models

Other models of deformable bodies have been proposed in computer graphics. Even if such methods have seldom been applied to medical simulation, they are relevant to the development of real time simulation of soft tissue.

For instance, implicit surfaces defined by potential fields attached to skeletons is a representation well-suited for collision detection computation and the modeling of very soft objects. In [CGD97], Gascuel shows that material stiffness is related to the gradient of potential field and that such models can support cutting or suturing. Other global transformations such as

extended free-form deformations [Coq90] or modal analysis [SP91] are potentially of interest, for instance for modeling the effect on abdominal tissues of breathing.

4 Conclusions

We have isolated three main problems for achieving realistic soft tissue models :

- **Acquisition of Biomechanical Information.** A major impediment to building accurate soft tissue models is the lack of quantitative biomechanical information suitable for finite element computation. The required information not only refers to the inner mechanical property of a given soft tissue but also includes contact with the surrounding tissues. In terms of computation, the former corresponds to the constitutive law of motion linking the stress tensor with the strain tensor whereas the latter corresponds to the boundary conditions.

The acquisition of elastic or viscoelastic properties of a tissue is usually performed by rheological experiments. Existing rheometers require that experiments are performed *in vitro* on uniaxial samples. This raises two problems. First, the *in vitro* properties may vary substantially from the true *in vivo* properties, specifically with permeable tissues containing incompressible fluids (such as blood or CSF). Second, experiments with uniaxial samples are only valid if the tissue is homogeneous and isotropic. Finally, rheometers do not allow characterization of the force/deformation contact between neighboring tissues.

In the future, medical imaging could provide *in vivo* biomechanical tissue measurements. Widely-used imaging modalities such as CT-scanners or MRI, already provide approximate information about the density and the relative water content of tissue. Such information can then be used to infer approximately the biomechanical tissue properties. For instance, Koch *et al.* [KGCvB96] derives the stiffness values of spring models from the Hounsfield units of a CT image. Similar reasoning [HC92] were applied for the recovery of the Young's modulus of bones from CT scans. Brain MRI images could be used to approximate the stiffness of brain tissues since its compliance has been shown to be correlated with water content of *in vivo* brain tissue [Mas80]. More accurate experiments have been reported by Manduca *et al.* [MMRG96] using Magnetic Resonance Elastography (MRE). By propagating acoustic strain waves, MRE images provide an estimate of elastic stiffness for small displacements.

- **Efficient Computation.** Computation time is an important constraint for surgery planning or surgical procedure simulation. To achieve a given computation rate, it is necessary to make a compromise between the mesh resolution and the complexity of the biomechanical model. The exponential increase in computing and graphics hardware performance should lead naturally to denser soft tissue models. However, the addition of more sophisticated models of deformation and interaction requires even more computation power. It is therefore necessary to develop more efficient algorithms, specifically for the following three tasks :
 - **deformation of non linear viscoelastic tissue models,**
 - **collision detection between deformable bodies,**
 - **computation of contact forces between deformable bodies.**

It is likely that improved algorithms will stem from the both biomechanics and computer graphics communities.

- **Medical Validation.** Validation of soft tissue deformation is a crucial step in the development of soft tissue modeling in medical simulators. It requires the comparison of deformations between computerized models and *in vivo* tissues. The shape variation of tissues can be measured through tridimensional imagery such as CT-scanners or MRI images. By combining image segmentation with non rigid registration, displacement fields of tissues can be recovered and then compared with predicted displacements of soft tissue models. Physical markers or tagged MRI could help solving the matching problem. For a complete validation, the measurement of stress and applied forces on actual tissues should be performed and compared with predicted values.

	Deformation Accuracy	Computation Time	Cutting	Visualization
Finite Element Models	***	*	*	***
Spring Models	*	**	**	**
Implicit Models	*	**	***	*

Table 1: Comparison of soft tissue models (***) indicates a good adequacy of the model and * a poor adequacy).

The performance of current soft tissue models are summarized in table 1. Finite element models are mostly used in biomechanics because they aim at modeling accurate deformations. On the other hand, spring models have been developed in computer graphics for their simple and efficient implementation. In the past few years, there have been several attempts to improve the computational efficiency of finite elements [CDA96, BNC96] and to improve the realism of spring models [KE96]. It is likely in the future, that interaction between biomechanics and computer graphics will contribute to a major improvement in soft tissue modeling.

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References

- [Ber96] Michel Bernadou. *Finite element methods for thin shell problems*. Masson, 1996. ISBN 0-471-95647-3/2-225-84940-4.
- [BN95] M. Bro-Nielsen. Modelling elasticity in solids using active cubes - application to simulated operations. In *Computer Vision, Virtual Reality and Robotics in Medicine*, pages 535–541, Springer, April 1995.
- [BN96] M. Bro-Nielsen. *Medical Image Registration and Surgery Simulation*. Ph.D. dissertation, IMM Technical University of Denmark, Lingby, Denmark, March 1996. IMM-PHD-1996-25.
- [BNC96] M. Bro-Nielsen and S. Cotin. Real-time Volumetric Deformable Models for Surgery Simulation using Finite Elements and Condensation. In *Proceedings of Eurographics'96 - Computer Graphics Forum*, pages 57–66, 1996.
- [CDA96] S. Cotin, H. Delingette, and N. Ayache. Real Time Volumetric Deformable Models for Surgery Simulation. In K. Hohne and R. Kikinis, editors, *Visualization in Biomedical Computing*, pages 535–540, Springer, 1996.
- [CEO93] S. A. Cover, N. F. Ezquerro, and J. F. O'Brien. Interactively Deformable Models for Surgery Simulation. *IEEE Computer Graphics and Applications*, 68–75, 1993.
- [CGD97] M-P. Cani-Gascuel and M. Desbrun. Animation of deformable models using implicit surfaces. *IEEE Transactions on Visualization and Computer Graphics*, 3(1), March 1997.
- [CM96] K. Chinzei and K. Miller. Compression of swine brain tissue ; experiment in vitro. *Journal of Mechanical Engineering Laboratory*, 106–115, July 1996.
- [Coq90] Sabine Coquillart. Extended free-form deformation: A sculpturing tool for 3D geometric modeling. In Forest Baskett, editor, *Computer Graphics (SIGGRAPH '90 Proceedings)*, pages 187–196, August 1990.
- [Cot97] S. Cotin. *Modèles déformables temps-réel pour la simulation de chirurgie avec retour d'effort*. Ph.D. dissertation, Université de Nice Sophia-Antipolis, Nice, France, November 1997. in French.
- [CYTT92] Michel Carignan, Ying Yang, Nadia Magnenat Thalmann, and Daniel Thalmann. Dressing animated synthetic actors with complex deformable clothes. In Edwin E. Catmull, editor, *Computer Graphics (SIGGRAPH '92 Proceedings)*, pages 99–104, July 1992.
- [CZ92] David T. Chen and David Zeltzer. Pump it up: Computer animation of a biomechanically based model of muscle using the finite element method. In Edwin E. Catmull, editor, *Computer Graphics (SIGGRAPH '92 Proceedings)*, pages 89–98, July 1992.

- [DKMC95] D. DeCarlo, J. Kaye, D. Metaxas, J.R. Clarke, B. Webber, and N. Badler. *Interactive Technology and the New Paradigm for Healthcare*, chapter 14: Integrating Anatomy and Physiology for Behavior Modeling, pages 81–87. IOS Press and Ohmsha, 1995.
- [DKT95] O. Deussen, L. Kobbelt, and P. Tucke. Using simulated annealing to obtain a good approximations of deformable bodies. In *Proc. Eurographics Workshop on Animation and Simulation*, 1995.
- [DL95] S. Delp and J. Loan. A graphics-based software system to develop and analyse models of musculoskeletal structures. *Journal of Comput. Biol. Med.*, 25(1):21–34, 1995.
- [DSCP94] Hervé Delingette, Gérard Subsol, Stéphane Cotin, and Jérôme Pignon. A Craniofacial Surgery Simulation Testbed. In *Visualization in Biomedical Computing (VBC'94)*, octobre 1994.
- [Fun93] Y. C. Fung. *Biomechanics - Mechanical Properties of Living Tissues*. Springer-Verlag, second edition, 1993.
- [GSMF97] S. Gibson, J. Samosky, A. Mor, C. Fyock, E. Grimson, T. Kanade, R. Kikinis, H. Lauer, and N. McKenzie. Simulating arthroscopic knee surgery using volumetric object representations, real-time volume rendering and haptic feedback . In J. Troccaz, E. Grimson, and R. Mosges, editors, *Proceedings of the First Joint Conference CVRMed-MRCAS'97*, pages 369–378, March 1997.
- [GTT89] Jean-Paul Gourret, Nadia Magnenat Thalmann, and Daniel Thalmann. Simulation of object and human skin deformations in a grasping task. In Jeffrey Lane, editor, *Computer Graphics (SIGGRAPH '89 Proceedings)*, pages 21–30, July 1989.
- [HC92] R. Hodgkinson and J.D. Currey. Young modulus, density and material properties in cancellous bone over a large density range. *Journal of Materials Science: Materials in Medicine*, 3:377–381, 1992.
- [Hil44] D. Hilbert. *Geometry and the Imagination*. Dover, 1944.
- [HS88] P.J. Hunter and B.H. Smail. The analysis of cardiac function : A continuum approach. *Journal of Prog. Biophysic. molec. Biology*, 52:101–164, 1988.
- [JL95] David Crochemore Jean Louchet, Xavier Provot. Evolutionary identification of cloth animation model. In *Workshop on Computer Animation and Simulation (Eurographics'95)*, pages 44–54, 1995.
- [Kai96] Kaiss, M. and Le Tallec, P. La Modélisation numérique du contact œil-trépan. *Revue Européenne des Eléments Finis*, 5(3):375–408, February 1996.
- [KE96] Girod B. Keeve E., Girod S. Craniofacial surgery simulation. In *Proceedings of the 4th International Conference on Visualization in Biomedical Computing VBC'96*, pages 541–546, Hamburg, Germany, September 1996.

- [KGCvB96] R. M. Koch, M. H. Gross, F. R. Carls, D. F. von Büren, G. Fankhauser, and Y. Parish. Simulating facial surgery using finite element methods. In Holly Rushmeier, editor, *SIGGRAPH 96 Conference Proceedings*, pages 421–428, ACM SIGGRAPH, Addison Wesley, August 1996.
- [KGEb75] R. Kenedi, T. Gibson, J. Evans, and J. Barbenel. Tissue mechanics. *Physics in Medicine and Biology*, 20(5):163–169, 1975.
- [KKKN96] Ch. Kuhn, U. Kühnapfel, H.-G. Krumm, and B. Neisius. A 'virtual reality' based training system for minimally invasive surgery. In *Proc. Computer Assisted Radiology (CAR '96)*, pages 764–769, Paris, June 1996.
- [Lar86] W. Larrabee. A finite element model of skin deformation: Biomechanics of skin and tissue : a review. *Laryngoscope*, 96:399–405, 1986.
- [LJFC91] A. Luciani, S. Jimenez, J.L. Florens, C. Cadoz, and O. Raoult. Computational physics: a modeler simulator for animated physical objects. In *Eurographics Workshop on Animation and Simulation*, pages 425–437, Vienna, 1991.
- [Mar96] B. Marcus. Hands on : Haptic feedback in surgical simulation. In *Proc. of Medicine Meets Virtual Reality IV (MMVR IV)*, pages 134–139, San Diego, CA, January 1996.
- [Mas80] A. Masauzawa. *Brain and Nerve*, chapter Compliance of the Brain, pages 47–56. Volume 32, No to Skinkei, 1980.
- [MC97] P. Meseure and C. Chaillou. Deformable Body Simulation with Adaptive Sub-division and Cuttings. In *Proceedings of the WSCG'97*, pages 361–370, February 1997.
- [Meg96] D. Meglan. Making surgical simulation real. In *Computer Graphics*, pages 37–39, Addison Wesley, November 1996.
- [MMRG96] A. Manduca, R. Muthupillai, P. Rossman, J. Greenleaf, and L. Ehman. Visualization of tissue elasticity by magnetic resonance elastography. In *Proc of Visualization in Biomedical Imaging (VBC'96)*, pages 63–68, Hamburg, Germany, 1996.
- [MRFV96] William Mark, Scott Randolph, Mark Finch, James Van Verth, and Russell M. Taylor II. Adding force feedback to graphics systems: Issues and solutions. In Holly Rushmeier, editor, *SIGGRAPH 96 Conference Proceedings*, pages 447–452, ACM SIGGRAPH, Addison Wesley, August 1996.
- [Pro95] Xavier Provot. Deformation constraints in a mass-spring model to describe rigid cloth behavior. In Wayne A. Davis and Przemyslaw Prusinkiewicz, editors, *Graphics Interface '95*, pages 147–154, Canadian Information Processing Society, Canadian Human-Computer Communications Society, May 1995. ISBN 0-9695338-4-5.
- [PRZ92] Steven Pieper, Joseph Rosen, and David Zeltzer. Interactive graphics for plastic surgery: A task-level analysis and implementation. *Computer Graphics (1992 Symposium on Interactive 3D Graphics)*, 25(2):127–134, March 1992.

- [RRF96] R.J.Stone and McCloy R. F. Virtual environment training systems for laparoscopic surgery; activities at the uk's wolfson centre for minimally invasive therapy. *The journal of Medicine and Virtual Reality*, 1(2):42–51, 1996.
- [Sat96] R. Satava. Medical virtual reality : The current status of the future. In *Proc. of 4th conf.Medecine Meets Virtual Reality (MMVR IV)*, pages 100–106, 1996.
- [SBMH94] M. A. Sagar, D. Bullivant, G. Mallinson, P. Hunter, and I. Hunter. A Virtual Environment and Model of the Eye for Surgical Simulation. In *Computer Graphics (SIGGRAPH'94)*, pages 205–212, 1994.
- [Sec96] *Second International Workshop on Robotics and Computer Assisted Medical Interventions (RCAMI'96)*, Bristol, UK, June 1996.
- [SP91] Stan Sclaroff and Alex Pentland. Generalized implicit functions for computer graphics. In Thomas W. Sederberg, editor, *Computer Graphics (SIGGRAPH '91 Proceedings)*, pages 247–250, July 1991.
- [SR95] G. J. Song and N. P. Reddy. Tissue Cutting In Virtual Environment. In *Medecine Meets Virtual Reality IV*, pages 359–364, IOS Press, 1995.
- [Ter86] D. Terzopoulos. Regularization of inverse visual problems involving discontinuities. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 413–424, 1986.
- [TF88a] D. Terzopoulos and K. Fleische. Deformable models. *The Visual Computer*, 4:306–311, 1988.
- [TF88b] D. Terzopoulos and K. Fleischer. Modeling inelastic deformation: Viscoelasticity, plasticity, fracture. *Computer Graphics (SIGGRAPH'88)*, 22(4):269–278, 1988.
- [TPBF87] Demetri Terzopoulos, John Platt, Alan Barr, and Kurt Fleischer. Elastically deformable models. In Maureen C. Stone, editor, *Computer Graphics (SIGGRAPH '87 Proceedings)*, pages 205–214, July 1987.
- [TWK87] D. Terzopoulos, A. Witkin, and M. Kass. Symmetry-seeking models for 3d object reconstruction. *International Journal of Computer Vision*, 1(3):211–221, 1987.
- [Wat92] Keith Waters. A physical model of facial tissue and muscle articulation derived from computer tomography data. In *Visualization in Biomedical Computing (VBC'92)*, Chappel Hill, NC, 1992.
- [Zie77] O.C. Zienkiewicz. *The finite element method*. McGraw-Hill, London, 3 edition, 1977.



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