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Abstract: We define an Aloha type access control mechanism for large mobile, multihop, wireless networks. This mechanism is based on a new representation of the interferences and hence of the collisions adapted to the context of large networks with random mobility. The mechanism is designed for the multihop context, where it is important to find a compromise between the spatial density of communications and the range of each transmission. More precisely, it is possible to optimize the product of the number of simultaneously successful transmissions per unit of space (spatial reuse) by the average range of each transmission. The optimization is obtained via an averaging over all Poisson configurations for the location of interfering mobiles. The main mathematical tools stem from stochastic geometry and are spatial versions of the so called additive and max shot noise processes. The resulting MAC protocol can be implemented in a decentralized way provided some local geographic informations are available to the mobiles. Its transport capacity is proportional to the square root of the density of mobiles; under certain mobility and stability conditions discussed in the paper, the delay for transporting information from one mobile to any another is proportional to the distance between them and to the square root of the density of mobiles.

Key-words: MAC protocols, multiple access, IEEE 802.11, Hiperlan, CSMA, interference, collision, signal to noise ratio, Shannon's capacity, transport capacity, wireless communication, ad hoc network, multihop routing, progress, stochastic geometry, Poisson shot noise, Poisson max shot noise.

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Un protocole MAC de type Aloha à réutilisation spatiale, pour les réseaux de mobiles sans fil et avec relais

Résumé : Nous définissons un mécanisme de contrôle d'accès pour les grands réseaux de mobiles utilisant des communications sans fil et avec plusieurs relais. Ce mécanisme est fondé sur une nouvelle représentation des interférences (et donc des collisions) adaptée au contexte de grands réseaux avec mobilité aléatoire. Il est conçu pour le contexte du relais des communications par l'ensemble des mobiles, où il est important de trouver un compromis entre la densité spatiale des communications et la portée des transmissions. Plus précisément, nous montrons comment optimiser le produit du nombre moyen des transmissions qui réussissent simultanément dans une unité d'espace (nombre lié à la notion de réutilisation spatiale) et de la portée moyenne de chaque transmission. L'optimisation qui est proposée est fondée sur des moyennes réalisées sur toutes les configurations poissonniennes des mobiles. Les principaux outils mathématiques sont ceux de la géométrie aléatoire, et tout particulièrement des versions spatiales des processus poissonniens additif et maximal. Le protocole MAC associé peut être implémenté de manière décentralisée si les mobiles disposent d'informations géographiques locales. Sa capacité de transport est proportionnelle à la racine carrée de la densité des mobiles. Sous certaines conditions de mobilité et de stabilité décrites dans l'article, le délai moyen de transport d'un paquet d'un mobile vers un autre est proportionnel à la distance qui les sépare et à la racine carrée de la densité des mobiles.

Mots-clés : Protocoles MAC, accès multiple, IEEE 802.11, Hiperlan, CSMA, interférence, collision, rapport signal sur bruit, capacité de Shannon, capacité de transport, communication sans fil, réseau ad hoc, routage à plusieurs sauts, progrès, géométrie stochastique, processus poissonnien additif et maximal.

1 Introduction

This paper concentrates on the medium access control (MAC) of wireless networks with several mobile emitters and receivers sharing a common Hertzian medium, like in e.g. certain classes of mobile ad hoc networks or sensor networks. One of the main difficulties for tuning MAC within this context stems from the mobility and the resulting unpredictability of the geometrical properties of the emission patterns. Mobility may in particular lead to random spatial clustering rendering some sets of simultaneous transmissions impossible due to high interferences.

Within this context, the MAC protocols aim at defining policies where mobiles access the shared medium in such a way that spatial or temporal clustering does not happen or only rarely happens. This is done by some exclusion mechanism that prevents mobiles that are close to some emitting mobile (and also receiving mobile in the case of IEEE 802.11 with the RTS-CTS option) from emitting at the same time. In wired networks, the MAC algorithm is supposed to prevent simultaneous transmissions from happening, as much as possible, since such transmissions are bound to produce collisions. Another and contradictory requirement is that MAC protocols should nevertheless allow as many simultaneous and successful transmissions as possible in different parts of the network. This ability of mobile wireless networks is known as *spatial reuse*.

Aloha is a widely deployed and studied access protocol. The initial paper presenting Aloha has been published in 1970 [1] and Aloha is now used in most cellular networks to request access. Initial studies [2, 3] sought methods to stabilize the protocol. Keeping the “random access” spirit of the Aloha protocol, numerous works tried to present more efficient protocols, an excellent summary can be found in [4]. Two main ways have been investigated; the first one consists in improving the control of the channel by carrier sensing: that is the CSMA (Carrier Sense Multiple Access) technique. The second one consists in taking advantage of the history of the channel in order to adopt a better retransmission strategy than the blind Aloha re-emission strategy. The first paper studying Aloha in a multihop context is [5]. In this work Nelson and Kleinrock computed the probability of successful transmission in a random planar Aloha packet radio network with a simple model where interferences only propagate two hops away.

In 1988 Ghez, Verdu and Schwartz introduced a model for slotted Aloha with multipacket reception capability in a widely referenced paper [6]. To the best of the authors knowledge, this paper introduced the reference model for Aloha in a network with spatial reuse. Research on Aloha is still quite active. A new trend concerns the analysis of Aloha when nodes have special behaviors [7].

In multihop networks, hidden collisions are an important issue. After the initial work of Tobagi [8], numerous papers mostly in the 90s proposed dedicated protocols to cope with this problem [9, 10, 11, 12]. The general idea of these protocols is to implement a mechanism in the receiver to protect its reception. This can be done via a packet handshake prior to the actual transmission. This technique had been adopted by the IEEE 802.11 standard as an option called RTS/CTS (Request To Send / Clear To Send), which is the acronym of the two packets exchanged between the source and the destination before the actual transmission. The benefit of this method is questionable. First it implies a significant overhead due to radio switching and synchronization delays. In addition, a recent publication mentions congestion with this scheme [13].

The present article revisits the spatial reuse Aloha MAC mechanism in the context of multihop mobile wireless networks. Compared to [6], the main new contributions are

- an exact representation of the the signal to interference ratio for each transmission and hence of the collisions of the Aloha scheme, taking into account all interferers;
- various optimizations of the protocol:
  - SR-Aloha (for spatial reuse Aloha), which concerns the case where some predefined range of transmission is set;
  - MSR-Aloha (for multihop spatial reuse Aloha), which is meant for the multihop context and which aims at transmitting a packet "as far as possible" while taking into account the reception probability.

For this, we introduce an abstract geometric model allowing one to address the key concerns outlined above concerning MAC, namely spatial reuse and range of transmission can be simultaneously addressed as properties of simple random geometrical objects.

To simplify the considerations we have assumed a slotted Aloha model. The spatial exclusion mechanism alluded to above is enforced by a random access to the medium where each station tosses a coin with some bias p that will be referred to as the Medium Access Probability (MAP). Notice that here, power control follows a 0-1 law: a station either emits at maximal power, or does not emit at all. Note that this creates a random exclusion area around each mobile (emitter or receiver) with a mean radius proportional to $1/\sqrt{p}$.

All interference is taken into account in an exact way and the success of any transmission will be decided in function of the Signal to Interference Ratio (SIR) at the receiver, as would be the case under the classical Gaussian channel model of information theory.

In multihop networks, the network is in charge of transmitting information far away via several hops. A snapshot of the network at any given time consists of stations having to transmit informations in some direction far away and attempting to do so in a minimal number of hops. So each station attempts to transmit information via a jump of range R (as large as possible) in the desired direction.

For multihop networks, one may then pose the MAC optimization problem as follows: find MAP p such that at any given time, the product of the number of the simultaneously successful transmissions per unit of space by the average jump made by each transmission (a notion that we call *mean spatial density of progress* in the paper) is maximized. The resulting optimization leads to a new instance of spatial reuse Aloha protocol that will be referred to Multihop, Spatial Reuse (MSR) Aloha in what follows.

Our spatial density of progress is related to Gupta and Kumar's [14] *transport capacity*. It is shown in [14] that for a network with randomly distributed stations with spatial density λ, the amount of *bit-meters* successfully pumped in a given time interval by unit area of the network is of the order of $\sqrt{\lambda}$. Moreover, this law holds true even if stations are optimally placed, and transmission ranges as well as traffic routing are optimally organized, based on full information on the network both in space and time.

We show that MSR-Aloha gives a density of progress of the form $K(p)\sqrt{\lambda}$; we give a closed form for $K(p)$, allowing for an optimization with respect to p, which is the main result of the paper.

We also consider the time dynamic of the protocol under the following assumptions:

- each mobile initiates a stationary flow of packets of intensity τ to be transported to some random destinations;
- the origin-destination (o-d) pairs are isotropic and the distance between each o-d pair is a random variable with finite mean;
- each mobile moves according to way point model.

We present arguments showing that if τ is smaller than a threshold that is given in closed form, MSR-Aloha should be dynamically stable and the delay to transmit a packet between any o-d pair should be proportional to the distance between origin and destination.

The paper is organized as follows. Section 2 introduces the mathematical model. Section 3 focuses on SR-Aloha, namely on the optimization of the MAP p when each station expects to make a hop of length R, or on the best hop length R when p is fixed. As we shall see, this simple optimization fails to determining the optimal MAC setting. In § 3.3, we also compare SR-Aloha with the CSMA (Carrier Sense Multiple Access) technique which is the basis of the MAC protocol for the WLANs standards IEEE 802.11 [15] and Hiperlan type 1 [16]. The main result of the paper is introduced in Section 4 where the optimization of the MSR-Aloha is addressed. In Section 5 we discuss capacity and stability issues for the MSR-Aloha. Implementation issues are discussed in Section 6, with a particular emphasis on a decentralized implementation of the protocol.

2 A Stochastic Geometry Model for Spatial Reuse Aloha

We consider an infinite planar network. Let $\Phi = \{(X_i, (e_i, S_i, T_i))\}$ be a marked Poisson point process with intensity λ on the plane $\mathbb{R}^2$, where

- $\Phi = \{X_i\}$ denotes the locations of stations,
- $\{e_i\}_i$ the medium access indicator of station i; $e_i = 1$ for the stations which is allowed to emit and $e_i = 0$ means the station is (a potential) receiver. Here, the random variables e_i are independent, with $\mathbf{P}(e_i = 1) = p$.
- $\{S_i\}$ denote powers emitted by stations (stations for which $e_i = 1$); the random variables $\{S_i\}$ will here be assumed independent and exponentially distributed with mean $1/\mu$,
- $\{T_i\}$ are the SINR thresholds corresponding to some channel bit rates or bit error rates; here, we will take $T_i \equiv T$ are constant.

In addition to this marked point process, the model is based on a function $l(x, y)$ that gives the attenuation (path-loss) from y to x in $\mathbb{R}^2$. We will assume that the path-loss depends only on the distance; i.e, with a slight abuse of notation $l(x, y) = l(|x - y|)$; As an important special case of the *simplified* attenuation function we will take $l(u) = (Au)^{-\beta}$, for $A > 0$ and $\beta > 2$. Note that such $l(u)$ explodes at $u = 0$, and thus in particular *is not* correct for a small distance r and large intensities λ; cf Remark 3.4.

We also consider an independent external noise (i.e. independent of Φ, e.g. thermal) and denote it (at a given location) by W.

Note first that Φ can be represented as a pair of independent Poisson p.p. representing emitters $\Phi^1 = \{X_i : e_i = 1\}$, and receivers $\Phi^0 = \{X_i : e_i = 0\}$, with intensities, respectively, λp and $\lambda(1-p)$.

Suppose there is a station located at x that emits with power S and requires SINR T. Suppose there is a user located at $y \in \mathbb{R}^2$. The station can establish a channel to this user with a given bit-rate (which will be taken as the unit throughput in what follows) iff

$$\frac{Sl(|x-y|)}{W + I_{\Phi^1}(y)} \geq T\,, \tag{2.1}$$

where I_{Φ^1} is the shot-noise process of Φ^1:

$$I_{\Phi^1}(y) = \sum_{X_i \in \Phi^1} S_i l(|y - X_i|)\,. \tag{2.2}$$

Denote by $\delta(x, y, \Phi^1)$ the indicator that (2.1) holds. Note that by stationarity of Φ^1, the probability $\mathbf{E}[\delta(x, y, \Phi^1)]$ depends only on the distance $|x - y|$ and *not* on the specific locations of (x, y); so we can use the notation $p_{|x-y|}(\lambda p) = \mathbf{E}[\delta(x, y, \Phi^1)]$, where λp is the intensity of the emitters Φ^1.

Result 2.1 *For exponentially distributed S with mean $1/\mu$,*

$$p_R(\lambda) = \exp\left\{ -2\pi\lambda \int_0^\infty \frac{u}{1 + l(R)/(Tl(u))}\, du \right\} \psi_W(\mu T/l(R))\,, \tag{2.3}$$

where $\psi_W(\cdot)$ is the Laplace transform of W.

Proof: Note that by (2.1)

$$\begin{aligned} p_R(\lambda) &= \mathbf{P}(S \geq T(W + I_\Phi)/l(R)) \\ &= \int_0^\infty e^{-\mu s T/l(R)}\, \mathrm{d}\Pr(W + I_\Phi \leq s) \\ &= \psi_{I_\phi}(\mu T/l(R))\, \psi_W(\mu T/l(R))\,, \end{aligned}$$

where $\psi_{I_\Phi}(\cdot)$ is the Laplace transform of I_Φ. Note that ψ_W does not depend on λ, whereas it is known that for a general Poisson shot-noise

$$\psi_{I_\Phi}(\xi) = \exp\left\{ -\lambda \int_{\mathbb{R}^2} 1 - \mathbf{E}\left[e^{-\xi S l(|x|)}\right] \mathrm{d}x \right\}.$$

Since S is exponential with mean $1/\mu$

$$\begin{aligned}\psi_{I_\Phi}(\xi) &= \exp\Big\{-\lambda \int_{\mathbb{R}^2} 1 - \frac{\mu}{\mu + \xi l(|x|)} \, \mathrm{d}x\Big\} \\ &= \exp\Big\{ - 2\pi\lambda \int_0^\infty \frac{u}{1 + \mu/(\xi l(u))} \, \mathrm{d}u\Big\}\end{aligned}$$

that concludes the proof. ■

3 Spatial Density of Successful Transmissions and Spatial Reuse

In this section we suppose that each mobile $X_i \in \Phi$ attempts to transmit to one receiver Y_i located at a distance $R = |X_i - Y_i|$ to it via a channel based on the principle (2.1).

3.1 SR-Aloha: Best MAP Given Some Range

The first question that we investigate assumes that the range of all transmissions is given and looks for the value of MAP p that maximizes the mean number of emitters (and thus emitter-receiver pairs) that can successfully transmit, per unit area. The main result is that there exists and optimal MAP and thus a way to optimize Aloha once transmission range is fixed. The associated protocol will be referred to as SR-Aloha in what follows.

In fact, we don't ask here whether there is a receiver Y_i located at a distance R as we will do in the next section. This is why there is actually only one point process of intensity $\lambda^1 = \lambda p$ in the model of this section, and the optimization in p can actually be seen as that in λ^1. In order to simplify notation, we will drop the upper index 1 in this section and call here Φ the point process of emitters with intensity λ and look for the optimal λ.

We have the following simple formula for the *spatial density of successful transmissions* in the network.

Result 3.1 *The mean number of emitters per unit area that can successfully transmit at distance R is equal to $\lambda p_R(\lambda)$.*

Proof: For $B \subset \mathbb{R}^2$ of unit area

$$\begin{aligned}\mathbf{E}\Big[\#\{X_i \in B : \delta(X_i, Y_i, \Phi) = 1\}\Big] &= \mathbf{E}\Big[\sum_{X_i \in \Phi} \mathbb{1}(X_i \in B)\delta(X_i, Y_i, \Phi)\Big] \\ &= \lambda \int_{\mathbb{R}^2} \mathbb{1}(x \in B) p_R(\lambda) \, \mathrm{d}x \\ &= \lambda p_R(\lambda)\,.\end{aligned}$$

■

Now we look for $\lambda_{\text{max}} = \text{maxarg}_{\lambda \geq 0}\{\lambda p_R(\lambda)\}$ that maximizes the *spatial density of successful transmissions* in the network.

Result 3.2 *The maximal density of successful transmissions is attained for the following optimal spatial density of emitters:*

$$\lambda_{\text{max}} = \frac{1}{2\pi \int_0^\infty \frac{u}{1+l(R)/(Tl(u))}\, du} \tag{3.4}$$

and it is equal to

$$\lambda_{\text{max}} p_R(\lambda_{\text{max}}) = e^{-1} \lambda_{\text{max}}\, \psi_W(\mu T/l(R))\,, \tag{3.5}$$

where $\psi_W(\cdot)$ *is the Laplace transforms of* W.

Proof: By Result 2.1, and the differentiation of the function $\lambda p_R(\lambda)$ w.r.t. λ, it is easy to see that its unique maximum is attained at λ_{max} equal to (3.4) and that the maximal value is given by (3.5). ■

For the attenuation function $l(u) = (Au)^{-\beta}$, $\beta > 2$ we have

$$\begin{aligned} 2\pi \int_0^\infty \frac{u}{1+l(R)/(Tl(u))}\, \mathrm{d}u &= \frac{2\pi R^2 T^{2/\beta}}{\beta} \int_0^\infty u^{2/\beta - 1} \frac{1}{1+u}\, \mathrm{d}u \\ &= \frac{2\pi R^2 T^{2/\beta}}{\beta} \Gamma(2/\beta)\Gamma(1-2/\beta)\,, \end{aligned}$$

where $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\, \mathrm{d}t$ is the Gamma function. Thus we have the following result.

Result 3.3 *For the simplified attenuation function* $l(u) = (Au)^{-\beta}$

$$p_R(\lambda) = e^{-\lambda R^2 T^{2/\beta} C}\,, \tag{3.6}$$

where $C = C(\beta) = \Big(2\pi\Gamma(2/\beta)\Gamma(1-2/\beta)\Big)/\beta$. *Consequently,*

$$\lambda_{\text{max}} = \frac{1}{R^2 T^{2/\beta} C} \tag{3.7}$$

$$\lambda_{\text{max}} p_R(\lambda_{\text{max}}) = \frac{1}{R^2 T^{2/\beta} eC} e^{-\mu T W/l(R)}\,, \tag{3.8}$$

where in the second formula we assumed deterministic external noise W.

Note that under this optimal choice of λ, the mean distance progressed by transmissions per unit space is

$$R\lambda_{\text{max}} p_R(\lambda_{\text{max}}) = \frac{1}{RT^{2/\beta} eC} e^{-\mu T W/l(R)}\,, \tag{3.9}$$

which is maximal for $R = 0$. The apparent conclusion is here that smaller ranges are preferable. We will come back to this in the next section.

3.2 Spatial Reuse

We can also interpret (3.7) in terms of the so-called *spatial reuse factor* defined as the distance to the receiver R divided by the (mean) distance between adjacent emitters. For this last quantity, we take the mean distance between neighboring points in Poisson-Voronoi tessellation (more precisely, the mean edge length of the typical triangle in the Poisson-Delaunay triangulation), which is $32/(9\pi\sqrt{\lambda_{\max}})$. Thus we get

$$\text{Spatial reuse} = T^{-1/\beta}\frac{9\pi}{32\sqrt{C}}\,.$$

Analogous parameter for the network based on the perfect triangular mesh given in [17] is

$$T^{-1/\beta}\frac{\sqrt{3}}{2(6\zeta(\beta-1))^{1/\beta}}\,,$$

where $\zeta(s) = \sum_{n=1}^{\infty} 1/n^s$ is the Riemann zeta function. Figure 1 gives comparison of the values for $T = 10$dB and different β. Note, that analogous parameter in FDMA hexagonal networks, with super-hexagonal frequency reuse, would be from $1/6 = 0.133$ to $\sqrt{3}/12 = 0.144$ (depending on whether we take the receiver to be located in the middle of hexagonal cell edge or at its end.

Remark 3.4 *Note that in order to obtain explicit formulas (3.7)–(3.5) we assumed a* simplified attenuation *function $l(u) = (Au)^{-\beta}$ which explodes at $u = 0$, and thus in particular* is not *correct for a small distance r and large intensities λ. More accurate models of the short-range attenuation may by, e.g., $l(u) = (A\max(r_0, u))^{-\beta}$ or $l(u) = (1 + Au)^{-\beta}$ for some $A > 0$, $r_0 > 0$ and $\beta > 2$. Then the formulas (3.4)–(3.5) can be easily evaluated numerically. However, numerical experiments show, that for both forms of the more accurate attenuation function, with large A and small r_0 with respect to R (e.g. $A \geq 100r, r_0 \leq 0.01r$) and reasonable T (eg. $T > 1$), formulas (3.7)–(3.8) give relatively good approximations*

3.3 Tentative comparison of SR-Aloha and CSMA

The aim of this section is a tentative comparison between SR-Aloha and a generic CSMA protocol. We assume a random Poisson network, an attenuation function of the form $l(u) = (Au)^{-\beta}$, $\beta > 2$ and $W = 0$. We suppose that the radius of the carrier sense range R_{cs} is set at

$$R_{cs} = RT^{1/\beta}\frac{2(6\zeta(\beta-1))^{1/\beta}}{\sqrt{3}}\,,$$

where R denotes the targeted transmission range. According to [17] we are sure that with this value, there will be no collision for a receiver in a radius of range R if the transmitters in the network are on a triangular regular network with neighboring transmitters separated by R_{cs}. It is in a triangular regular network that the density of nodes being at least at R_{cs} away is maximum. It follows that in a random network, whatever the pattern of simultaneous emitters respecting the CSMA rule with R_{cs}, a transmission to a receiver within radius R will always be collision free.

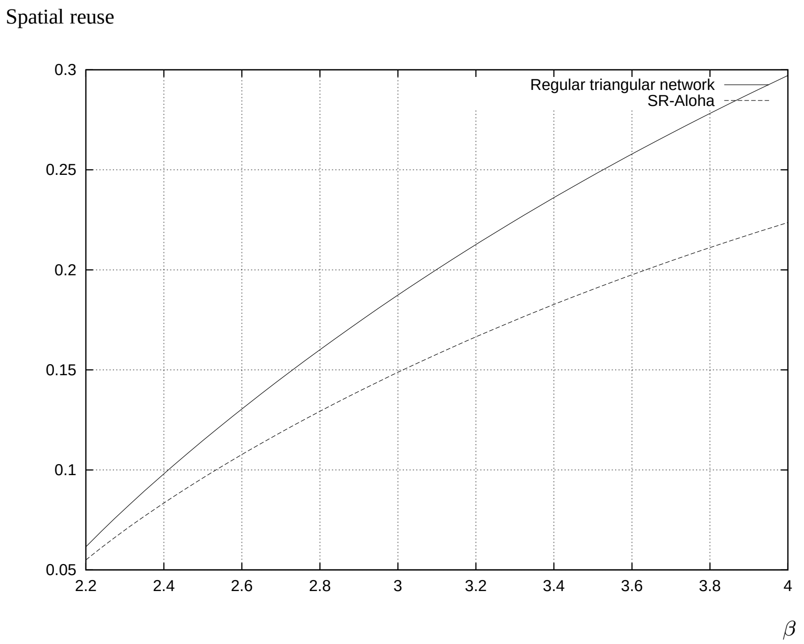

Figure 1: Comparison of the spatial reuse factor for Poisson (lower curve) and perfect triangular network (upper curve) for $T = 10$dB and different β. In hexagonal TDMA networks, with super-hexagonal frequency reuse, this parameter is between 0.133 and 0.144 regardless of β.

In order to compare SR-Aloha to CSMA protocol, we have to compute the intensity of an extracted point process satisfying the CSMA exclusion rule. Of course the intensity of this process will depend on the selection algorithm. An intuitive algorithm consists in picking nodes randomly and adding them to the CSMA transmission set if they are not in the carrier sense range of an already selected node. This algorithm is close to the effective behavior of a simple CSMA system. However this model does not seem to be easily tractable mathematically. Another selection algorithm is that based in the Matern hard-core process [18, 19]. This process is a thinning of the initial Poisson point process in which points are selected according to random marks. A point of the process is selected if its mark is larger than all marks in a radius of range R_{cs}. It is easy to check that the selected points follow the CSMA rule. The spatial intensity $\lambda_{R_{cs}}$ of the Matern hard-core process can be obtained in function of the spatial intensity of the initial Poisson point process by the formula:

$$\lambda_{R_{cs}} = \frac{1 - e^{-\pi\lambda R_{cs}^2}}{\pi R_{cs}^2};$$

see [19].

Simulations show that the intensity of this process is smaller than the intensity obtained through the random pick algorithm alluded to above, while giving results of the same order of magnitude. We can notice that the Matern hard-core process is a natural model for the access scheme of HiPERLAN type 1. The MAC of HiPERLAN type 1 actually uses an advanced version of CSMA. A signaling burst is sent before the packet; the (random) length of this elimination burst will be the mark which allows one to derive the Matern process.

Since we know R_{cs}, it is then easy to compute the transmission density for a CSMA scheme and to compare it with the spatial density of successful transmission of our SR-Aloha scheme given by Equation (3.8).

This comparison is given in Figure 2. Figure 2 (top) compares the spatial intensity of CSMA (selection of active nodes as in a Matern hard-core process) and the spatial density of successful transmission of SR-Aloha scheme in function of β, for $T = 10$dB. The curve on the top gives the spatial intensity of CSMA in a regular triangular network. On the bottom we have a zoom for β between 2 and 3. We see that, near 2, the optimized Aloha scheme actually outperforms the CSMA scheme.

Figure 2 shows that under these assumptions, the performance of SR-Aloha is very close to that of the CSMA scheme. This observation is consistent with [5] ,where a similar result reports that Aloha and CSMA have close performance. However the study in [5] uses a simplified transmission model (interference is only considered to propagate two hops away) and the carrier sense range and transmission range are supposed to be the same. In [20] a convenient tuning of the carrier sense range is shown to be important for the global performance of the network.

As a result of this tentative comparison we can conclude that SR-Aloha and a generic CSMA algorithm will have comparable result, a better framework and further studies will be necessary to precise this comparison.

3.4 Best Range Given Some MAP

Assuming some intensity λ of emitters given, we will use the following notation and definition:

$$r_{\max}(\lambda) = \text{maxarg}_{r\geq 0}\{rp_r(\lambda)\} \tag{3.10}$$

$$\rho(\lambda) = \max_{r\geq 0}\{rp_r(\lambda)\}. \tag{3.11}$$

We call $r_{\max}(\lambda)$ *the best range attempt* for λ and $\rho(\lambda)$ the *best mean range*. Note that by Result 2.1, $0 < r_{\max}, \rho < \infty$.

In the special case with the simplified attenuation function $l(r) = (Ar)^{-\beta}$ and when assuming that there is no external noise, using Result 2.1, we have

$$r_{\max}(\lambda) = \frac{1}{T^{1/\beta}\sqrt{2\lambda C}}, \tag{3.12}$$

$$\rho(\lambda) = \frac{1}{T^{1/\beta}\sqrt{2\lambda e C}}, \tag{3.13}$$

where $C = C(\beta) = \Big(2\pi\Gamma(2/\beta)\Gamma(1-2/\beta)\Big)/\beta$.

Here again, trying to maximize the cumulated mean range of all transmissions initiated per unit of space w.r.t. λ, namely trying to maximize $\lambda\rho(\lambda)$ in λ, leads to a degenerate answer since the maximum is for $\lambda = \infty$ which again gives $R = 0$.

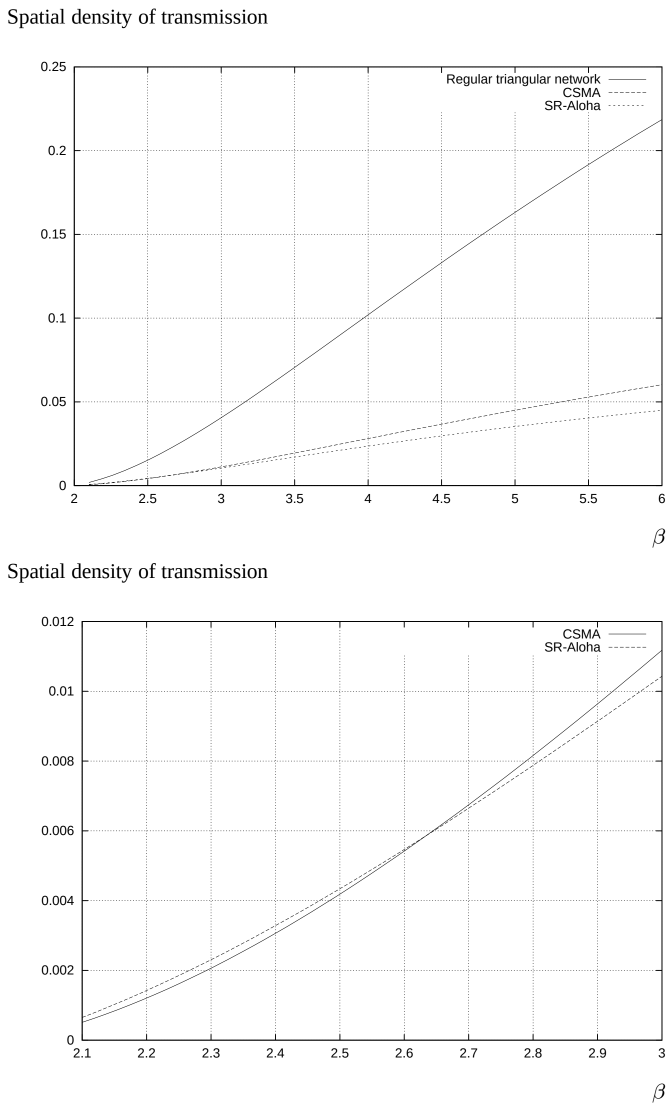

Figure 2: Top: spatial intensity of successful transmissions for CSMA (Matern selection model) and for SR-Aloha scheme in function of β, $T = 10$dB. The top curve gives the throughput of a regular triangular network. Bottom: Zoom of the comparison CSMA-SR-Aloha for β between 2 and 3.

4 Multihop Networks and Spatial Density of Progress

We now return to the general model of Section 2 with emitters Φ^1 and receivers Φ^0 and focus on the multihop context. This leads to a new optimal range problem this time with a non-degenerate solution.

4.1 Progress

Suppose an emitter X_0 located at the origin (so $X_0 = 0$) has to send information in some given direction (say along the x axis) to some destination located far from it (say at infinity). Since the destination is too far from the source to be able to receive the signal in one hop, the source tries to find a non-emitting station in Φ^0 such that the hop to this station maximizes the distance traversed towards the destination, among those which are able to receive the signal. This station will later forward the data to the destination or next intermediary station.

In this model, the "effective" distance traversed in one hop, which we will call the progress, is equal to

$$D = \max_{X_j \in \Phi^0} \left(\delta(0, X_j, \Phi^1) |X_j| \left(\cos(\arg(X_j))\right)^+ \right), \tag{4.14}$$

where $\arg(y)$ is the argument of the vector $y \in \mathbb{R}^2$ ($-\pi < \arg(y) \leq \pi$) and $\delta(x, y, \Phi^1)$ the indicator that (2.1) holds. We are interested in the expectation $d(\lambda, p) \equiv \mathbf{E}[D]$ that only depends on λ and

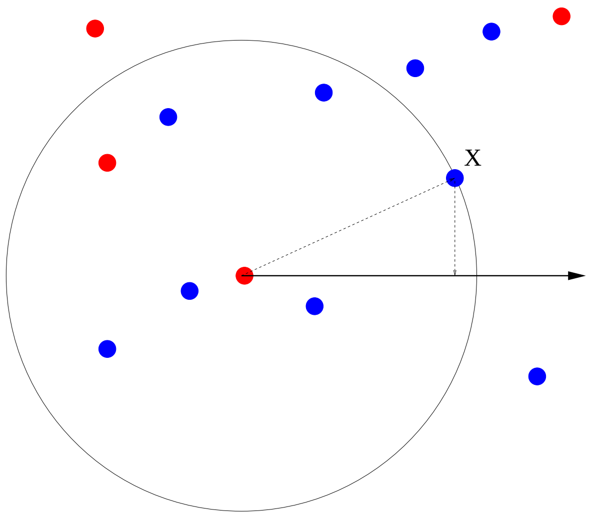

Figure 3: Progress: receivers in blue, emitters in red.

on the MAP p, once given the parameters concerning emission and reception, Note that similarly to Result 3.1, we have the following formula for the *(spatial) density of progress*:

Result 4.1 *The mean total distance traversed in one hop by all transmissions initialized in some unit area (density of progress) is equal to* $\lambda p d(\lambda, p)$.

4.2 MSR-Aloha and Optimal Progress

Note that for given λ, there is the following trade-off in p between the spatial density of communications and the range of each transmission. For a small p, there are few emitters only per unit area, but they can likely reach a very remote receiver because I_{Φ^1} is small. On the other hand, a large p means many emitters per unit area that create interference and thus prevent each other from reaching a remote receiver. Another feature associated with large p is the paucity of receivers, which makes the chances of a jump in the right direction smaller. In the sequel we try to quantify this tradeoff and find p that maximizes the density of progress. Since this optimization is adapted to the multihop context, the corresponding MAC protocol will be referred to as MSR-Aloha in what follows.

For for mathematical convenience and also for the reasons that will be discussed in Section 6 we will not study $d(\lambda, p)$ directly but rather a lower bound of this quantity which we now introduce. Let

$$\tilde{D} = \max_{X_j \in \Phi^0} \left(p_{|X_j|}(\lambda p)|X_j| \Big(\cos(\arg(X_j))\Big)^+ \right) \tag{4.15}$$

and let $\tilde{d}(\lambda, p) = \mathbf{E}[\tilde{D}]$.

Result 4.2 *For all* λ, p, $d(\lambda, p) \geq \tilde{d}(\lambda, p)$.

Proof: Let $\mathbf{E}^1, \mathbf{E}^0$ denote expectation w.r.t Φ^1 and Φ^0, respectively. Note that $\mathbf{E}[\tilde{D}] = \mathbf{E}^1\mathbf{E}^0[\tilde{D}]$ due to the independence between Φ^1 and Φ^0. The result now follows from Jensen's inequality, since the functional $\varphi(f) = \mathbf{E}^0[\max_{X_j \in \Phi^0}(f(X_j)|X_j|(\cos(\arg(X_j)))^+)]$ is convex on the space of real functions $f : \mathbb{R}^2 \to \mathbb{R}$. ∎

The aim of the remaining part of this section is to determine the value of MAP p that optimizes $\lambda p \tilde{d}(\lambda, p)$.

We will use the notation (cf §3.4) $r_{\max} = r_{\max}(\lambda p) = \mathrm{maxarg}_{r\geq 0}\{r p_r(\lambda p)\}$ and $\rho = \rho(\lambda p) = \max_{r\geq 0}\{r p_r(\lambda p)\}$.

For $z \in [0, 1]$, let

$$G(z) = \frac{2}{r_{\max}^2} \int_{\{r\geq 0: \rho z/(r p_r)<1\}} r \arccos\Big(\frac{\rho z}{r p_r}\Big)\, \mathrm{d}r\,. \tag{4.16}$$

In general this function depends on λp since $r_{\max} = r_{\max}(\lambda p)$, $\rho = \rho(\lambda p)$ and $p_r = p_r(\lambda p)$. We will see later on in this section that at least for the simplified attenuation $l(r) = (Ar)^{-\beta}$ and no external noise $W = 0$, the function G does not depend on any parameter of the model.

We now study the distribution function of $\tilde{D}$.

Result 4.3 *We have*

$$F_{\tilde{D}}(z) = \mathbf{P}(\tilde{D} \leq z) = e^{-\lambda(1-p)(r_{\max}(\lambda p))^2 G(z/\rho(\lambda p))}\,.$$

Proof: Note in (4.15) that $\tilde{D}$ has the form of the so-called *extremal shot-noise* $\max_{X_i \in \Phi^0} g(X_i)$ with the response function $g(x) = |x| p_{|x|}(\cos(\arg(x)))^+$. Its distribution function can be expressed by the Laplace transform of the (additive) shot noise

$$\mathbf{P}(\max_{X_i\in\Phi^0} g(X_i)\le z) = \mathbf{E}\Big[\exp\Big[\sum_{X_i\in\Phi^0} \ln(\mathbb{1}(g(X_i)\le z))\Big]\Big]$$

and thus, for Poisson p.p. Φ^0 with intensity $\lambda(1-p)$

$$\mathbf{P}(\tilde{D}\le z) = \exp\Big[-\lambda(1-p)\int_{\mathbb{R}^2} \mathbb{1}(g(x)>z)\,\mathrm{d}x\Big]\,.$$

Passing to polar coordinates in the integral $\int_{\mathbb{R}^2}\ldots\mathrm{d}x$, we get

$$\begin{aligned}\int_{\mathbb{R}^2}\mathbb{1}(g(x)>z)\,\mathrm{d}x &= 2\int_0^\infty\int_0^{\pi/2} r\mathbb{1}\Big(rp_r\cos(\theta)>z\Big)\,\mathrm{d}\theta\mathrm{d}r\\ &= 2\int_{\{r\ge 0: z/(rp_r)<1\}} r\arccos\Big(\frac{z}{rp_r}\Big)\,\mathrm{d}r\\ &= r_{\max}^2 G(z/\rho)\,,\end{aligned}$$

which completes the proof. ∎

Immediately from the Result 4.3 we have the following one.

Result 4.4 *The expectation of $\tilde{D}$ is equal to*

$$\tilde{d}(\lambda,p) = \mathbf{E}[\tilde{D}] = \rho(\lambda p)\int_0^1 1 - e^{-\lambda(1-p)(r_{\max}(\lambda p))^2 G(z)}\,dz\,.$$

We continue studying the special case of the model with the simplified attenuation function $l(r) = (Ar)^{-\beta}$. Moreover, we assume that there is no external noise $W = 0$.

Result 4.5 *For the model with the simplified attenuation function and $W = 0$ we have the following explicit formulas*

$$r_{\max}(\lambda p) = \frac{1}{T^{1/\beta}\sqrt{2\lambda pC}} \tag{4.17}$$

$$\rho(\lambda p) = \frac{1}{T^{1/\beta}\sqrt{2\lambda peC}} \tag{4.18}$$

and

$$\tilde{d}(\lambda,p) = \frac{1}{T^{1/\beta}\sqrt{2\lambda peC}}H(p)\,, \tag{4.19}$$

$$\lambda p\tilde{d}(\lambda,p) = \frac{\sqrt{\lambda p}}{T^{1/\beta}\sqrt{2eC}}H(p)\,, \tag{4.20}$$

where

$$H(p) = \int_0^1 1 - \exp\Big[\Big(1 - \frac{1}{p}\Big)\frac{G(z)}{2T^{2/\beta}C}\Big]\,dz \qquad (4.21)$$

and

$$G(z) = 2\int_{\{t: e^t/\sqrt{2et} \le 1/z\}} \arccos\Big(\frac{ze^t}{\sqrt{2et}}\Big)\,dt\,. \qquad (4.22)$$

Thus the maximal density of progress is attained for MAP p^* satisfying

$$\int_0^1 \Big(1 + \frac{G(z)}{p^* T^{2/\beta}C}\Big)\exp\Big[\Big(1 - \frac{1}{p^*}\Big)\frac{G(z)}{2T^{2/\beta}C}\Big]\,dz = 1\,. \qquad (4.23)$$

Note that (4.23) does not depend on λ.

The successful numerical calculation of $\tilde{d}$ and of the solution p of (4.23) maximizing the density of progress requires an efficient way of calculating the function G given by (4.22). Below we show some properties of G that involve the so called *Lambert W* functions LW^0 and LW^1. These functions can be seen as the inverses of the function te^t in the domains $(-1,\infty)$ and $(-\infty,-1)$ respectively; i.e., for $s \ge -1/e$, $\mathrm{LW}^0(s)$ is the unique solution of $\mathrm{LW}^0(s)e^{\mathrm{LW}^0(s)} = s$ satisfying $\mathrm{LW}^0(s) \ge -1$, whereas for $0 > s \ge -1/e$, $\mathrm{LW}^1(s)$ is the unique solution of $\mathrm{LW}^1(s)e^{\mathrm{LW}^1(s)} = s$ satisfying $\mathrm{LW}^1(s) \le -1$. Let

$$L^0(s) = -\frac{1}{2}\mathrm{LW}^0(-s^{-2}e^{-1})$$

and

$$L^1(s) = -\frac{1}{2}\mathrm{LW}^1(-s^{-2}e^{-1})\,.$$

The following representation of G is equivalent to that in (4.22):

$$\begin{aligned} G(z) &= 2\int_{L^0(1/z)}^{L^1(1/z)} \arccos\Big(\frac{ze^t}{\sqrt{2et}}\Big)\,\mathrm{d}t\,, \\ &= 2z\int_1^{1/z} \frac{1}{\sqrt{1-z^2s^2}}\Big(L^1(s) - L^0(s)\Big)\,\mathrm{d}s\,, \\ &= 2\int_{\arcsin(z)}^{\pi/2} \Big(L^1\big(\frac{\sin s}{z}\big) - L^0\big(\frac{\sin s}{z}\big)\Big)\,\mathrm{d}s\,. \end{aligned}$$

Moreover, the following function

$$G_\sim(z) = \pi(1-z) - 2\ln(z)\arccos(z)$$

approximates G very well on the whole interval $0 < z < 1$; see Figure 4 (top). Figure 4 (bottom) shows the density of progress calculated by means of $G_\sim$ for $\beta = 3$, $\lambda = 1$ and three values of the SIR threshold $T = \{10, 13, 15\}$dB. On the plot, we can identify the fraction p which maximizes the density of progress for a given T in the case of *unlimited* reception area.

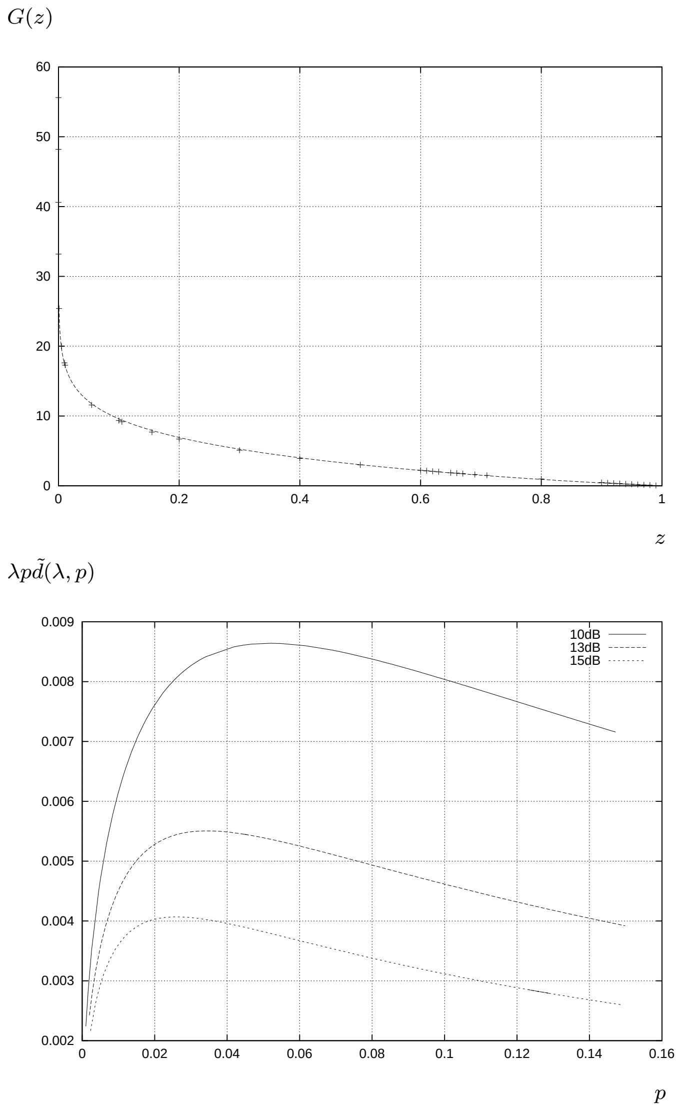

Figure 4: Top: Plot of the function $G_\sim$ with points representing values of G. The total relative error $\frac{1}{36}\sqrt{\sum \frac{(G(z)-G_\sim(z))^2}{G^2(z)}}$, where the summation is taken over 36 points marked on the plot, is less then 1.27%. Bottom: Density of progress for the model with simplified attenuation function with $\beta = 3$, $\lambda = 1$ and $W = 0$, with $T = \{10, 13, 15\}$dB (curves from top to bottom). The optimal values $(\text{maxarg}, \max)$ are respectively $\{(0.052, 0.0086), (0.034, 0.0055), (0.026, 0.0040)\}$.

4.3 Optimal Progress for Restricted Reception

It would also be interesting to know whether the optimal density of progress can be approached in a model with a restricted domain of reception; by this we mean that we exclude in the definition of D and $\tilde{D}$ the receivers laying outside the disk with some given, fixed radius R. Note first that we have the following straightforward generalization of our previous results.

Result 4.6 *The Results 4.1, 4.2, 4.3 and 4.4 remain true if we take* $\max_{X_i\in\Phi^0,|X_j|\le R}(\dots)$ *the definitions (4.14) and (4.15). In this case the function* G *has to be modified by taking the integral in (4.16) over the region* $\{0 \le r \le R : \rho z/(rp_r) < 1\}$.

We now try to find a reception radius R such that for a given p the density of progress in the limited model is close enough to that of the unlimited model. It is convenient to relate the reception radius R with the intensity λ of emitters. As we will see later on, its even more convenient to take $R = Kr_{\text{max}}$ for some constant $K \ge 0$ (recall, that $r_{\text{max}} = r_{\text{max}}(\lambda p)$ is the distance at which the mean range $rp_r(\lambda p)$ is maximal). Denote by G_K the function defined by (4.16) with the integral taken over $\{0 \le r \le Kr_{\text{max}} : \rho z/(rp_r) < 1\}$.

We will continue with our special form of the simplified attenuation function $l(u) = (Au)^{-\beta}$, $\beta > 2$ and $W = 0$. In our example this function is equal to

$$G_K(z) \quad = \quad 2\int_{\{0\le t\le K^2/2:\, e^t/\sqrt{2et}\le 1/z\}} \arccos\Big(\frac{ze^t}{\sqrt{2et}}\Big)\,\mathrm{d}t$$

For $K \ge 1$, this integral is equal to

$$2\int_{L^0(1/z)}^{\min(L^1(1/z),K^2/2)} \arccos\Big(\frac{ze^t}{\sqrt{2et}}\Big)\,\mathrm{d}t, \tag{4.24}$$

whereas for $0 \le K \le 1$, it is equal to

$$2\int_{\min(L^0(1/z),K^2/2)}^{K^2/2} \arccos\Big(\frac{ze^t}{\sqrt{2et}}\Big)\,\mathrm{d}t, \tag{4.25}$$

Denote now by $\tilde{d}_K = \tilde{d}_K(\lambda, p)$ the mean progress (4.15) in the model with the reception area restricted to Kr_{max}. We can prove now the following *continuity result* for the restricted model.

Result 4.7 *For the simplified attenuation function and* $W = 0$ *and* $K \ge 1$

$$\tilde{d} - \tilde{d}_K \le \rho z_K\,, \tag{4.26}$$

where $z_K = Ke^{1/2-K^2/2}$.

Proof: From (4.24), for $K \ge 1$, $G(z) = G_K(z)$ for z such that $L^1(1/z) \le K^2/2$ that is for $z \ge z_K$.Thus, from the Results 4.4 and 4.6

$$\tilde{d} - \tilde{d}_K \quad \le \quad \rho\int_0^1 e^{-\lambda(1-p)(r_{\text{max}}(\lambda p))^2 G_K(z)} - e^{-\lambda(1-p)(r_{\text{max}}(\lambda p))^2 G(z)}\,\mathrm{d}z \le \rho z_K\,,$$

where the last inequality follows from $G_K(z) = G(z)$ for $z \geq z_K$. ∎

In order to guarantee that the relative difference between unrestricted and restricted model is less than a given margin ϵ:

$$\frac{\tilde{d} - \tilde{d}_K}{\tilde{d}} \leq \epsilon,$$

it suffices to find the (minimal) K such that $Ke^{1/2-K^2/2} \leq \epsilon\tilde{d}/\rho$. Take for example, $p = 0.035$, which for $T = 13$dB gives the mean progress in the unbounded model (about) $\tilde{d} = 0.0055/0.035 = 0.157$ (cf. Figure 4), the best mean range is attained for the range attempt $r_{\text{max}} = 0.506$ and is equal to $\rho = 0.307$. In order to have a relative difference $\epsilon = 0.01$ we find the minimal $K \geq 1$ such that $Ke^{1/2-K^2/2} \leq 0.01 \cdot 0.157/0.307 = 0.00513$, which is $K = 3.768$. This means that in the model with the reception radius $R = Kr_{\text{max}} = 3.768 \cdot 0.506 = 1.905$ the mean progress (and the density of progress) is at most 1% less then the optimal one, obtained in the unrestricted model.

Remark 4.8 *Note that in this section we studied the optimization of the function*

$$\lambda p\mathbf{E}[\max_{X_j \in \Phi^0} (p_{|X_j|}(\lambda p) g(X_j))]\,,$$

w.r.t. the MAP p, where $g(x) = |x| p_{|x|}(\cos(\arg(x)))^+$. The reasons for optimizing this rather than

$$\lambda p\mathbf{E}[\max_{X_j \in \Phi^0} \delta(X_j, 0, \Phi^1) g(X_j))]$$

will be discussed in Section 6. The optimization of the last functional and of functionals of the form

$$\lambda p\mathbf{E}[\max_{X_j \in \Phi^0} (f(|X|_j|) p_{|X_j|}(\lambda p) g(X_j))]\,,$$

where the function $f(r)$ takes into account other aspects of the transmission at distance r than the probability of success also seems feasible and of potential interest.

5 Capacity and Stability

5.1 Transport Capacity

The spatial density of progress introduced above is related to Gupta and Kumar's [14] notion of *transport capacity*. In [14], it is shown how to construct a spatial and temporal scheme for scheduling transmissions in a bounded region such that the number of bit meters pumped by the network every second is of the order of $O\left(\sqrt{\lambda}\right)$ when the intensity $\lambda \to \infty$.

Our MSR-Aloha protocol also pumps a certain number of bit meters every second. If the bit rate corresponding to the threshold T is b, then the density of progress is $\lambda p d(\lambda, p)$ and MSR-Aloha pumps $C_t = b\lambda p d(\lambda, p)$ bit meters per second and per unit area. From the Result 4.2 and formula (4.20), we can lower-bound this transport capacity per unit area by

$$C_t = b\left(\frac{\sqrt{p^*}}{T^{1/\beta}\sqrt{2eC}} H(p^*)\right)\sqrt{\lambda} = O\left(\sqrt{\lambda}\right), \tag{5.27}$$

where $H(p)$ is given by (4.21), letting p^* to be the solution of (4.23). So, we conclude that MSR-Aloha, achieves the optimal transport capacity of Gupta and Kumar. In Section 6, we will describe some conditions under which MSR-Aloha can be implemented in a purely decentralized way. Under these conditions MSR-Aloha can then be seen as a way of achieving optimal transport capacity in a decentralized way.

5.2 Stability of MSR Aloha

Up to now, we analyzed spatial properties of the MSR-Aloha mechanism. We cannot really address stability issues unless we define temporal evolution of the model. A detailed spatio-temporal analysis of the model is beyond the scope of this paper, but we gather a few comments on the matter in the remaining part of this section.

Suppose each mobile has the following transmission dynamics: it has a queue of packets to be transmitted at the bit rate specified by the SIR threshold T. This queue is fed by *packets* which are either *fresh packets* originating from this mobile or arriving from another mobile and to be *relayed*. Each mobile tries to transmit the packet head of the line according to the MSR-Aloha scheme, namely tries to transmit this packet with probability p and either succeeds or keeps this packet head of line in case of collision (to be identified with the instant progress $D = 0$).

Each packet transport consists in several transmission hops between a random source and a random destination. We assume that the set of such packet transports is homogeneous (for instance forming a random segment process with uniform orientation and mean length L). Then, assuming MSR-Aloha mechanism, the transport of each new packet requires an average of $L/d(\lambda, p)$ individual transmissions. Let τ denote the mean number of fresh packets initiated per time slot and per mobile. Thanks to the homogeneity assumptions, the average number of transmissions that are created by the network per slot and per unit of space is therefore $\lambda\tau L/d(\lambda, p)$.

We also know that when all stations have packets to transmit, the mean number of packets that are allowed to transmit per unit area and per slot is λp.

These two observations lead to the following conclusion: if the time intensity τ of fresh packets per station is larger than $pd(\lambda, p)/L$, then there is no way for the protocol to cope with the traffic load during periods where most stations have to transmit in some area. Thus the quantity

$$C_d = \frac{pd(\lambda, p)}{L} \tag{5.28}$$

is an upper bound on the mean number of fresh packets per station and per slot that MSR-Aloha can handle at a given MAP p.

The question whether any time intensity of communications smaller than C_d per mobile leads to a stable dynamics for a network controlled by MSR-Aloha is quite natural by analogy with what we know of Aloha or Ethernet.

We show below that under simple independence and non-degeneracy assumptions on mobility, this dynamic stability can be conjectured.

The slotted mobility model is as follows: mobiles are numbered in some way (e.g. using the distance to the origin at slot 0). Mobile i, which is located at X_i^n in slot n, has a random and independent motion m_i^n during this slot, so that its position at slot $n+1$ is $X_i^{n+1} = X_i^n + m_i^n$. If the $\{m_i^n\}$ sequence is made of independent and identically distributed (i.i.d.) random variables in n and i, then $\{X_i^n\}$ is a Poisson point process at all time n if it is at time 0. The law of m_i^n is assumed to be non-degenerate (i.e. the norm of m_i^n is assumed to be positive with a positive probability). This implies that the sequence of configurations seen by mobile i over time (by configuration seen by mobile i at time n, we understand the family of points $\{X_j^n - X_i^n\}_j$) is stationary *and* ergodic (see [21] for these definitions). The distribution of each such configuration is the Palm probability of a planar Poisson point process of intensity λ.

Given the ergodicity of the configurations seen by mobile i over time and the homogeneity assumptions, it makes sense to assume that the (time) point process of packets (fresh or to be relayed) arriving into the queue of station i is stationary and ergodic, with a time intensity τ' equal to $\tau' = \tau L/d(\lambda, p)$. The ergodicity assumption would not be justified in case of a 0 motion as mobile i might then be a bottleneck having to relay a larger number of packets or experiencing a larger collision rate due to its particular location in configuration 0. The worst case scenario for mobile i (or equivalently an upper bound to the content of its queue) is obtained when considering the case where all other queues are always full (which is the analogue of the situation where all stations are backlogged in standard Aloha). In contrast with what happens in standard Aloha, where the probability of success in an infinite population model is 0 when all stations are blocked, in our model, thanks to spatial reuse, the probability for mobile i to transmit is still positive, equal to p, even when all mobiles have infinite backlogs (note that we count all time slots when the node is authorized to transmit, including the collisions, namely the transmission at distance $D = 0$; this is consistent with taking $L/\mathbf{E}[D]$, where $D =$ "distance" $\times$ "indicator of the success" as the mean number of transmissions generated by each fresh packet). The sequence of successful transmission times that mobile i would experience if backloged is also a stationary and ergodic since it is based on the stationary and ergodic sequence of configurations seen by i over time and the i.i.d. sequences of transmission coin tosses in all mobiles. It makes sense to assume that the sequence of successful transmission of mobile i when backloged and the arrival process in the queue of station i, are *jointly* stationary and ergodic since they are both functionals of the same sequence of configurations seen by station i. Loynes' theorem [21] can then be invoked to show that under assumption $\tau' < p$, that is equivalent to $\tau < C_d$, mobile i (and hence any mobile) has a queue size that is upper bounded by a finite stationary and ergodic process, which is a satisfactory definition of dynamic stability.

Of course, the above argument does not extend to the case with no mobility at all, where one can fear a bad behavior of the plain MSR Aloha protocol in some parts of the plane due to long lasting bottleneck local situations. For this or for low mobility, an estimate of the local density and an adaption of the MAP to this estimate is required. Such local estimates are discussed in § 6 below.

Remark 5.1 *Connectivity in mobile ad hoc networks is most often addressed as a* static *percolation question. One typically considers a snapshot of such a network and one says that two nodes are connected if a successful transmission is possible between them within this snapshot. The simultaneous success of several transmissions in the considered snapshot is based on local values of SIR. One then defines connectivity either as the property that all nodes belong to the same connected component (e.g. [14]) or as the existence of an infinite connected component (e.g. [22]). Note that this snapshot* *connectivity condition is one of the requirements in Gupta and Kumar's transport capacity estimate [14].*

The setting of this section can actually be viewed as a dynamic *framework for addressing connectivity: within the framework described above, the network has to transport an infinite flow of fresh packets originating from all nodes, each with its own destination. The existence of a sequence of successful transmissions over time allowing the network to transport each fresh packet of this infinite flow from origin to destination in a finite number of slots is a quite natural definition of connectivity. Notice that this new definition does not require that any given snapshot of the network be connected in the static sense (and hence could possibly allow one to go beyond the limits derived in [14]).*

Let us now look at the average end to end delays. We concentrate on the case where $\tilde{d}$ is used and on the simplified attenuation model. When all queues are stable and reach a stationary regime, as alluded to above, then this new definition of connectivity is satisfied and in steady state, the mean delay for transporting a packet from origin to destination ought to be proportional to $L/\tilde{d}(\lambda, p^*)$. The multiplicative constant is the average steady state queueing delay through one relay. Each relay is a slotted queue with arrival rate $\tau L/\tilde{d}(\lambda, p^*)$ and service rate p^* per slot. Assume now that λ varies in a range where L is large compared to $1/\sqrt{\lambda}$, which is required for the multihop model of the last sections to make sense. Also assume that τ is chosen in such a way that the load factor $\tau L/(p^* \tilde{d}(\lambda, p^*))$ of each such queue is equal to some positive $\theta < 1$ when λ varies, which is required for dynamic stability. This last assumption can be rephrased by stating that we adapt τ to the density of nodes according to the formula $\tau = \theta p^* \tilde{d}(\lambda, p^*)/L = O\left(\frac{1}{\sqrt{\lambda}}\right)$. Then it makes sense to assume that the average stationary delay through one relay is approximately constant in λ. Since $\tilde{d}(\lambda, p^*) = \kappa/\sqrt{\lambda}$ for some κ (see 4.19), we conclude that under the assumptions made above, this average stationary origin to destination delay ought to be proportional to $L\sqrt{\lambda}$.

6 Implementation Issues

This section contains a list of questions that have to be addressed for the design of a complete MSR-Aloha MAC protocol based on the notion of progress. First MSR-Aloha being a random access MAC protocol, we have to cope with collisions. Of course MAC collisions can be handled above the MAC layer but it can be easily shown that this leads to inefficient communication systems. This is why a good implementation of MSR-Aloha should use MAC acknowledgments for point to point packets as it is done in MAC protocols used for WLANs standards [15, 16]. We have assumed that MSR-Aloha is slotted, the slot can be divided in two parts: a data part (the main part) used by the emitter to send the packet and an acknowledgment part used by the receiver to indicate that it has correctly received the packet. In case a packet is not acknowledged, MSR-Aloha will just have to send the packet again still using p as transmission probability.

It beyond the scope of this article to describe routing algorithms or to fully study how routing algorithms could work with MSR-Aloha. However since MSR-Aloha is optimized for multihop network, MSR-Aloha must be closely related to a routing protocol. A routing protocol is in charge of computing a route to any destination node in the network. Most existing routing protocols do not use the geographical locations of nodes to compute routes, but research has shown that geographical location information can improve routing performance in mobile multihop networks. Given the genuine optimization of MSR-Aloha, it is easily understandable that this protocol will easily work with geographical position information assisted routing protocols. We just want to give a few hints concerning the use of MSR-Aloha in such conditions. Although less demanding assumptions are possible, for the sake of simplicity it will be assumed in the following that each network node knows the locations of all network nodes including itself. We can imagine two possibilities: the next hop to the final destination is directly computed or the next hop is the result of a real transmission.

Direct computation of the next hop under the previous assumptions, the following information is know by the emitter: its location (say 0), the direction of the final destination and the locations X_i of the emitter's neighbors expressed in the referential centered in the emitter in 0 and such that the x axis points to the destination. It can hence evaluate the functions $p_{|X_i|}v(X_i)$ for all i, where $v(x) = |x|(\cos(\arg(x)))^+$ and determine which is the best neighbor to be the next hop towards the final destination. It can be noticed that this algorithm can also be implemented by the receivers. As a matter of fact the functions $p_{|X_i|}v(X_i)$ can also be (pre)computed by the receivers. The receiver who realizes the maximum of this function can elect oneself as the next hop to the final destination. In either cases (the emitter selects the next hop or the next hop elects oneself), an acknowledgment packet must be sent by the receiver to the emitter. Notice that

- such a direct computation of the next hop realizes the mean optimal progress (4.15);
- the function $r \mapsto p_r(\lambda p)$ must be known;
- the actual optimization requires not only the knowledge of the location of the nodes, but also their actual MAC states (either receiver or emitter), which is a non realistic assumption. Notice however that the lack of information on the MAC state of other stations may only be problematic when the station that is elected for relaying a packet happens to be an emitter in the considered slot. Given that p is rather small, this is a relatively rare event that should perhaps simply be interpreted as a collision.

Next hop selected in a real transmission this second mechanism can be, in principle, implemented by the following algorithm: The emitter starts transmitting in a first small part of the slot asking other neighbor stations to respond. In a second part of the slot, the nodes having received the initial part of the transmission respond, which makes it possible for the emitter to decide which of them is the optimal forwarder (remind that node locations are supposed to be known; if it were not the case, then this information could be included in the responses). Then the emitter selects the best receiver, for instance the one defined by (4.14). In the remaining part of the slot (the essential part of the time-slot), the emitter sends the packet to the chosen receiver.

An interesting property of this mechanism is that for any given MAP p, it realizes the mean progress $d(\lambda, p) \geq \tilde{d}(\lambda, p)$. Moreover the whole protocol does not require that $p_r(\lambda p)$ be known. And we have also shown that the throughput is strictly positive. This could be seen as a breakthrough since to the best knowledge of the authors, no other protocol has this property. However this mechanism hides a non realistic assumption; actually we cannot assume that recipient nodes could all answer to the emitter in a fixed given part of a slot. This paragraph has been presented more for the peculiar properties of the mechanism than for giving a really implementable algorithm.

We have just seen that the function $r \mapsto p_r(\lambda p)$ must be known to implement some versions of MSR-Aloha. We address this issue in the next paragraph.

Estimation of p_r: Over time, emitters keep a record of the stations that successfully receive its emitted signals and the distance at which they are located. In order to establish this record, the emitter would ideally have to learn about: (i) all stations that successfully received its signal, which is easy to do via record-acks that would also contain information allowing the emitter to determine the emitter-receiver distance (ii) all stations that did not receive its signal, that is the complement of the set of stations having sent record-acks.

The practicality of the last requirement is of course questionable if one would really require to determine the set of all stations having failed receiving the signal. However, under the above assumption that each station knows at any given time the location of its neighboring stations, some exchange of information between stations would easily allow every emitter to build a full knowledge of the set of all stations in a ball of radius R_0 centered on it. Hence, an emitter can in fact deduce a complete sample of all successes and failures within a ball of radius R_0 from the set of record-acks.

Then the function p_r can be estimated by the empirical mean

$$\frac{1}{N}\sum_{i=1}^{N} s_i(r)\,,$$

where $s_i(r)$ is the number of successful transitions on the distance r, provided the N is large enough and *provided this sequence of samples is ergodic*.

Note that in case no assumption can be made on the mobility of stations (and in particular in the case where none of them moves), then the samples gathered over time by a *single* station have no reason to be ergodic, namely nothing then guarantees that $\lim_{N\to\infty} 1/N\sum_{i=1}^{N} s_i(r) = p_r(\lambda p)$. Remind that the mean value $p_r(\lambda p)$ is a *spatial* mean and that although such a spatial mean can be retrieved by spatial averaging (thanks to the fact that the Poisson point process is spatially ergodic), it cannot be retrieved via local averaging. A local averaging might be biased (e.g. by the local environment that does not change over time in the case with no motion at all).

In case all stations move independently, then over time, any given station will be in a position to collect samples that are built upon a set of Poisson configurations large enough to guarantee ergodicity. This follows from the so-called displacement theorem which states that if one gives independent motions to each point of a Poisson point process, then the Poisson law is preserved at any instant of time.

7 Conclusion

We have introduced a spatial reuse Aloha multiple access protocol adapted to large random homogeneous mobile networks using multihop transport mechanisms. Thanks to a direct representation of the interference process and of the progress made by each transmission, we have shown how the transport capacity of the network could be maximized by selecting the probability of channel access appropriately. We have shown that the transport capacity of such a network is proportional to the square root of the density of mobiles; the mean transport delay should be proportional to distance and to the square root of the density of mobiles, provided certain mobility assumptions hold. The specification of a fully distributed implementation of this MAC mechanism is one of the first questions that should be addressed. Other possible applications of our analytic framework could also be considered like the optimization of mobile wireless systems using directional antennas

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